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Experimental Verification of Saturation Reducing, Zero Vibration Command Shapers

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Abstract— **Command shapers are used to create inputs that will move flexible systems without residual vibration. Originally developed for linear systems, command shapers use superposition of the inputs to cancel vibration and control the system response. Nonlinearities in the system degrade the performance of traditional command shapers. The presence of "hard" nonlinearities such as actuator saturation, distort the desired commands before they reach the plant. Command shapers that avoid saturating the actuators, thus allowing the desired command to reach the plant in the original form, have been developed for a mass under PD control. These shapers are easier to compute than previously shown results. Simulations and experiments verify the efficacy of the new shapers.**

I. Introduction

Developing controllers that move flexible systems quickly with little residual vibration work has been an active research area for many decades. However, design and implementation of these controllers is often difficult and time consuming. Intelligent design of the command generator, often an overlooked portion of the complete control system, can aid in controlling the effects of the system's flexibility [1-5]. A diagram of a typical control system is shown in Figure 1. The command generator is used to convert the desired motion to a reference command. It should be noted that sensor dynamics and random disturbances are absent from this control system model. These effects have been left off for the sake of simplicity, but should always be considered when designing a control system.

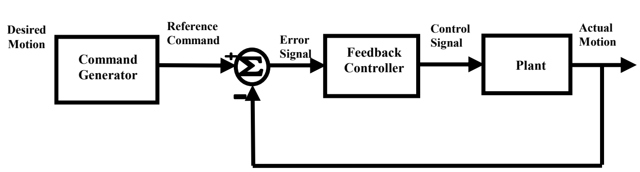

Fig. 1. Diagram of Typical Control Sytem.

To demonstrate the effect of the command generation on performance, consider a system that can be modeled as an undamped harmonic oscillator given by

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \tag{1}$$

The desired motion for this system is a change in position. The top graph of Figure 2 shows the system response to a step input. The system responds quickly, but a large amount of oscillation is induced. In an effort to eliminate the residual vibration, a smooth profile can be given to the system as shown by the middle graph of Figure 2. Using this command reduces the amplitude of the oscillation by a small amount, but it increases the rise time of the system. Finally, a staircase command can be given to the system. This command consists of two equal amplitude steps with the second step delayed by half the vibration period. The staircase command eliminates the residual vibration at the expense of a small increase in the rise time. Notice that the rise time of the system is delayed by half a period of the vibration.

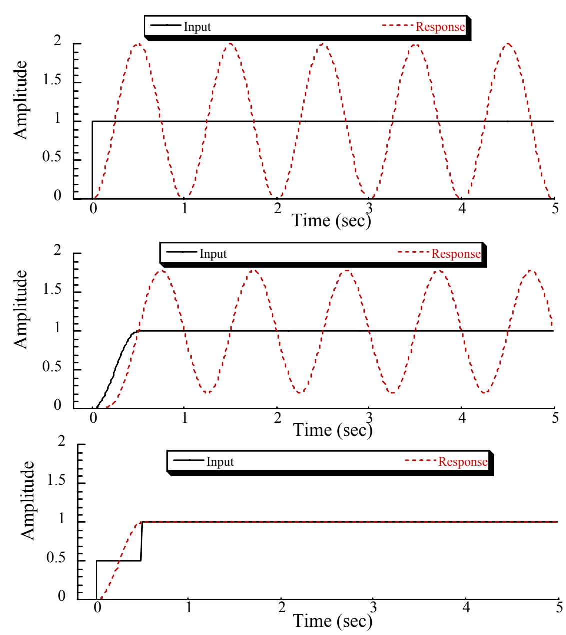

Fig. 2. Response of Undamped Harmonic Oscillator.

It is obvious from the example discussed above that the command given to the system can have a tremendous effect on the performance of the system. While we can design feedback controllers to accomplish a low-vibration move, its derivation and implementation may be mathematically complex and require the use of sensors. In addition the

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feedback controller is often used to adhere to the reference command, stabilize the system and reduce the effects of disturbances. Developing these vibration-free reference commands would eliminate one design criterion for the feedback controller, thus simplifying its design or choice of controller gains. The following section will outline how to create commands that will eliminate residual vibration and it will also detail the characteristics of these commands.

A. Input Shaping Review

Input shaping is a form of command generation that is designed to reduce *command-induced* vibration [3,4]. Input shaping can be implemented on any computer-controlled system with fairly well-known vibrational characteristics, such as number of modes of vibration, natural frequencies and damping ratio. Unlike traditional forms of command generation, it considers the system's natural tendency to vibrate when it develops the reference command for a system.

Input shaping is implemented by convolving a sequence of impulses, known as the input shaper, with a desired system command to produce a shaped input that is then used to drive the system. This process is demonstrated in Figure 3. The amplitudes and time locations of the impulses are determined by solving a set of constraint equations that attempt to control the dynamic response of the system. Examples of these constraint equations are limits on residual vibration, robustness to modeling errors and shaper gain.

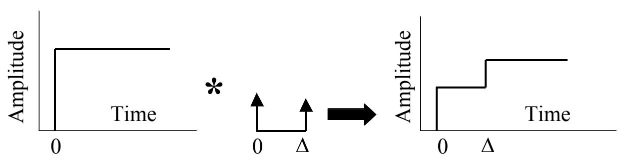

Fig. 3. Input Shaping Example.

The constraint on residual vibration amplitude can be expressed as the ratio of residual vibration amplitude with shaping to that without shaping. The percentage vibration can be determined by using the expression for residual vibration of a second-order harmonic oscillator of frequency ω and damping ratio ζ, which is given in [6]. The vibration from a series of impulses is divided by the vibration from a single impulse to get the percentage vibration:

$$V(\omega,\zeta) = e^{-\zeta\omega t_n} \sqrt{[C(\omega,\zeta)]^2 + [S(\omega,\zeta)]^2}\ , \tag{2}$$

where,

$$C(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i} \cos\left(\omega\sqrt{1-\zeta^2}\, t_i\right), \tag{3}$$

and

$$S(\omega,\zeta) = \sum_{i=1}^{n} A_i e^{\zeta\omega t_i} \sin\left(\omega\sqrt{1-\zeta^2}\, t_i\right). \tag{4}$$

If V(ω,ζ) is set equal to zero at the modeling parameters, (ω_m, ζ_m), then a shaper that satisfies (1) is called a Zero Vibration (ZV) shaper and is given by [3,4]:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \dfrac{1}{1+K} & \dfrac{K}{1+K} \\ 0 & \Delta T \end{bmatrix} \tag{5}$$

where

$$K = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right) \tag{6}$$

and

$$\Delta T = \frac{\pi}{\omega\sqrt{1-\zeta^2}} \tag{7}$$

A ZV shaper will not work well on many systems because it is sensitive to modeling errors [4]. Figure 4 shows the response of the ZV shaper when the actual frequency of the system is different for the modeling frequency. A 10% error in modeling frequency leads to a large amount of vibration (approximately 16% of the vibration that would result from a unit step input).

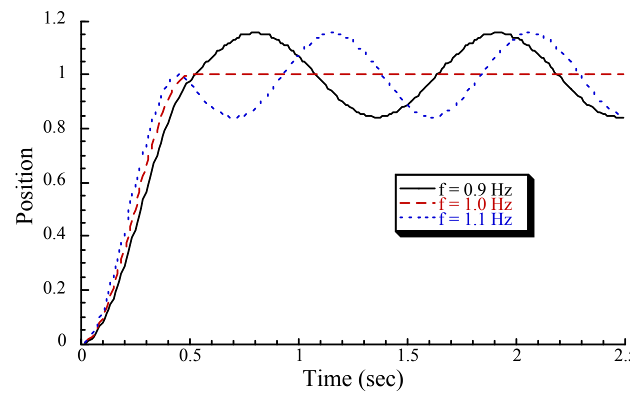

Fig 4. Sensitivity Plot.

For input shaping to work well on most real systems, the constraint equations must ensure robustness to modeling errors. Singer and Seering developed a form of robust input shaping by setting the derivative with respect to the frequency of the residual vibration given in (2) equal to zero [4]. The price paid for this increase in robustness is the increase in shaper duration (Δ in Figure 3).

There has been a significant amount of work focused on developing command shapers for systems with “soft” nonlinearities such as nonlinear dynamics or configuration-dependent frequencies [7-9]. However, systems with “hard” nonlinearities such as actuator such as deadzone or backlash have not received as much attention. One notable exception is the design of command shapers for systems with coulomb friction [10]. The remainder of this work will focus on systems with actuator saturation.

II. THE EFFECT OF ACTUATOR SATURATION ON COMMAND SHAPING PERFORMANCE

Command shapers are developed for simple systems because they can be made robust to modeling errors. Benchmark systems are often used to compare various solutions because they can be extended or expanded to fit a wide range of systems. A mass under PD control is a useful benchmark system because it results in a 2nd-order harmonic oscillator. Also, it is the closed-loop result of many feedback linearization schemes. Figure 6 shows a sketch of the benchmark system with actuator limits. For demonstrative purposes, we set the mass equal to unity and the proportional and derivative gains are set to 39.44 and 0.628, respectively. This produces a closed-loop system with a natural frequency of 1 Hz and a damping ratio of 0.05. Figure 7 shows the unshaped and shaped step responses of this system in the absence of actuator saturation. Command shaping has successfully eliminated the residual vibration shown in the unshaped response.

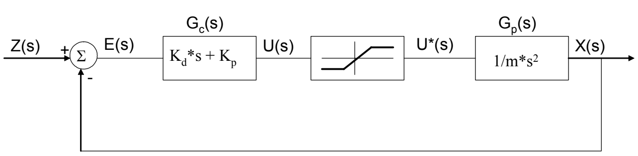

Fig 6. Control System with Actuator Limits.

Actuator saturation can greatly degrade the effect of command shaping. Figure 8 shows the unshaped and ZV-shaped system responses when there is a saturation limit of 12 (approximately 30% of the maximum amount used in Figure 7). As previously reported [11], the unshaped, saturated response has a lower frequency than the unsaturated response. The shift in the apparent "natural frequency" of the vibration diminishes the effectiveness of the command shaper. When the saturation limit is added, the ZV shaper only reduces the vibration by 75%. It should also be noted that the unshaped and shaped response vibrations have different frequencies.

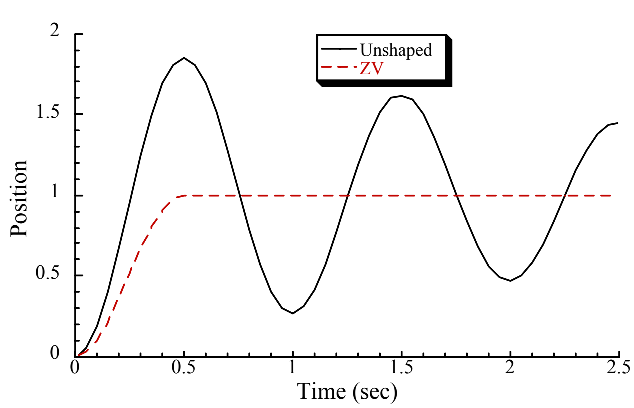

Fig 7. Unsaturated Results.

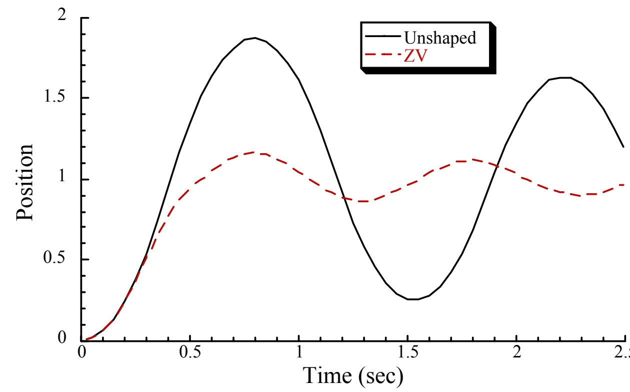

Fig 8. Saturated Results.

Figure 9 shows the unsaturated and saturated shaped inputs to the system for the first ½ second. The distortion of the commands is clearly shown and is greatest at the beginning of the move. This is the source of the decreased effectiveness of the shaped command.

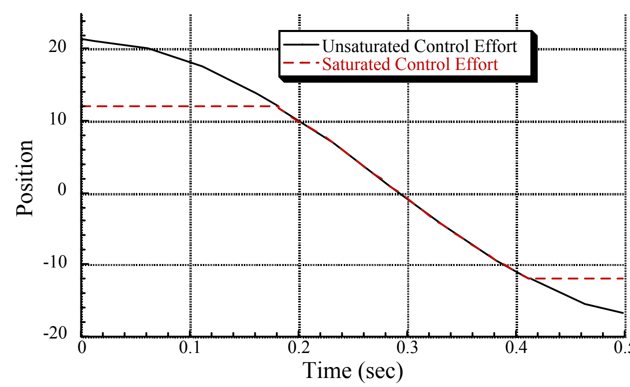

Fig 9. Saturated and Unsaturated Control Effort.

Experiments were performed on the Quanser High Fidelity Linear Cart (HFLC) system [12]. The HFLC consists of a solid aluminum cart driven by a high-power 400 Watt 3-phase brushless DC motor sliding along two steel guide rails with self-aligning linear bearings. The resulting linear positioning system can obtain high speed, high acceleration, and excellent repeatability characteristics. A PD controller with proportional and derivative gains of 420 A/m and 0.15 A*s/m, respectively, was used to drive the 4.9 kg cart.

Figure 10 shows the cart's position in response to a move distance of 2.5 cm. The natural frequency of the response is estimated to be 8 Hz and the damping ratio is estimated to be 0.05. A ZV shaper was developed for the estimated parameters and used to modify the reference command. As shown in the figure, the ZV command greatly reduced the residual vibration. The system settles in 0.06 seconds and has a percent overshoot of 15%.

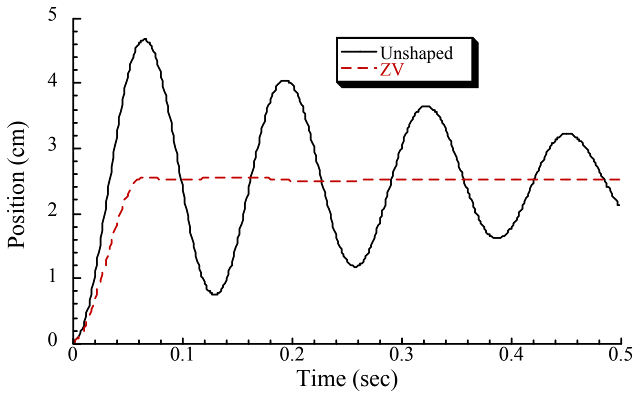

Fig. 10. HFLC Step Response of unshaped and ZV-shaped commands.

To investigate the effects of actuator saturation, the current was artificially limited to 30% of the unshaped maximum. The step responses for the unshaped and ZV-shaped commands are shown in Figure 11. The current-limited unshaped response appears to have a lower natural frequency and a higher damping ratio than the response shown in Figure 10. Actuator saturation has greatly reduced the effectiveness of the ZV shaper. The shaped response has 30% overshoot and takes over 0.5 seconds to settle.

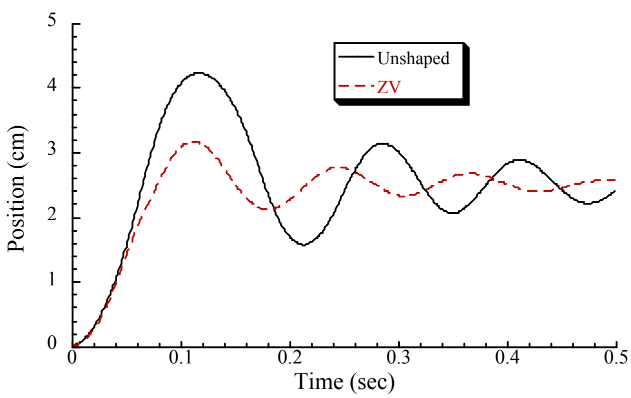

Fig. 11. HFLC Saturated Step Response of unshaped and ZV-shaped commands.

Several methods have been proposed to create shaped commands for systems with actuator saturation. Eloundou and Singhose [11] created shapers via an optimization with a simulation inside the loop for a mass under PD control. Sorensen and Singhose [13] created shapers for a variety of hard nonlinearities including actuator saturation. In the next section, a shaping method is proposed that creates commands that do not require inputs greater than the saturation limit, thereby avoiding any distortion of the command and do not require the use of complex optimization routines.

III. Command Shapers for Systems with Actuator Saturation

Recently a method for creating command shapers that both eliminate residual vibration and avoid actuator saturation have been presented [14]. An abbreviated derivation will be repeated here. For the system shown in Figure 6 (ignoring actuator saturation), an expression for the control effort as a function of the reference input can be derived. The transfer function is given by

$$\frac{X(s)}{Z(s)} = \frac{G_c(s)*G_p(s)}{1+G_c(s)*G_p(s)} \tag{8}$$

The error is given by

$$E(s) = Z(s) - X(s) = \frac{s^2}{s^2 + \frac{k_d}{m}s + \frac{k_p}{m}} * Z(s) \tag{9}$$

And the actuator effort is given by

$$U(s) = E(s)*G_c(s) = \frac{k_d s^3 + k_p s^2}{s^2 + \frac{k_d}{m}s + \frac{k_p}{m}} Z(s) \tag{10}$$

$U*(s) = U(s)$ when $|u(t)| \le |u_{\max}|$, so if we choose $Z(s)$ such that $U(s)$ stays below a known value, we can insure that the commands sent to the plant are not altered. For example, consider the case when $Z(s)$ is a step of magnitude $z_{\max}$. Then the actuator effort is given by

$$\begin{aligned} U(s) &= z_{\max} \frac{k_d s^2 + k_p s}{s^2 + \frac{k_d}{m}s + \frac{k_p}{m}} \\ &= z_{\max}\left[\frac{C_1(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2+\omega_d^2} + \frac{C_2\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2} + 2m\zeta\omega_n\right] \end{aligned} \tag{11}$$

where

$$\omega_n = \sqrt{\frac{k_p}{m}} \tag{12}$$

$$\zeta = \frac{k_d}{2\sqrt{k_p m}} \tag{13}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{14}$$

$$C_1 = m\omega_n^2(1+\zeta^4)$$

and

$$C_2 = \frac{m\zeta(\omega_n + 4\zeta^2\omega_n + 2)}{\sqrt{1-\zeta^2}} \tag{15}$$

In the time domain this yields

$$u(t) = z_{\max}\left[A_{total}\exp(-\zeta\omega_n t)\cos(\omega_d t + \phi) + 2\zeta m\omega_n\delta(t)\right] \tag{16}$$

where

$$A_{total} = \sqrt{C_1^2 + C_2^2} \tag{17}$$

and

$$\begin{aligned}\phi &= \arctan\frac{-C_2}{C_1}\\ &= \arctan\frac{-\zeta(\omega_n + 4\zeta^2\omega_n + 2) + 2}{\omega_n^2(1+4\zeta^2)\sqrt{1-\zeta^2}}\end{aligned} \tag{18}$$

For lightly damped systems with a natural frequencies greater or equal to 1 Hz, the initial step size at t=0 that will not saturate the actuator is given by

$$z_i = \frac{u_{\max}}{A_{total}} \tag{19}$$

Combining (2), (16) and (19) give the constraints needed to solve for command shapers that eliminate residual vibration and avoid actuator saturation.

A. Saturation-Reducing Command Shapers

A command shaper was created for the benchmark system with the mass set equal to unity and the proportional and derivative gains set to 39.44 and 0.628, respectively, and a saturation limit equal to 12 using (2) and (19). Assuming the input is a unit step, the resulting Saturation-Reducing, Zero Vibration (SRZV) shaper is given by

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 0.3 & 0.4352 & 0.2648 \\ 0 & 0.3819 & 0.7797 \end{bmatrix} \tag{20}$$

Figure 12 shows the ZV-shaped and SRZV-shaped system responses when there is a saturation limit of 12. The SRZV shaper greatly reduces the residual vibration. The desired control effort reaches the plant without being modified or distorted.

Figure 13 shows the effect of modeling error on the performance of the SRZV shaper. There is a 10% overshoot when the actual natural frequency of 0.9 Hz and a 5% overshoot when the actual frequency is 1.1 Hz. The vibration is approximately 50% less than the ZV shaper's vibration for the same modeling error even though no robustness constraints were used when developing the SR-ZV shaper.

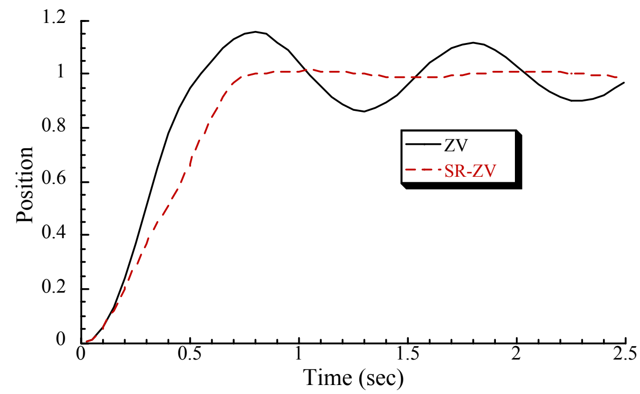

Fig. 12. SRZV Shaper Response.

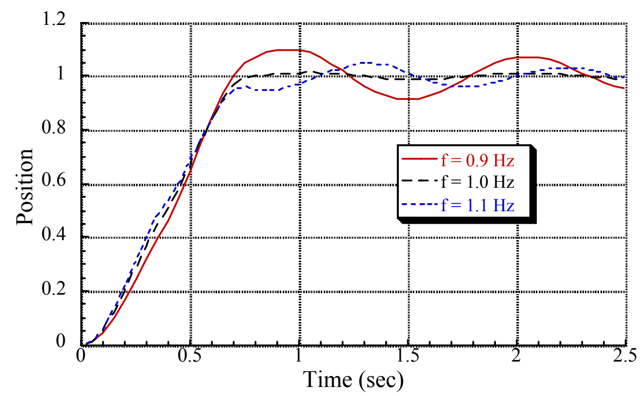

Fig. 13. Effect of Modeling error on SRZV Shaper performance.

Figure 14 shows the robustness of the command shaper to variations in the actuator limit. As expected, when the actual saturation limit is higher than then the modeled or expected limit, there is very little overshoot or residual vibration. The command shaper is remains effective for saturation limits that are 20% less than the expected limit. However, its performance begins to rapidly degrade once the actual limit is less than 10% of the expected value.

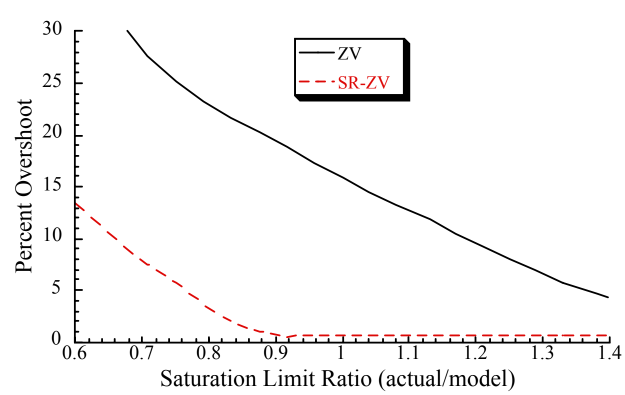

Fig. 14. Effect of Modeling error on SRZV Shaper performance.

Figure 15 shows the experimental responses for the ZV-shaped and SRZV-shaped systems when the current is limited to 30% of the maximum level used in Figure 10. The SRZV shaper greatly reduces the residual vibration. There is approximately 2% overshoot (90% reduction of the ZV overshoot) and the system settles in less than 0.15 seconds (over 70% reduction of the ZV settling time). The desired control effort reaches the plant without being modified or distorted.

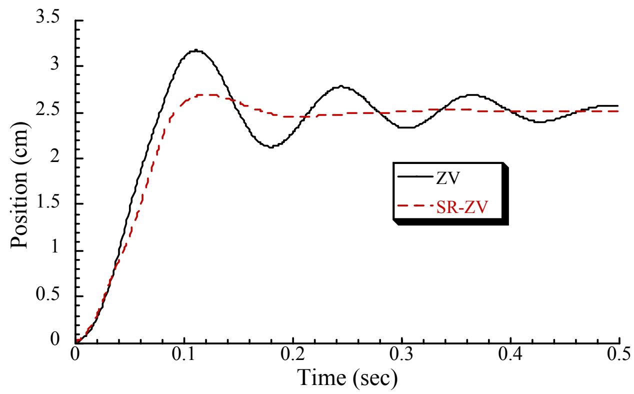

Fig. 15. Experimental SRZV Shaper performance.

Figure 16 shows the experimental response robustness for the ZV and SR-ZV shapers to variations in the actuator limit. As predicted by the simulations, when the actual saturation limit is higher than then the modeled or expected limit, there is very little overshoot or residual vibration for the SR-ZV shaper and less than 15% overshoot for the ZV shaper. The SR-ZV shaper produces overshoot of less than 5% for saturation limits that are 20% less than the expected limit. However, its performance begins to degrade once the actual limit is less than 10% of the expected value and rapidly increases for large errors in available actuator effort.

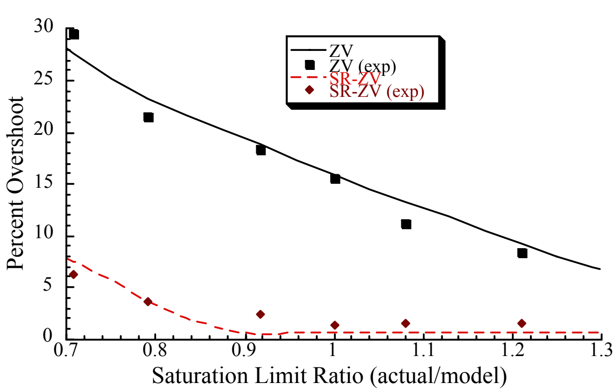

Fig. 16. Experimental SRZV Shaper performance.

IV. Conclusion

Saturation-Reducing, Zero Vibration (SRZV) shapers were developed for systems with actuator limits. Developed assuming that the unshaped command is a step of a known amplitude, these command shapers limit the amplitude of the error sent to the controller, thereby reducing the likelihood of saturating the actuators. While these shapers have longer durations than command shapers that do not account for actuator effort, they successfully reduce the residual vibration for the benchmark system and are more robust than traditional command shapers. Experimental results on a linear cart system verified the effectiveness of the approach.

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