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An Iterative Learning Control Design for Self-Servowriting in Hard Disk Drives Using L_1 Optimal Control

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Abstract—This paper presents an Iterative Learning Controller (ILC) design for Self-Servowriting (SSW) process in Hard Disk Drives (HDDs). In SSW, the position and timing information are written onto the disk surface by referring to the previously written servo information. This process is repeated until the whole disk is completely written. The main issue in this process is Radial Error Propagation (REP), which refers to the accumulation of the written-in errors as tracks are written sequentially. In this paper, an ILC scheme is designed by using the position error information in the previous track to prevent REP and improve the quality of servowriting. Analytic conditions for stability and monotonic convergence are discussed. The learning filter design is formulated as an L_1 (or induced L_∞) optimal control problem, which is solved by optimization programming. Simulation results show that REP is contained and a good quality of servowriting is assured.

I. Introduction

In Hard Disk Drives (HDDs), the servo patterns are embedded by means of writing the burst information onto the disk surface at discrete locations (i.e., the servo sectors). The burst information is demodulated to determine the position off-track information, i.e., Position Error Signal (PES), which is used for track-following on a track or for seeking to a target track.

In conventional servowriting, the position (servo burst) and timing (synchronization mark) information are written by using an external costly laser-guided push-pin mechanism such that the write head is positioned on the desired tracks. As the aerial density of HDD increases, this process has become extremely time consuming. For mass production of HDD units in the factory, a large number of expensive servo writers and clean-room space are required. Therefore, it is important to reduce the cost and time of servowriting process.

Self-Servowriting (SSW) is an attractive approach because it not only saves the cost and time of manufacturing, but also effectively maintains the servowriting quality. In one embodiment of SSW, one or a few tracks (called the seed tracks) are written at a predetermined area on the disk surface by the servo writer. After that, the servo patterns are propagated to other tracks from the seed tracks without using the servo writers but using the recording head of the disk unit. During this process, the read head position follows the previously written track while the write head writes the servo information to next track. This process is repeated until the servo patterns on all tracks have been written. By self propagating the servo patterns using the internal heads and by minimizing the number of servo writers required in the factory, a considerable saving of manufacturing cost is achieved. Moreover, SSW does not have to be performed in the clean-room environment, and it saves clean-room space for the factory.

However, new challenges such as containment of the radial propagation of the written-in errors and ensuring the servowriting quality arise in SSW process. See Ye et al. [1], and Melkote et al. [2] among others for further details. Iterative Learning Control (ILC) based control structure (Melkote and McNab [3], Wu and Tomizuka [4]) has been proved to be an effective approach to contain the radial error propagation.

In this paper, a methodology of designing ILC scheme is presented. It is followed by analyzing the conditions for asymptotic convergence and monotonic convergence. Based on these conditions, the ILC design is formulated as an L_1 optimal control problem that can be solved by optimization programming.

The remainder of this paper is organized as follows. In section II, the issues and control objectives in SSW servo system are briefly reviewed. Section III designs the ILC control scheme and an L_1 optimal control problem is formulated based on the conditions for asymptotic convergence and monotonic convergence. A simulation study is presented in Section IV. Finally, conclusions are given in Section V.

II. Problem Statements

A. Radial Error Propagation (REP)

The general steps for SSW process were described by Melkote and McNab [3] and Wu and Tomizuka [4], among others.

SSW regenerates timing and position information from the previously written track. During this process, each writing step writes a ‘memory’ of all proceeding track errors including position errors and timing errors. As illustrated in Fig. 1, the position errors cause noncircuity of servo tracks, and the timing errors result in warping of servo sectors on the disk.

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These written-in errors propagate along radial direction. Such propagation is called Radial Error Propagation (REP) and it is the main issue in SSW. If REP is not adequately controlled, SSW process will fail.

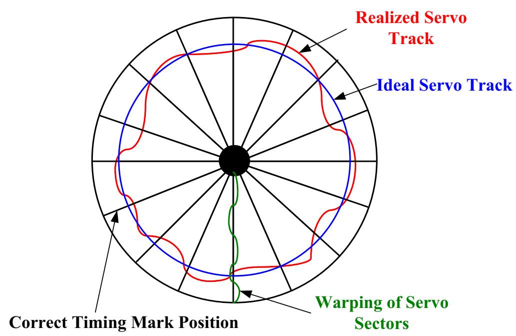

Fig. 1 Illustration of Written-in Errors on the Disk

The block diagram of SSW servo system is illustrated in Fig. 2, which contains a typical track following servo loop with Voice Coil Motor (VCM) $P(z)$ and the feedback compensator $C(z)$. The read head follows the i^{th} track with profile $y_i(k)$ while the write head writes the next track with profile $y_{i+1}(k)$. This is possible because of an offset in the radial direction between the read head and write head. $e_i(k)$ denotes the position error signal.

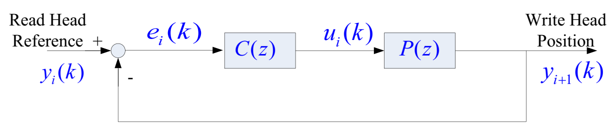

Fig. 2 Block Diagram of SSW Servo System

It is noted that the transfer function from $y_i(k)$ to $y_{i+1}(k)$ is the complementary sensitivity function, i.e.,

$$y_{i+1}(k) = T(z)y_i(k) \quad (1)$$

where, $T(z) = P(z)C(z)(1+P(z)C(z))^{-1}$

In general systems, the gain of $T(z)$, as shown in Fig. 3, is larger than 1 in some range of frequencies, which amplifies the frequency components of $y_i(k)$ and causes the written-in errors to build up in the radial direction across the disk. So in order to contain the REP, we need to contain the magnitude of $y_i(k)$. Furthermore, it is desired to design a correction signal to make $\lim_{i\to\infty} y_i(k) = 0$, as illustrated in Fig. 4.

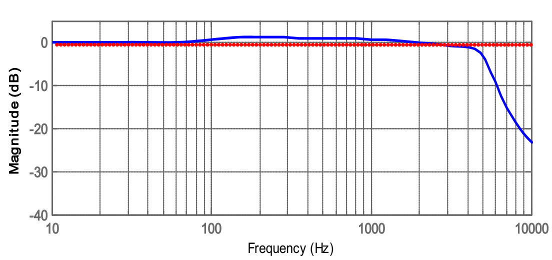

Fig. 3 Typical Response of Complementary Sensitivity Function

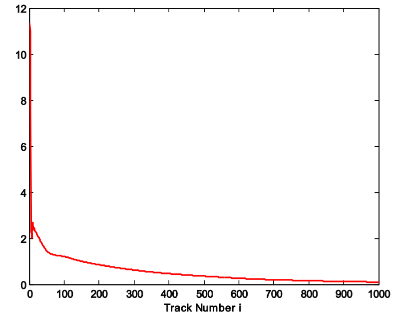

Fig.4 Ideal Track Profile (i.e., without disturbances)

B. Servo Writing Quality

In addition to containing REP, another significant goal in SSW process is to assure good servowriting quality. An important quantity for this performance is the AC Track Squeeze, which is defined as the minimum spacing between two adjacent servo tracks as illustrated in Fig. 5. The definition of AC Track Squeeze is,

$$T_{sq}(i) = \min_{k\in[0,N-1]} \{1 + y_{i+1}(k) - y_i(k)\} \times 100\% \quad (2)$$

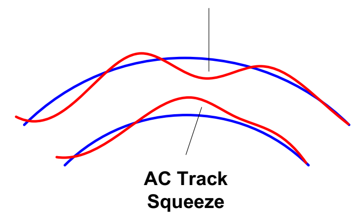

Fig. 5 Illustration of AC Track Squeeze in HDD

Equation (2) can be further written as,

$$\begin{aligned} T_{sq}(i) &= \left\{1 - \max_{k\in[0,N-1]} \left|y_i(k) - y_{i+1}(k)\right|\right\} \times 100\% \\ &\le \left\{1 + \|y_{i+1}(\bullet)\|_\infty - \|y_i(\bullet)\|_\infty\right\} \times 100\% \end{aligned} \quad (3)$$

where $\|y_i(\bullet)\|_\infty$ is defined as the maximum magnitude of the head positions along one track, i.e., $\|y_i(\bullet)\|_\infty = \max_{0\le k\le N-1}(|y_i(k)|)$, for $i = 0,1\ldots$. Therefore it is desired to control $\|y_i(\bullet)\|_\infty$ in order to improve the AC Track Squeeze.

III. ILC System Analysis

In this section, the feedforward correction signal $u_i(k)$ is injected to the servo loop, as shown in Fig. 6. This configuration is same as the one in Wu and Tomizuka [4].

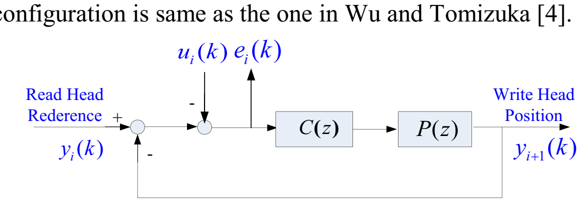

Fig. 6 Block Diagram of SSW Servo System with ILC

In SSW process, the head position $y_i(k)$ is not measurable, but the error signal $e_i(k)$ and its previous track values $e_{i-1}(k)$

are available for us to estimate $y_i(k)$. Hence, it is intuitive to use $e_{i-1}(k)$ to generate the correction signal $u_i(k)$,

$$u_i(k) = F(z)\cdot e_{i-1}(k) \tag{4}$$

where $F(z)$ is a learning filter to be designed. $F(z)$ has both causal and non-causal parts, as shown in the following equation:

$$F(z) = \cdots + f_2^N z^2 + f_1^N z + f_0 + f_1^C z^{-1} + f_2^C z^{-2} + \cdots \tag{5}$$

Namely $\{e_i(k): 0 \le k \le N-1\}$ is all available from the beginning of the i^{th} iteration. $u_i(k)$ may depends on $e_i(l)$, for $l \le k$ as well as $l > k$.

Notice that after injecting the feedforward signal $u_i(k)$, the error signal $e_i(k)$ in Fig. 6 is written as $e_i(k) = y_i(k) - y_{i+1}(k) - u_i(k)$, which is different from that in Fig. 2.

A. Analysis in Lifted Domain

First, the following super-vectors are defined,

$$y_i = \begin{bmatrix} y_i(0) \\ y_i(1) \\ y_i(2) \\ \vdots \\ y_i(N-1) \end{bmatrix} \quad e_i = \begin{bmatrix} e_i(0) \\ e_i(1) \\ e_i(2) \\ \vdots \\ e_i(N-1) \end{bmatrix} \quad \text{and} \quad u_i = \begin{bmatrix} u_i(0) \\ u_i(1) \\ u_i(2) \\ \vdots \\ u_i(N-1) \end{bmatrix}$$

where N denotes the total servo sector number in one track, the evolution equation for track profile can be derived as,

$$\begin{aligned} y_{i+1} &= Ty_i - Tu_i = Ty_i - TFe_{i-1} \\ &= Ty_i - TF(PC)^{-1} y_i = [T - SF]\, y_i \end{aligned} \tag{6}$$

where S and T are markov matrices of $S(z)$ and $T(z)$, respectively; and $S(z) = 1/(1 + P(z)C(z))$ is the sensitivity function.

Note that the impulse response of $S(z)$ can be represented as,

$$S(z) = s_0 + s_1 z^{-1} + s_2 z^{-2} + \dots + s_{N-1} z^{-N+1} + \dots \tag{7}$$

where s_0, s_1,..., s_{N-1} are called its markov parameters.

By using these parameters, we construct S as an $N \times N$ matrix:

$$S = \begin{bmatrix} s_0 & 0 & 0 \dots 0 \\ s_1 & s_0 & 0 \dots 0 \\ s_2 & s_1 & s_0 \dots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ s_{N-1} & s_{N-2} & s_{N-3} \dots s_0 \end{bmatrix}$$

Matrices T and F are constructed in the same way.

$$T = \begin{bmatrix} t_0 & 0 & 0 \dots 0 \\ t_1 & t_0 & 0 \dots 0 \\ t_2 & t_1 & t_0 \dots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ t_{N-1} & t_{N-2} & t_{N-3} \dots t_0 \end{bmatrix}, \quad F = \begin{bmatrix} f_0 & f_1^N & f_2^N & \cdots & & \\ f_1^C & f_0 & f_1^N & f_2^N \cdots & & \\ f_2^C & f_1^C & f_0 & f_1^N \cdots & & \\ \vdots & f_2^C & f_1^C & f_0 \cdots & & \\ & \vdots & \vdots & \vdots & f_0 & f_1^N \\ & & & f_2^C & f_1^C & f_0 \end{bmatrix}$$

Remark: Equation (6) implies that matrix $[T - SF]$ is essential in relating the current track profile to next track profile.

B. Asymptotic Convergence Analysis [4]

Definition: (Asymptotic Convergence for SSW process) SSW process is said to be asymptotically convergent if the head position y_i converges to zero, i.e., $\lim_{i\to\infty} y_i = 0$. It is easy to see that the asymptotic convergence condition for system in Equation (6) is,

$$\left|\rho\left(T - SF\right)\right| < 1 \tag{8}$$

where $\rho\left(T - SF\right)$ denotes the spectral radius of matrix $[T - SF]$.

Asymptotic convergence guarantees that the REP is well contained.

C. Monotonic Convergence Analysis

Definition: (Monotonic Convergence for SSW process): SSW system is said to be monotonically convergent if the maximum magnitude of track profile y_i, i.e., $\|y_i\|_\infty$ decreases successively.

The above definition can be equivalently expressed as,

$$\|y_{i+1}(\bullet)\|_\infty < \|y_i(\bullet)\|_\infty \tag{9}$$

For the system in Equation (6), the monotonic convergence requires

$$\|T - SF\|_\infty < 1 \tag{10}$$

Notice that condition (10) is stronger than condition (8). Based on the monotonic convergence definition (9), the maximum magnitude of track profile decreases monotonically; this results in monotonic decrease in peak value of PES. Therefore, the performance of servowriting (i.e., AC track squeeze) is improved.

Remark: The condition for monotonic convergence in the inequality (10) is stronger than that has been discussed in [4]: $\|y_{i+1}(k)\|_2 < \|y_i(k)\|_2$, for $k = 0,1,...,N-1$, which guarantees monotonic decrease in terms of energy for track profile $y_i(k)$. Notice that $\|y_{i+1}(k)\|_\infty < \|y_i(k)\|_\infty$ is a sufficient condition for $\|y_{i+1}(k)\|_2 < \|y_i(k)\|_2$. Furthermore, $\|y_{i+1}(k)\|_2 < \|y_i(k)\|_2$ requires $\|T - SF\|_2 < 1$, which is an H_∞ control problem, while inequality (10) defines an L_1 control problem.

D. L_1 Optimal Control Formulation

In this paper, the learning filter $F(z)$ is considered as a FIR filter with three causal terms and two non-causal terms, i.e.,

$$F(z) = f_2^N z^2 + f_1^N z + f_0 + f_1^C z^{-1} + f_2^C z^{-2} \tag{11}$$

Now we introduce the following new transfer functions and their markov matrices:

(a) $S_1(z)$ is defined as a one-step advanced $S(z)$, i.e., $S_1(z) = z \cdot S(z)$ and S_1 is its markov matrix. S_2 is defined as the markov matrix of $S_2(z)$, which is two-step advanced $S(z)$, i.e., $S_2(z) = z^2 \cdot S(z)$

$$S_1=\begin{bmatrix} s_1 & s_0 & 0 \dots 0 \\ s_2 & s_1 & s_0 \dots 0 \\ s_3 & s_2 & s_1 \dots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ 0 & s_{N-1} & s_{N-2} \dots s_1 \end{bmatrix} \quad S_2=\begin{bmatrix} s_2 & s_1 & s_0 \dots 0 \\ s_3 & s_2 & s_1 \dots 0 \\ s_4 & s_3 & s_2 \dots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ 0 & 0 & s_{N-1} \dots s_2 \end{bmatrix} \tag{12}$$

(b) Similarly, $S_3(z)$ and $S_4(z)$ are one-step delayed and two-step delayed of $S(z)$, and their corresponding markov matrices are S_3 and S_4.

$$S_3=\begin{bmatrix} 0 & 0 & 0 \dots 0 \\ s_0 & 0 & 0 \dots 0 \\ s_1 & s_0 & 0 \dots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ s_{N-2} & s_{N-3} & s_{N-4} \dots 0 \end{bmatrix}, \quad S_4=\begin{bmatrix} 0 & 0 & 0 \dots 0 \\ 0 & 0 & 0 \dots 0 \\ s_0 & 0 & 0 \dots 0 \\ \vdots & \vdots & \vdots \quad \vdots \\ s_{N-3} & s_{N-4} & s_{N-5} \dots 0 \end{bmatrix} \tag{13}$$

Consequently, the matrix SF is expressed as,

$$SF = f_0 \cdot S_0 + f_1^N S_1 + f_2^N S_2 + f_1^C S_3 + f_2^C S_4 = \sum_{\kappa=0}^{4} S_k f_k \tag{14}$$

where $S_0 = S$, $f_1 = f_1^N$, $f_2 = f_2^N$, $f_3 = f_1^C$ and $f_4 = f_2^C$

It is followed by the expression of $\|T - SF\|_\infty$:

$$\|T-SF\|_\infty = \left\|T - \sum_{\kappa=0}^{4} S_k f_k\right\|_\infty = \max_i \left(\sum_{j=1}^{N} \left| T_{ij} - \sum_{\kappa=0}^{4} S_{k\_ij} f_k \right| \right) \tag{15}$$

where S_{k_ij} is the $(i,j)^{th}$ element of matrix S_k, for $k = 0,1,...,4$.

By introducing Equation (15), the condition in inequality (10) is equivalently written as,

$$\max_i \left(\sum_{j=1}^{N} \left| T_{ij} - \sum_{\kappa=0}^{4} S_{k\_ij} f_k \right| \right) < 1 \tag{16}$$

Notice that the inequality (16) defines an optimization problem.

E. Consideration of Disturbance and Noise

In actual design, we need to consider the effect of disturbance and noise during SSW process as shown in Fig. 7. In this block diagram, $d_i(k)$ denotes the disk flutter and vibration and $n_i(k)$ denotes the sensor measurement noise.

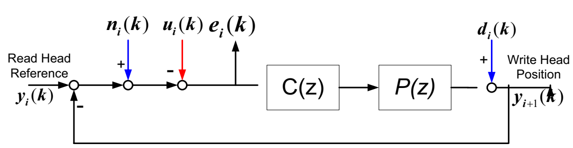

Fig. 7 SSW Servo System with Disturbance and Noise

By considering $d_i(k)$ and $n_i(k)$, we have new evolution equation for track profile,

$$\begin{aligned} y_{i+1} &= Ty_i - Tu_i + Tn_i + Sd_i = Ty_i - TFe_{i-1} + Tn_i + Sd_i \\ &= Ty_i - TF(PC)^{-1} y_i + TF(PC)^{-1} d_{i-1} + Tn_i + Sd_i \\ &= [T - SF] y_i + Tn_i + Sd_i + SFd_{i-1} \end{aligned} \tag{17}$$

From Equation (17), it can be seen that the next track profile y_{i+1} is affected not only by the current track disturbance n_i and d_i, but also by the previous track disturbance d_{i-1}. Since the response from $n_i(k)$ to $y_{i+1}(k)$ is $T(z)$, and that from $d_i(k)$ to $y_{i+1}(k)$ is $S(z)$, the rejection performance on n_i and d_i is decided by $T(z)$ and $S(z)$, which are considered when the feedback compensator $C(z)$ was designed. On the other hand, the rejection performance on d_{i-1} is determined by the transfer function $S(z)F(z)$. Thus, in order to effectively attenuate d_{i-1}, the filter $F(z)$ should be designed such that, $\|SF\|_\infty \le \|S\|_\infty$, or equivalently,

$$\|F\|_\infty \le 1 \tag{18}$$

The proposed learning filter in Equation (11) implies that matrix F can be constructed as,

$$F=\begin{bmatrix} f_0 & f_1^N & f_2^N & & & \\ f_1^C & f_0 & f_1^N & f_2^N & & \\ f_2^C & f_1^C & f_0 & f_1^N \cdots & & \\ & f_2^C & f_1^C & f_0 \cdots & & \\ & & \vdots & \vdots & f_0 & f_1^N \\ & & & f_2^C & f_1^C & f_0 \end{bmatrix} \tag{19}$$

From the structure of matrix F, we further conclude that $\|F\|_\infty \le 1$ can be rewritten as,

$$\left(|f_2^C| + |f_1^C| + |f_0| + |f_1^N| + |f_2^N| \right) \le 1 \tag{20}$$

Now the two conditions in (16) and (20) are formulated into one optimization problem for feasible values of filter coefficients. This problem is solved by using Yalmip package [5].

IV. SIMULATION RESULTS

In the simulations, the servo sector number is $N = 220$, and the spindle rotation speed is 7200 *rpm*; which sets the sampling frequency to be $220 \times (7200/60) = 26400$ *Hz*. The Bode plot of HDD plant is as shown in Fig. 8.

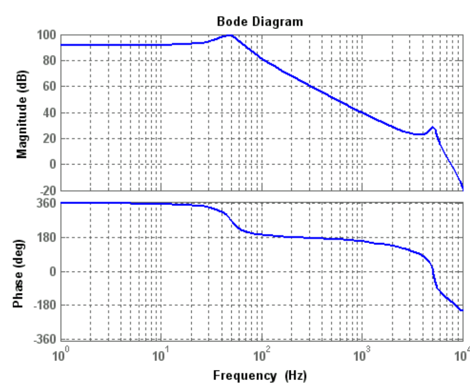

Fig. 8 Bode Plot of HDD plant

The performance of learning filter $F(z)$ and $S(z)F(z)$ are firstly examined.

Fig. 9 is the frequency response of learning filter $F(z)$. We can see that its magnitude is below 0*db* at all frequencies. The comparison of $S(z)$ and $S(z)F(z)$ is shown in Fig. 10. It is learned that $S(z)F(z)$ not only preserves the disturbance attenuation property of $S(z)$ at low frequencies, but also has much lower peak value than $S(z)$; in other words, $S(z)F(z)$ has better rejection performance on d_{i-1} than $S(z)$.

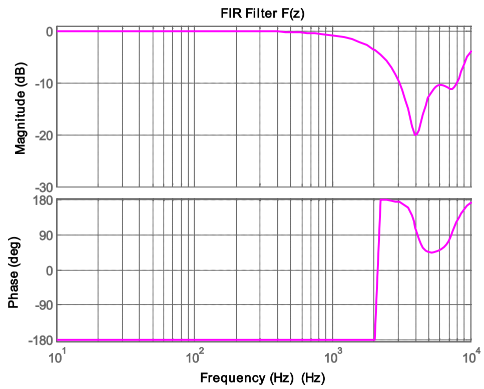

Fig. 9 Frequency Response of $F(z)$

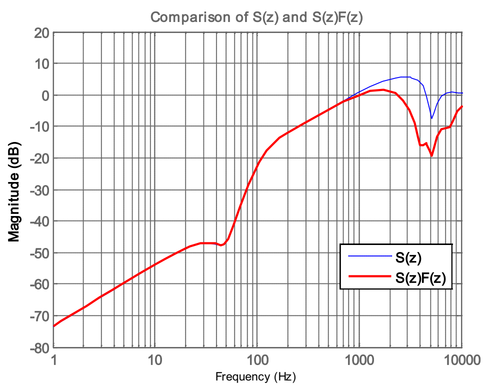

Fig. 10 Frequency Response of $S(z)$ and $S(z)F(z)$

Fig. 11 and Fig. 12 compare the closed-loop responses before and after applying the proposed ILC control law. It is noted that with ILC, the maximum gain of the closed-loop response, i.e., $T(z)-S(z)F(z)$ is less than 1. This condition ensures that the effect of non-circular tracks is diminishing as the track number i increases.

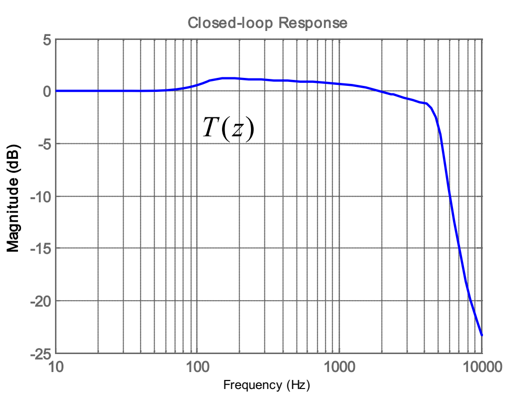

Fig. 11 Closed-loop Response before applying ILC

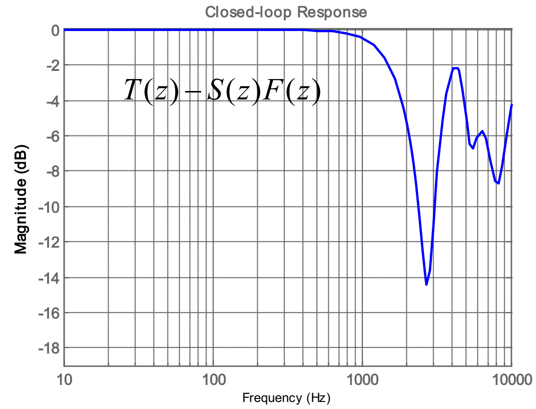

Fig. 12 Closed-loop Response with ILC

Fig. 13 is the seed track profile with one sigma value of *11%* track pitch. In the simulation study, total 1000 servo tracks data was collected. Fig. 14 shows the track profiles with the effect of disturbance and noise. It is observed that the REP is well contained since the head position y_i converges to some small value. Its average one sigma value is about *2.93%* track pitch, which is comparable to that of the seed track $\boldsymbol{y}_0$.

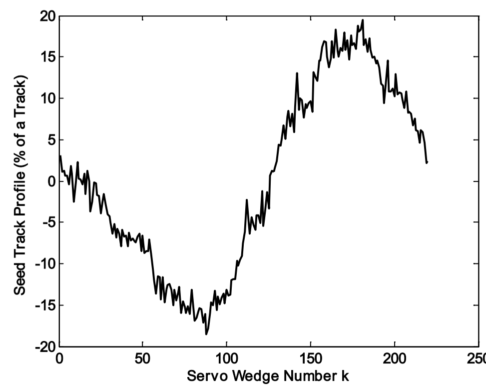

Fig. 13 Seed Track Profile

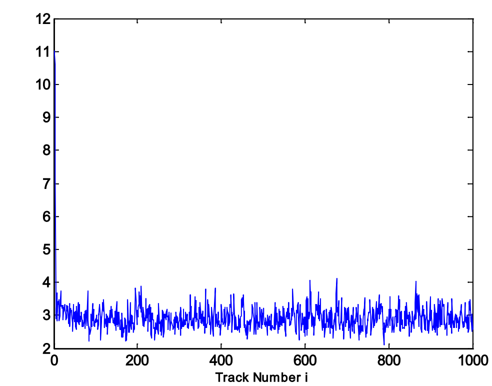

Fig. 14 Track Profiles (1000 tracks)

Another important performance measure is the AC track squeeze. The ideal AC track squeeze is one track width. When the AC track squeeze value is too small, two adjacent tracks with narrow track spacing may cause some data corruption. The simulation results in Fig. 15 and Fig. 16 show that the average AC track squeeze is about *91.6%* track pitch, which is within the acceptable limit.

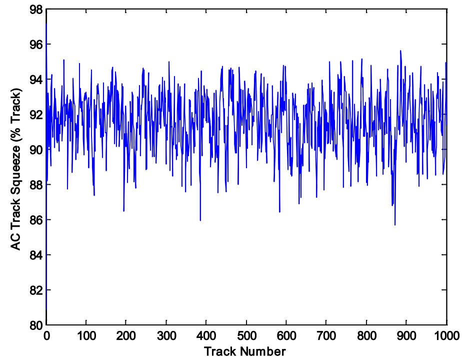

Fig. 15 AC Track Squeeze

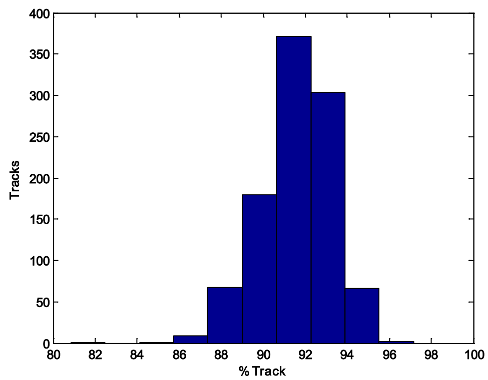

Fig. 16 Histogram of AC Track Squeeze

V. CONCLUSION

The issues and control objectives of SSW servo system in HDD have been described in this paper. An ILC control scheme was proposed to prevent REP and improve servowriting quality. The learning filter was designed based on the conditions for monotonic convergence. It was formulated as an L_1 optimal control problem by means of containing the peak gain of track profile and PES. The simulation results showed that the *REP* is contained and servowriting quality is satisfying.

VI. ACKNOWLEDGMENTS

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