2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Improved Bumpless Transfer with Slow-fast Controller Decomposition

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Abstract— **Previously introduced slow-fast decomposition bumpless transfer is extended to 2-degree-of-freedom configuration of candidate controllers. An easy implementation with observable canonical form of slow modes controllers is introduced. For controllers that have only fast modes, an uncontrollable-slow-modes state augmentation technique is provided. Simulations demonstrate comparative advantages over other bumpless transfer methods.**

I. Introduction

Situations in which switching among multiple controllers is required are frequently observed in various fields of control engineering. In nonlinear control, it is common to switch gains to attenuate the effects of control actuator saturation, as in anti-windup compensator design [1], [2]. Controller switching is also used for adaptive control of uncertain or changing plants. It is possible that controller output signal mismatch can occur at times controllers switched [3], leading to discontinuities and abrupt transients called ‘bumps’. These bumps on controller output are not desired in most cases. Therefore, researchers have been encouraged to develop ‘bumpless transfer’ methods to overcome this problem since the 1980’s.

In adaptive switching control applications, the plant is not precisely known at the outset in general, and the goal of adaptive control is to change the controllers for stabilization or performance improvement as measured data begins to reveal information about the plant. Due to this situation of controller adaptation an exact plant model is regarded unavailable at the time of switching. This requires for bumpless transfer switching controller to have a particular property, which does not depends on a plant model. While existing bumpless transfer literatures successfully define the problem and shows good performance [3]–[5], some of them can be adequately applied to adaptive switching controls. The conditioning technique [3], the continuous switching method [6], and linear quadratic optimal bumpless transfer method [4] are examples of methods in this category of bumpless transfer methodologies. Furthermore, recent study covers controller uncertainty beyond perfect knowledge of controller model [7].

Another method which suggested particularly for adaptive switching controls is the slow-fast decomposition bumpless transfer by us [8]. By appropriately re-initializing the states of the slow and fast modes controllers at switching times, this method can ensure that not only will the controller output be continuous, but also that it avoids fast transient bumps after switching. This can be considered as an improvement over the method in [6], which is one of the simplest methods by assuring continuity [9] but may allow abrupt fast transients after switching. Our slow-fast decomposition bumpless transfer method removes the possibilities of the abrupt transients.

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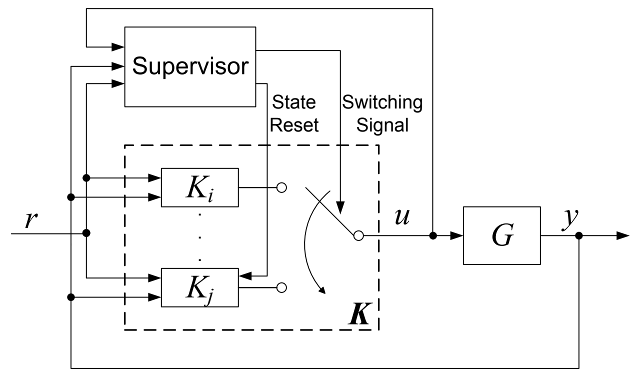

Fig. 1. Switching control system with 2-degree-of-freedom controllers

The purpose of this paper is to present improvements over [8]. The previous results are extended to include 2-degree-of-freedom controllers. For this extension a switching control structure with 2-DOF candidate controllers in Fig. 1 is introduced and a proof is also modified. Afterwards, we develop a straightforward way to build slow mode controllers based on the standard observable canonical form. This makes manipulating slow-fast decomposed configuration easy in practice. Special cases of which a controller has only slow modes or only fast modes are addressed. A controller with only fast modes uses state augmentation technique to create a slow modes controller. Furthermore, simulation results comparing the method presented in this paper with the other in [6] while earlier examples in [8] demonstrated differences between controller switching transients with and without bumpless transfer. The example verifies slow-fast decomposition bumpless transfer produces better performance than [6] when controllers have fast parts.

The organization of the paper is as follows. Notation and the switching control system configuration are introduced in Section II. The bumpless transfer problem formulation is presented in Section III. The slow-fast bumpless transfer theory and method is presented in Section IV. Section V shows simulation results and conclusions are in Section VI .

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II. PRELIMINARIES

A. Switching control system

We consider the switching control system as shown in Fig. 1. The system includes a plant and a set of controllers

$$\boldsymbol{K} = \{K_1, \cdots, K_i, \cdots, K_n\} \quad (i = 1, 2, \cdots, n). \tag{1}$$

Assume that the plant output is continuous when input is continuous; A linear time invariant plant with a proper transfer function is a good example. The input of the plant is $u(t)$ and the output is $y(t)$. Plant input is directly connected to the controller output. Controller inputs are $r(t)$ and $y(t)$ where $r(t)$ is a reference signal.

When a controller K_i is in the feedback loop, the controller is said to be *on-line*, and the other controllers are said to be *off-line*. The i-th controller K_i is supposed to have state-space realization

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i z \\ y_{Ki} &= C_i x_i + D_i z \end{aligned} \tag{2}$$

where $z = [r^T \ y^T]^T$ is the input and y_{Ki} is the output of K_i. Equivalently, we write

$$K_i(s) \overset{s}{=} \left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array} \right]. \tag{3}$$

We are interested in the situation in which the on-line controller is switched from K_i to K_j at time t_s, so that

$$u = \begin{cases} y_{Ki} & \text{for } t < t_s \\ y_{Kj} & \text{for } t \geq t_s \end{cases}. \tag{4}$$

Note that time t_s is called *switching time* or *switching instant*.

Since the controller output y_{Ki} is replaced by y_{Kj} at the switching instant t_s, the control signal u can have bumps in the neighborhood of $t = t_s$ if y_{Ki} and y_{Kj} have different values. Times immediately before and after t_s are denoted as t_s^- and t_s^+, respectively.

The objective of bumpless transfer is to ensure continuity in the control signal and to smooth 'bumpy' transients at, and immediately following, the switching instant.

B. Slow-fast decomposition

We now consider controllers that can be additively decomposed into slow and fast parts as follows:

$$K(s) = K_{slow}(s) + K_{fast}(s) \tag{5}$$

with respective minimal realizations

$$K_{slow}(s) \overset{s}{=} \left[\begin{array}{c|c} A_s & B_s \\ \hline C_s & D_s \end{array} \right] \text{ and } K_{fast}(s) \overset{s}{=} \left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array} \right]. \tag{6}$$

The poles of $K_{slow}(s)$ are of smaller magnitude than the poles of $K_{fast}(s)$, i.e.,

$$|\lambda_i(A_s)| \leq |\lambda_j(A_f)| \text{ for all } i, j$$

where $\lambda_i(\cdot)$ denotes the i-th eigenvalue.

The $K_{slow}(s)$ and $K_{fast}(s)$ of the slow-fast decomposition may be computed by various means, e.g., the MATLAB `slowfast` algorithm, which is based on the stable-antistable decomposition algorithm described in [10].

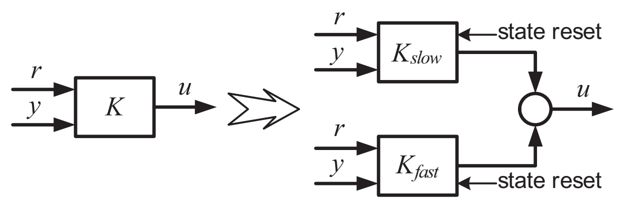

Fig. 2. Slow-fast controller decomposition

The slow-fast decomposition of the i-th controller K_i in the set $\boldsymbol{K}$ is denoted with the subscript i as (6).

$$K_{islow}(s) \overset{s}{=} \left[\begin{array}{c|c} A_{is} & B_{is} \\ \hline C_{is} & D_{is} \end{array} \right] \text{ and } K_{ifast}(s) \overset{s}{=} \left[\begin{array}{c|c} A_{if} & B_{if} \\ \hline C_{if} & D_{if} \end{array} \right] \tag{7}$$

Further details on how the controller modes are divided as slow or fast will be described in a later section.

III. PROBLEM FORMULATION

As we mentioned in Section I, bumpless transfer should perform not only continuous control signal but also smooth transient after switching. To address both of problems, we define bumpless transfer as follows.

Definition 1: (Bumpless Transfer) A switching controller with slow-fast decomposition (5) is said to perform a *bumpless transfer* if, whenever controller is switched, the new controller state is reset so as to satisfy both of the following two conditions:

(a) The control input signal $u(t)$ is continuous at t_s whenever $r(t) \in C^0$, and
(b) the state of fast part of controller $K_{fast}(s)$ is reset to zero at t_s. ♢

Condition (a) in Definition 1 is frequently observed in other bumpless transfer literature [6], [11]. Condition (b) in Definition 1 concerns control signal after switching. This additional requirement for our bumpless transfer is needed to ensure that there are no rapid transients immediately following controller switching. How controller state reset be performed to simultaneously satisfy both conditions will be described in the following section.

IV. BUMPLESS TRANSFER IMPLEMENTATION

The idea of using slow-fast decomposition of the controller as the basis for bumpless transfer generalizes a related idea introduced by [12] for adaptive PID controller switching. The details of the bumpless transfer method for PID controllers were described in [8]. We shall now describe the details of bumpless transfer with slow-fast decomposition and suggest one particular way to implement it.

A. Bumpless transfer with slow-fast decomposition

Our bumpless transfer method which will be stated in this section requires the following assumption hold for each of the candidate controllers.

Assumption 1: For each candidate controller K_i, the slow part K_{islow} in (7) has at least $m = \dim(u)$ states. $\diamondsuit$

The Assumption 1 is sufficient to allow the state of the slow controller $K_{islow}(s)$ to be reset at switching times to ensure both continuity and smoothness of the control signal $u(t)$, as we shall explain.

In general, even if all the controllers have the same order and all share a common state vector, when the controller switching occurs, any or all of the slow and fast controller state-space matrices will be switched, which can lead to bumpy transients or discontinuity in the control signal $u(t)$ at switching times. However, if only A_{is} or B_{is} are switched and there is common state vector before and after the switch, then the control signal will be continuous and furthermore no 'bumpy' fast modes of the controller will be excited. Fast transient 'bumps' or discontinuities, when they occur, may arise from switching the D_{is} matrix of the slow controller or from switching any of the state-space matrices $(A_{if}, B_{if}, C_{if}, D_{if})$ of the fast controller. In the case of switching the matrices A_{if} or B_{if} switches do actually not result in discontinuous jumps in $u(t)$, but nevertheless can result 'bumpy' fast transients in the control signal which, if very fast, may appear to be nearly discontinuous.

Our goal in bumpless transfer is to avoid both discontinuity and fast transients induced by changing fast modes. We would like our methods to work even when the order of the controller changes at switching times, and to allow for the possibility that the true plant may be imprecisely known, we would like our switching algorithm not to depend on precise knowledge of the true plant. In our method, we can do this by initializing the state of the slow part of the new controller $K_{jslow}(s)$ after each switch to a value computed to ensure continuity, and setting the state of the fast part $K_{jfast}(s)$ to zero.

Theorem 1 (Main Result): Suppose that each of the candidate controllers have slow-fast decomposition (7) satisfying Assumption 1 and suppose that at time t_s the online controller is switched from controller K_i to controller K_j. At t_s, let the states of the slow and fast controllers be reset as follows

$$x_{fast}(t_s^+) = 0 \quad (8)$$
$$x_{slow}(t_s^+) = C_{js}^\dagger\{u(t_s^-) - (D_{js} + D_{jf})z(t_s^-)\} + \xi \quad (9)$$

where $z = [r^T \ y^T]^T$, $C_{js}^\dagger$ is the pseudoinverse matrix of C_{js}, and ξ is any element of the null space of C_{js};

$$C_{js}\xi = 0 \ . \quad (10)$$

Then, bumpless transfer is achieved at the switching time t_s. $\diamondsuit$

Proof: The control signal immediately after switching (time t_s^+) can be written, based on state space representation model (7) of the new controller $K_j(s)$, as

$$u(t_s^+) = C_{js}x_{slow}(t_s^+) + C_{jf}x_{fast}(t_s^+) + (D_{js} + D_{jf})z(t_s^+) \ . \quad (11)$$

By (8) – (9),

$$u(t_s^+) = C_{js}[C_{js}^\dagger\{u(t_s^-) - (D_{js} + D_{jf})z(t_s^-)\} + \xi] + (D_{js} + D_{jf})z(t_s^+)$$

By Assumption 1, $C_{js}C_{js}^\dagger = I_{m\times m}$ where m is larger than or equal to the number of states of K_j. This results in

$$u(t_s^+) = u(t_s^-) - (D_{js} + D_{jf})z(t_s^-) + (D_{js} + D_{jf})z(t_s^+) \ .$$

Since

$$z(t_s^-) = [r^T(t_s^-) \ \ y^T(t_s^-)]^T = [r^T(t_s^+) \ \ y^T(t_s^+)]^T = z(t_s^+) \ ,$$

we finally have

$$u(t_s^+) = u(t_s^-) \ .$$

The result follows immediately from the Definition 1. *Q.E.D.*

Comment: Since C_{js} is a full rank matrix which consists of m linearly independent vectors, $C_{js}C_{js}^T$ is invertible and

$$C_{js}^\dagger = C_{js}^T(C_{js}C_{js}^T)^{-1} \ .$$

For details, see [13]. $\diamondsuit$

Equations (8) and (9) now define our *slow-fast bumpless transfer algorithm*. An example using this algorithm will be presented in Section V.

B. Slow modes controller with observable canonical form

Now we introduce a way to build slow modes controller satisfying Theorem 1. Although there are various ways to make slow modes controller, using observable canonical form of K_{slow} is a good choice since C_{js} in (11) has the following form

$$C_{js} = \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix} . \quad (12)$$

in which case, $C_{js}^\dagger = C_{js}^T$ in (9). Additionally, one simple choice for the vector ξ in (10) is $\xi = 0$.

By having (12) for all j, one uses transpose matrices rather than pseudoinverse matrices of C_{js}s. This reduces complexity on state reset procedure.

Comment: Note that a bumpless transfer in [6] is a special case of our bumpless transfer method. A method in [6] requires that all controllers have the same number of states and all controllers must have a state-space realization which share a common C-matrix and D-matrix. Our slow-fast method does not impose these controller restrictions, and can be used whenever the minimal realization of slow part K_{islow} has order at least equal to the $\dim(u)$. However, if the controllers have slow part only (i.e., $K_{ifast} = 0$ for all i) and each controller has C-matrices with the form of (12) and a common D-matrix (e.g. $D_{is} = 0$ for all i), then, it is expected for both of methods to have the same result. The advantages of the slow-fast bumpless transfer method considered in this paper arise when the controllers have both slow and fast modes, in which case our method is able to exploit the additional flexibility for state re-initialization provided by the additional fast modes to eliminate the bumpy abrupt transients the might otherwise result. $\diamondsuit$

C. Controllers with single type of modes

Situations that some of candidate controllers have only slow modes or only fast modes need to be addressed as special cases of the slow-fast decomposition bumpless transfer. If candidate controllers have slow modes only, using the method in Section IV-B and applying (9) straightforwardly solve the problem.

On the other case, when candidate controllers have only fast modes $K = K_{fast}$, it is necessary that the controllers are modified in order to apply Theorem 1 because they do not contain slow parts to be re-initialized. One possible solution is augmenting the controller states with uncontrollable slow modes that was not originally contained in the controller. Then, by slow-fast decomposition, an additive slow mode controller K_{slow} is included in the controller so that $\tilde{K} = K_{slow} + K_{fast}$ where

$$K_{slow} \overset{s}{=} \left[\begin{array}{c|c} A_s & 0 \\ \hline C_s & 0 \end{array}\right] \quad (13)$$

and $A_s \in \mathcal{R}^{m\times n}$ has only slow modes. Note that the K_{slow} has zero matrices for its B_s and D_s, so its output is determined solely by initial state. The matrix C_s is chosen with the form of (12) for the easiest way. Now, (8) and (9) in Theorem 1 can be applied, exactly as when there was already a K_{slow}. Since $K(s) = \tilde{K}(s)$, measurements for performance are not effected by adding the slow mode controller (13) except transients after the switching times, as expected how x_{slow} works.

V. Simulation Results

In this section, we present simulation example to demonstrate the effect of our slow-fast bumpless transfer method in reducing bumpy abrupt transients that might otherwise occur.

Example: Comparison among non-bumpless transfer, continuity assuring bumpless transfer, and slow-fast decomposition bumpless transfer

The example verifies validity of the additional condition Definition 1(b) compared with a previously existing method in [6]. A PID controller has an infinitely slow pole and a very fast zero when ϵ is not zero with the following definition;

$$K(s) = K_{slow}(s) + K_{fast}(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\epsilon s + 1}. \quad (14)$$

Since a fast zero in its differentiator can make a large and fast transient even after controller switching, considering only continuity of controller output as in [6] might not be sufficient to perform bumpless transfer. In this example, we show that our method can suppress undesired transient right after the switching. The results compare our bumpless transfer method properly initializing both fast and slow modes controllers with non-bumpless transfer switching and another bumpless transfer in [6].

A plant for the comparison is

$$G(s) = \frac{s^2 + s + 10}{s^3 + s^2 + 98s - 100} \ .$$

Two controllers having the structure in (4) were used to show the results. Each controllers have three gains to be switched, and the gains are as follows;

Controller 1.	$K_{P1} = 80,$	$K_{I1} = 50,$	$K_{D1} = 0.5$
Controller 2.	$K_{P2} = 5,$	$K_{I2} = 2,$	$K_{D2} = 1.25$

A small number ϵ is 0.01 and the reference input is $r = 1$. The ϵ prevents the differentiator not to make an infinite peak when a discontinuity comes into the controller. A PID controller is naturally decomposed into a slow and a fast part. Since a proportional gain is memoryless component, it can be added to either part. Controller input is $z = [r^T \ y^T]^T$ for 2-DOF controllers. Subsequently, the controllers were decomposed into

$$K_{slow}(s) \overset{s}{=} \left[\begin{array}{c|c} 0 & [K_I \quad -K_I] \\ \hline 1 & [K_P \quad -K_P] \end{array}\right]. \quad (15)$$

And, in the same way, K_{fast} can be written by

$$K_{fast}(s) \overset{s}{=} \left[\begin{array}{c|c} -1/\epsilon & [1/\epsilon \quad -1/\epsilon] \\ \hline -K_D/\epsilon & [K_D/\epsilon \quad -K_D/\epsilon] \end{array}\right]. \quad (16)$$

Controller 1 and Controller 2 in this particular case were, respectively,

$$\begin{aligned} K_1(s) &= K_{1slow} + K_{1fast} = 80 + \frac{50}{s} + \frac{0.5s}{0.01s+1} \\ K_2(s) &= K_{2slow} + K_{2fast} = 5 + \frac{2}{s} + \frac{1.25s}{0.01s+1} \end{aligned}$$

$K_1(s)$ was designed to stabilize the plant, while $K_2(s)$ cannot stabilize the plant. In this experiment, K_2 is the on-line controller at first. Thus, the plant was not stabilized at early stage. After 2 seconds, the on-line controller was switched into K_1.

Comment: The bumpless transfer method in [6] does not include any initializing or state reset procedure at switching instants. Instead, it works allowing only controllers for which there exist state-space realizations such that share common C and D matrices; i.e.,

$$C_i = C_j \triangleq C \text{ and } D_i = D_j \triangleq D \text{ for all } i \neq j \quad (17)$$

where the matrices are as in (3). Note that this is not possible in general, unless all controllers have the same order and same D-matrics. Though our slow-fast method does not suffer this restriction on controllers, we have chosen for our simulation example controllers that do conform to this requirement in order be able to directly compare the two different methods in the same situation. ♢

Three simulation experiments were done. First, switching without any bumpless transfer method was performed. Next, for comparison one used the method of [6] and another used our slow-fast method based on Theorem 1. The upper part of Fig. 3 shows the controller output. The solid line (Cheong and Safonov's) of output $u(t)$ shows a smooth transient around the switching instant while the dashed line (Arehart and Wolovich's) shows a fast transient after switching. The dotted line indicates switching transient without bumpless transfer, which has extremely high peak value generated by

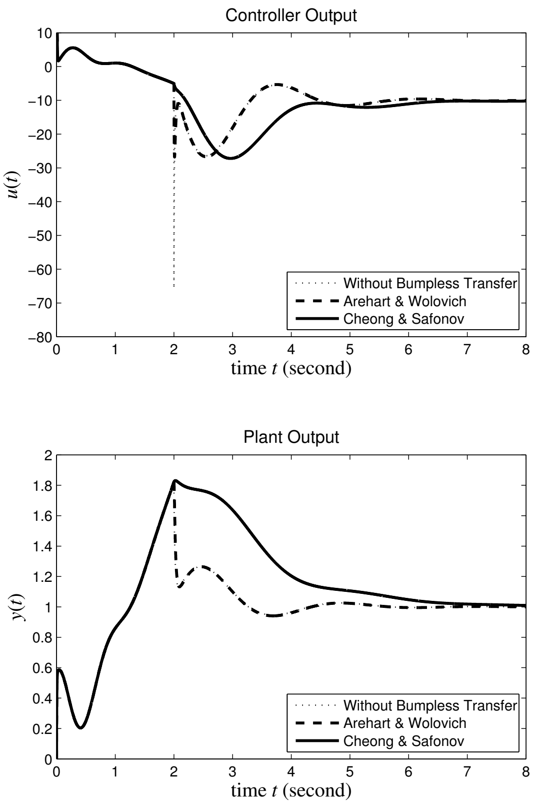

Fig. 3. Controller output $u(t)$ (upper figure); Plant output $y(t)$ (lower figure). Controller is switched at $t = 2$.

derivative controller. If ϵ approaches to zero, the peak value goes to infinity.

Fig. 4 shows $u(t)$ with the time axis magnified near the switching time. While the output without bumpless transfer has discontinuity at switching time 2 second, the outputs with bumpless transfer (dashed line and solid line) show continuous transient. Note that dashed line (Arehart and Wolovich's) satisfies Definition 1(a) which coincides with the definition of bumpless transfer used in [6]. However, comparing with the solid line, the dashed line exhibits a fast 'bumpy' transient after 2 second. It is excited by changing K_{fast}, which is clearly different result from our method.

The resultant plant output $y(t)$ shown in the lower part of Fig. 3 likewise exhibits an abrupt transient with the method of [6]. Evidently, both the control signal and the plant output in the case are significantly smoother with our slow-fast bumpless transfer method.

VI. Conclusion

In this paper, we have analyzed and improved the slow-fast controller decomposition bumpless transfer method in [8]. Our bumpless transfer imposes, in addition to the usual continuity demand, a requirement that there be no fast transients induced by controller switching. We extended the slow-fast bumpless transfer theory and method to 2-degree-of-freedom controllers and explained how to use the standard observable canonical form of the controllers to simplify practical implementation. Special cases that controllers have only slow modes or only fast modes were commented and possible solutions were provided. To clarify advantages of our bumpless transfer method, comparison with continuity based bumpless transfer and with non-bumpless transfer were performed. Simulation results demonstrate the effectiveness of our slow-fast bumpless transfer method in eliminating abrupt fast switching transients.

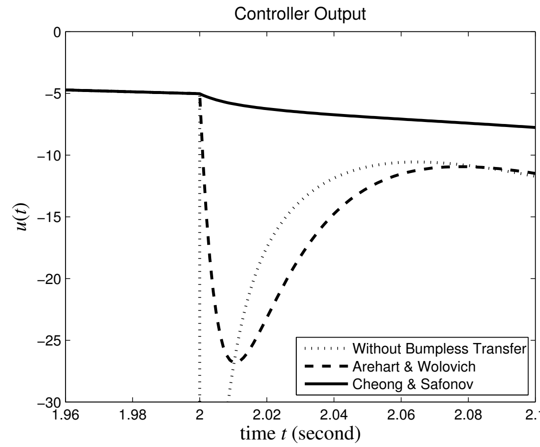

Fig. 4. Magnified $u(t)$ around the switching instant $(t = 2)$.

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