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On Robust Stability of Uncertain Neutral Systems with Discrete and Distributed Delays

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***Abstract*— This paper is concerned with the problem of robust stability of uncertain neutral systems with discrete and distributed delays. The uncertainties under consideration are assumed to be time-varying but norm bounded. By introducing a new form of Lyapunov-Krasovskii functional which contains some novel triple-integral terms, improved discrete-, distributed-, and neutral-delay-dependent stability conditions are obtained and formulated in terms of linear matrix inequality (LMI). Numerical examples are given to show that the proposed method is effective and leads to less conservative results than the existing ones.**

I. INTRODUCTION

Time delays are often encountered in many dynamic systems such as chemical or process control systems and networked control systems [1], [2]. Time delay is always one of the sources of instability and poor performance. The subject of analysis and synthesis of time-delay systems, thus, has attracted considerable attention during the past few years. Stability criteria for time-delay systems can be classified into two major categories, namely, delay-independent ones [3], [4] and delay-dependent ones [5], [6], [7], [8], [9], [10], [11], [12], [13]. Generally speaking, delay-dependent conditions are less conservative than delay-independent ones. So, many efforts have been paid to obtain less conservative delay-dependent conditions. An important index of measuring the conservativeness of the obtained conditions is the maximum upper bond on the delay. Finding some less conservative stability conditions motivates the present study.

Some practical applications can be modeled by neutral systems with distributed delays [14], [15]. So it is important both in theory and in application to study the stability of neutral systems with distributed delays. However, much fewer results have been proposed for the stability analysis of neutral systems with distributed delays compared with the rich results for neutral systems with only discrete delays [16], [17], [18], [19], [20]. Applying the discretized Lyapunov functional approach, stability conditions for systems with distributed delays were obtained in [21], [22], [23]. However, this method is difficult to be extended to deal with the synthesis problems. On the basis of the descriptor model transformation [7], [9] and the decomposition technique of discrete-delay term matrix, Han [24] put forward a stability test for neutral systems with discrete and distributed delays. Using a combination of the integral inequality technique and the descriptor model transformation, new delay-dependent stability conditions were proposed in [25]. Most recently, results in [24], [25] have been further improved in [26] where a modified Lyapunov-Krasovskii functional has been constructed and some free-weighting matrices [13], [18] have been introduced to reduce the conservativeness. However, there still exists room for further improvements.

In our previous work [27], a new form of Lyapunov-Krasovskii functional containing a triple-integral term was introduced to derive less conservative delay-dependent stability conditions for time-delay systems. In this paper, we extend this method to study the stability of neutral systems with distributed delays. Using some integral inequalities, improved discrete-, distributed-, and neutral-delay-dependent stability conditions are obtained without introducing any free-weighting matrices. Two numerical examples are presented to illustrate that our results are less conservative than the existing ones.

Notations: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes an n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$is the set of all $m \times n$ real matrices; $P > 0$ means that the matrix P is symmetric positive definite; I is an appropriately dimensional identity matrix.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following uncertain neutral system with discrete and distributed delays:

$$\begin{aligned} \dot{x}(t) - C(t)\dot{x}(t-\tau) &= A(t)x(t) + B(t)x(t-h) \\ &\quad + D(t)\int_{t-r}^{t} x(s)ds, \quad t > 0 \\ x(t) &= \phi(t), \quad t \in [-\rho,\ 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $\tau > 0$, $h > 0$ and $r > 0$ are constant neutral, discrete and distributed delay, respectively; $\rho = \max\{\tau,\ h,\ r\}$; The initial condition $\phi(t)$ is a continuously differentiable vector-valued function; $A(t)$, $B(t)$, $C(t)$ and $D(t)$ are uncertain matrices and can be

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described by

$$A(t) = A + \Delta A(t), \quad B(t) = B + \Delta B(t),$$
$$C(t) = C + \Delta C(t), \quad D(t) = D + \Delta D(t) \tag{2}$$

where A, B, C, D are known constant matrices; The admissible uncertainties are assumed to satisfy the following condition:

$$[\Delta A(t) \quad \Delta B(t) \quad \Delta C(t) \quad \Delta D(t)] = MF(t)\,[N_a \quad N_b \quad N_c \quad N_d] \tag{3}$$

where M, N_a, N_b, N_c and N_d are known constant matrices with appropriate dimensions and $F(t)$ is an unknown time-varying matrix satisfying:

$$F^{\mathrm{T}}(t)F(t) \le I, \quad \forall t \tag{4}$$

Throughout this paper, it is assumed that the matrix $C(t)$ is Schur stable.

The objective of this paper is to derive less conservative delay-dependent stability conditions in terms of LMI to ensure a larger maximum upper bound on the delay.

Before moving on, the following lemmas are introduced which play important roles in the development of the main results.

Lemma 1: For any constant matrix $Z = Z^{\mathrm{T}} > 0$ and a scalar $\tau > 0$ such that the following integrations are well defined, then

(1)

$$-\int_{t-\tau}^{t} \varrho^{\mathrm{T}}(s) Z \varrho(s) ds \le -\frac{1}{\tau} \int_{t-\tau}^{t} \varrho^{\mathrm{T}}(s) ds Z \int_{t-\tau}^{t} \varrho(s) ds$$

(2)

$$-\int_{-\tau}^{0}\int_{t+\theta}^{t} \varrho^{\mathrm{T}}(s) Z \varrho(s) ds d\theta \le -\frac{2}{\tau^2} \int_{-\tau}^{0}\int_{t+\theta}^{t} \varrho^{\mathrm{T}}(s) ds d\theta Z \int_{-\tau}^{0}\int_{t+\theta}^{t} \varrho(s) ds d\theta$$

Lemma 2: [28] For given matrices $Q = Q^{\mathrm{T}}$, M and N with appropriate dimensions, then

$$Q + MF(t)N + N^{\mathrm{T}}F^{\mathrm{T}}(t)M^{\mathrm{T}} < 0$$

for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$, if and only if there exists a scalar $\varepsilon > 0$, such that

$$Q + \varepsilon^{-1}MM^{\mathrm{T}} + \varepsilon N^{\mathrm{T}}N < 0$$

III. MAIN RESULTS

In this section, robust stability of system (1) is studied. Firstly, the following nominal system is considered.

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h) + D\int_{t-r}^{t} x(s)ds \tag{5}$$

The following theorem presents a stability condition for the nominal system (5).

Theorem 1: Given scalars $\tau > 0$, $h > 0$ and $r > 0$, the nominal neutral system (5) is asymptotically stable if there exist matrices $P = [P_{ij}]_{5\times5} > 0$, $Q = [Q_{ij}]_{2\times2} > 0$, $X = [X_{ij}]_{2\times2} > 0$, $R_i > 0$, $W_i > 0$, $S_i > 0$ ($i = 1, 2$) and $Z_j > 0$ ($j = 1, 2, 3$) with appropriate dimensions such that

$$\Xi + \Gamma_1^{\mathrm{T}} P \Gamma_2 + \Gamma_2^{\mathrm{T}} P \Gamma_1 + A_c^{\mathrm{T}} Y A_c < 0 \tag{6}$$

where

$$\Xi = \begin{bmatrix} \Xi_1 & \Xi_2 \\ \Xi_2^{\mathrm{T}} & \Xi_3 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} A & 0 & B & C & 0 & 0 & 0 & D & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 \end{bmatrix}$$

$$A_c = \begin{bmatrix} A & 0 & B & C & 0 & 0 & 0 & D & 0 \end{bmatrix}$$

$$\Xi_1 = \begin{bmatrix} \Xi_{11} & \frac{1}{\tau}R_2 & HB + \frac{1}{h}X_{22} \\ * & -\frac{1}{\tau}R_2 - Q_{11} & 0 \\ * & * & -W_1 - \frac{1}{h}X_{22} \end{bmatrix}$$

$$\Xi_2 = \begin{bmatrix} HC & 0 & \frac{1}{r}S_2 & \frac{2}{\tau}Z_1 & \Xi_{18} & \Xi_{19} \\ -Q_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{h}X_{12}^{\mathrm{T}} \end{bmatrix}$$

$$\Xi_3 = \mathrm{diag}\{-Q_{22}, -W_2, -\frac{1}{r}S_2, -\frac{1}{\tau}R_1 - \frac{2}{\tau^2}Z_1, -\frac{1}{r}S_1 - \frac{2}{r^2}Z_3, -\frac{1}{h}X_{11} - \frac{2}{h^2}Z_2\}$$

$$\Xi_{11} = Q_{11} + HA + A^{\mathrm{T}}H^{\mathrm{T}} + \tau R_1 + W_1 + hX_{11} + rS_1 - \frac{1}{\tau}R_2 - \frac{1}{h}X_{22} - \frac{1}{r}S_2 - 2Z_1 - 2Z_2 - 2Z_3$$

$$\Xi_{18} = HD + \frac{2}{r}Z_3$$

$$\Xi_{19} = \frac{2}{h}Z_2 - \frac{1}{h}X_{12}^{\mathrm{T}}$$

$$Y = Q_{22} + \tau R_2 + W_2 + hX_{22} + rS_2 + \frac{\tau^2}{2}Z_1 + \frac{h^2}{2}Z_2 + \frac{r^2}{2}Z_3$$

$$H = Q_{12} + hX_{12}$$

Proof: Choose a Lyapunov-Krasovskii functional candidate as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) \tag{7}$$

where

$$V_1(x_t) = \zeta^{\mathrm{T}}(t) P \zeta(t)$$

$$V_2(x_t) = \int_{t-\tau}^{t} \omega^{\mathrm{T}}(s) Q \omega(s) ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s) R_1 x(s) ds d\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_2 \dot{x}(s) ds d\theta$$

$$+\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)dsd\lambda d\theta$$

$$\begin{aligned}V_3(x_t) = &\int_{t-h}^{t}x^{\mathrm{T}}(s)W_1x(s)ds\\&+\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)W_2\dot{x}(s)ds\\&+\int_{-h}^{0}\int_{t+\theta}^{t}\omega^{\mathrm{T}}(s)X\omega(s)dsd\theta\\&+\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)dsd\lambda d\theta\end{aligned}$$

$$\begin{aligned}V_4(x_t) = &\int_{-r}^{0}\int_{t+\theta}^{t}x^{\mathrm{T}}(s)S_1x(s)dsd\theta\\&+\int_{-r}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)dsd\theta\\&+\int_{-r}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)Z_3\dot{x}(s)dsd\lambda d\theta\end{aligned}$$

where $\zeta^{\mathrm{T}}(t) = \Big[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau)\ x^{\mathrm{T}}(t-h)\ \int_{t-\tau}^{t}x^{\mathrm{T}}(s)ds\ \int_{t-r}^{t}x^{\mathrm{T}}(s)ds\Big]$, $\omega^{\mathrm{T}}(s) = \big[x^{\mathrm{T}}(s)\ \dot{x}^{\mathrm{T}}(s)\big]$.

Taking the time derivative of $V(x_t)$ along the trajectory of system (5) yields

$$\begin{aligned}\dot{V}(x_t) = &2\zeta^{\mathrm{T}}(t)P\dot{\zeta}(t) + \omega^{\mathrm{T}}(t)(Q+hX)\omega(t)\\&-\omega^{\mathrm{T}}(t-\tau)Q\omega(t-\tau) + x^{\mathrm{T}}(t)(\tau R_1 + W_1\\&+rS_1)x(t) + \dot{x}^{\mathrm{T}}(t)(W_2 + rS_2 + \tau R_2 + \frac{\tau^2}{2}Z_1\\&+\frac{h^2}{2}Z_2 + \frac{r^2}{2}Z_3)\dot{x}(t) - \int_{t-\tau}^{t}x^{\mathrm{T}}(s)R_1x(s)ds\\&-\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)ds - \int_{t-h}^{t}\omega^{\mathrm{T}}(s)X\omega(s)ds\\&-\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)dsd\theta\\&-\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)dsd\theta\\&-\int_{t-r}^{t}x^{\mathrm{T}}(s)S_1x(s)ds - \int_{t-r}^{t}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)ds\\&-\int_{-r}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_3\dot{x}(s)dsd\theta \qquad (8)\end{aligned}$$

Using Lemma 1 yields

$$-\int_{t-\tau}^{t}x^{\mathrm{T}}(s)R_1x(s)ds \le -\frac{1}{\tau}\int_{t-\tau}^{t}x^{\mathrm{T}}(s)ds\,R_1\int_{t-\tau}^{t}x(s)ds \qquad (9)$$

$$-\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)ds \le -\frac{1}{\tau}\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)ds\,R_2\int_{t-\tau}^{t}\dot{x}(s)ds \qquad (10)$$

$$-\int_{t-h}^{t}\omega^{\mathrm{T}}(s)X\omega(s)ds \le -\frac{1}{h}\int_{t-h}^{t}\omega^{\mathrm{T}}(s)ds\,X\int_{t-h}^{t}\omega(s)ds \qquad (11)$$

$$-\int_{t-r}^{t}x^{\mathrm{T}}(s)S_1x(s)ds \le -\frac{1}{r}\int_{t-r}^{t}x^{\mathrm{T}}(s)ds\,S_1\int_{t-r}^{t}x(s)ds \qquad (12)$$

$$-\int_{t-r}^{t}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)ds \le -\frac{1}{r}\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)ds\,S_2\int_{t-r}^{t}\dot{x}(s)ds \qquad (13)$$

$$\begin{aligned}&-\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)dsd\theta\\&\quad\le -\frac{2}{\tau^2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)dsd\theta Z_1\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta \qquad (14)\end{aligned}$$

$$\begin{aligned}&-\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)dsd\theta\\&\quad\le -\frac{2}{h^2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)dsd\theta Z_2\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta \qquad (15)\end{aligned}$$

$$\begin{aligned}&-\int_{-r}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)Z_3\dot{x}(s)dsd\theta\\&\quad\le -\frac{2}{r^2}\int_{-r}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)dsd\theta Z_3\int_{-r}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta \qquad (16)\end{aligned}$$

Substituting (9)-(16) into (8) yields

$$\dot{V}(t) \le \xi^{\mathrm{T}}(t)(\Xi + \Gamma_1^{\mathrm{T}}P\Gamma_2 + \Gamma_2^{\mathrm{T}}P\Gamma_1 + A_c^{\mathrm{T}}YA_c)\xi(t) \qquad (17)$$

where $\xi^{\mathrm{T}}(t) = \Big[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau)\ \ x^{\mathrm{T}}(t-h)\ \ \dot{x}^{\mathrm{T}}(t-\tau)\ \ \dot{x}^{\mathrm{T}}(t-h)\ \ x^{\mathrm{T}}(t-r)\ \ \int_{t-\tau}^{t}x^{\mathrm{T}}(s)ds\ \ \int_{t-r}^{t}x^{\mathrm{T}}(s)ds\ \ \int_{t-h}^{t}x^{\mathrm{T}}(s)ds\Big]$. Therefore, if $\Xi + \Gamma_1^{\mathrm{T}}P\Gamma_2 + \Gamma_2^{\mathrm{T}}P\Gamma_1 + A_c^{\mathrm{T}}YA_c < 0$, then $\dot{V}(x_t) < 0$ which guarantees system (5) is asymptotically stable. This completes the proof. ■

Remark 1: On the basis of Lyapunov-Krasovskii functional approach, a new discrete-, distributed-, and neutral-delay-dependent stability criterion is developed. It should be noted that two integral inequalities are used to derive Theorem 1 and no additional free-weighting matrices are introduced in the derivation except for Lyapunov matrices. So, the method proposed in this paper may have less decision variables than the well-known free-weighting matrix method.

Remark 2: The proposed augmented Lyapunov functional is more general than those in [18], [26]. In particular, our Lyapunov-Krasovskii functional contains some triple integral terms, that is, $\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)dsd\lambda d\theta$, $\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)dsd\lambda d\theta$ and $\int_{-r}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)Z_3\dot{x}(s)dsd\lambda d\theta$ which play key roles in the further reduction of conservativeness. Furthermore, we choose the augmented vector $\zeta(t)$ as $\zeta^{\mathrm{T}}(t) = \Big[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-\tau)\ x^{\mathrm{T}}(t-h)\ \int_{t-\tau}^{t}x^{\mathrm{T}}(s)ds\ \int_{t-r}^{t}x^{\mathrm{T}}(s)ds\Big]$. Through some numerical examples, we can see that each term in $\zeta(t)$ may contribute to the further reduction of the conservativeness, which means that if any terms are removed from $\zeta(t)$, a more conservative result will be obtained. So we think the proposed Lyapunov functional in this paper is different from existing ones and can lead to less conservative results.

Remark 3: If letting $P_{14} = 0$, $P_{24} = 0$, $P_{34} = 0$, $P_{15} = 0$, $P_{25} = 0$, $P_{35} = 0$, $P_{45} = 0$, $X_{12} = 0$, $P_{44} = \epsilon_1 I$, $P_{55} = \epsilon_2 I$, $R_1 = \epsilon_3 I$, $W_2 = \epsilon_4 I$, $X_{22} = \epsilon_5 I$, $Z_1 = \epsilon_6 I$, $Z_2 = \epsilon_7 I$, $Z_3 = \epsilon_8 I$ with ϵ_i $(i = 1, 2, \ldots, 8)$ being sufficiently small positive scalars in Theorem 1, a corollary can be directly obtained which is omitted here. Furthermore, following the similar line as in [29], it can be proved that this corollary is equivalent to Theorem 1 in [26] where the improvements over [24], [25] are demonstrated. So, results in [26] can be covered by Theorem 1 in this paper. This also proves theoretically that Theorem 1 is less conservative than the results in [26].

Consider the following neutral systems with mixed delays

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h) \tag{18}$$

A stability condition for such a system can be obtained on the basis of Theorem 1.

Corollary 1: Given scalars $\tau > 0$ and $h > 0$, neutral system (18) is asymptotically stable if exist $P = [P_{ij}]_{4\times 4} > 0$, $Q = [Q_{ij}]_{2\times 2} > 0$, $X = [X_{ij}]_{2\times 2} > 0$, $R_i > 0$, $W_i > 0$, $Z_i > 0$ $(i = 1, 2)$ with appropriate dimensions such that

$$\Theta + \Lambda_1^{\mathrm{T}} P \Lambda_2 + \Lambda_2^{\mathrm{T}} P \Lambda_1 + \hat{A}_c^{\mathrm{T}} \hat{Y} \hat{A}_c < 0 \tag{19}$$

where

$$\Theta = \begin{bmatrix} \Theta_1 & \Theta_2 \\ \Theta_2^{\mathrm{T}} & \Theta_3 \end{bmatrix}$$

$$\Lambda_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} A & 0 & B & C & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \\ I & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{A}_c = \begin{bmatrix} A & 0 & B & C & 0 & 0 & 0 \end{bmatrix}$$

$$\Theta_1 = \begin{bmatrix} \Theta_{11} & \frac{1}{\tau}R_2 & HB + \frac{1}{h}X_{22} \\ * & -\frac{1}{\tau}R_2 - Q_{11} & 0 \\ * & * & -W_1 - \frac{1}{h}X_{22} \end{bmatrix}$$

$$\Theta_2 = \begin{bmatrix} HC & 0 & \frac{2}{\tau}Z_1 & \frac{2}{h}Z_2 - \frac{1}{h}X_{12}^{\mathrm{T}} \\ -Q_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{h}X_{12}^{\mathrm{T}} \end{bmatrix}$$

$$\Theta_3 = \mathrm{diag}\{-Q_{22}, -W_2, -\frac{1}{\tau}R_1 - \frac{2}{\tau^2}Z_1, -\frac{1}{h}X_{11} - \frac{2}{h^2}Z_2,\}$$

$$\Theta_{11} = Q_{11} + HA + A^{\mathrm{T}}H^{\mathrm{T}} + \tau R_1 + W_1 + hX_{11} - \frac{1}{\tau}R_2 - \frac{1}{h}X_{22} - 2Z_1 - 2Z_2$$

$$\hat{Y} = Q_{22} + \tau R_2 + W_2 + hX_{22} + \frac{\tau^2}{2}Z_1 + \frac{h^2}{2}Z_2$$

$$H = Q_{12} + hX_{12}$$

Proof: Choose the Lyapunov-Krasovskii functional candidate as $V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)$ where $V_i(x_t)$ $(i = 1, 2, 3)$ are the same as those in Theorem 1 and follow the same line as in Theorem 1, and the proof will be completed. ∎

When consider the parameter uncertainties described by (2)-(4), the following theorem can be easily obtained on the basis of Theorem 1.

TABLE I

MAXIMUM UPPER BOUND ON r FOR $\tau = 0.1$ AND DIFFERENT h

h	0.1	0.5	1	1.5	1.6	1.7
[26]	6.64	5.55	1.62	—	—	—
Theorem 2	6.67	6.12	2.75	1.31	0.93	0.42

TABLE II

MAXIMUM UPPER BOUND ON h FOR $\tau = 0.1$ AND DIFFERENT r

r	1	2	3	4	5	6
[26]	1.12	0.93	0.77	0.65	0.55	0.43
Theorem 2	1.58	1.20	0.95	0.77	0.64	0.51

Theorem 2: Given a scalar $\tau > 0$, $h > 0$ and $r > 0$, neutral system (1) is robustly asymptotically stable if exists a scalar $\varepsilon > 0$ and $P = [P_{ij}]_{5\times 5} > 0$, $Q = [Q_{ij}]_{2\times 2} > 0$, $X = [X_{ij}]_{2\times 2} > 0$, $R_i > 0$, $W_i > 0$, $S_i > 0$ $(i = 1, 2)$ and $Z_j > 0$ $(j = 1, 2, 3)$ with appropriate dimensions such that

$$\begin{bmatrix} \Pi & A_c^{\mathrm{T}}Y & \Gamma_3^{\mathrm{T}}M \\ * & -Y & YM \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{20}$$

where

$$\Pi = \Xi + \Gamma_1^{\mathrm{T}} P \Gamma_2 + \Gamma_2^{\mathrm{T}} P \Gamma_1 + \varepsilon \Upsilon^{\mathrm{T}} \Upsilon$$

$$\Gamma_3 = [P_{11} + H \quad P_{12} \quad P_{13} \quad 0 \quad 0 \quad 0 \quad P_{14} \quad P_{15} \quad 0]$$

$$\Upsilon = [N_a \quad 0 \quad N_b \quad N_c \quad 0 \quad 0 \quad 0 \quad N_d \quad 0]$$

Proof: Replacing A, B, C and D in Theorem 1 with $A + MF(t)N_a$, $B + MF(t)N_b$, $C + MF(t)N_c$ and $D + MF(t)N_d$, respectively and using Lemma 2 completes the proof. ∎

IV. NUMERICAL EXAMPLES

In this section, two numerical examples are given to show that the proposed results are improvements over some existing ones.

Example 1: Consider the following uncertain neutral system:

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \quad D = \begin{bmatrix} -0.12 & -0.12 \\ -0.12 & 0.12 \end{bmatrix},$$

$$M = I, \quad N_a = N_b = N_c = N_d = 0.1I.$$

This system has been investigated in [26]. Assuming $\tau = 0.1$, the purpose is to calculate the maximum upper bound on r or h for different given h or r. To the best of authors' knowledge, results in [26] are the least conservative among the existing ones, so we compare our results with those in [26]. Table I lists the maximum upper bounds on r for different given h. It is seen from Table I that much larger value of r can be obtained using Theorem 2. In particular, when $h \geq 1.5$, method in [26] is unfeasible, while our results are 1.31, 0.93, and 0.42, respectively. Table II lists the maximum upper bounds on h for different given r. It can be seen that our results are less conservative than those in [26], that is, larger maximum upper bounds on h can be obtained by our method.

TABLE III

MAXIMUM UPPER BOUND ON h FOR DIFFERENT c AND r

Method	c	0.15	0.2	0.25	0.3
[26]	$r = 1$	0.95	0.82	0.70	0.59
Theorem 2	$r = 1$	0.97	0.84	0.72	0.60
[26]	$r = 0.8$	1.13	0.98	0.83	0.69
Theorem 2	$r = 0.8$	1.16	1.00	0.85	0.70
[26]	$r = 0.6$	1.41	1.22	1.02	0.83
Theorem 2	$r = 0.6$	1.49	1.28	1.06	0.86
[26]	$r = 0.4$	1.87	1.68	1.40	1.10
Theorem 2	$r = 0.4$	2.20	1.88	1.53	1.19
[26]	$r = 0.2$	2.75	2.69	2.44	1.98
Theorem 2	$r = 0.2$	8.30	6.54	4.59	2.80

Example 2: Consider the following uncertain neutral system with

$$A = \begin{bmatrix} -3.4 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1.1 & 0.1 \\ 0.1 & -1.2 \end{bmatrix},$$
$$C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & -0.2 \\ -0.1 & 0.3 \end{bmatrix}$$
$$M = I, \quad N_a = N_b = N_c = N_d = 0.2I.$$

Assume $\tau = 0.2$, the objective is to calculate the upper bound on h for different values of c and r. Table III lists the results together with those obtained using the method in [26]. Table III shows that the stability condition proposed in this paper yields less conservative results than those in [26].

V. CONCLUSIONS

In this paper, the stability of linear neutral systems with discrete and distributed delays has been investigated. New discrete-, distributed-, and neutral-delay-dependent criteria have been proposed. These criteria are derived based on the Lyapunov-Krasovskii approach and the integral inequality technique and are presented in terms of LMI. Due to the new construction of the introduced Lyapunov-Krasovskii functional, our results are less conservative than the existing ones. Two numerical examples have illustrated the effectiveness of the proposed method.

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