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Global Learning Controls for Uncertain Relative Degree One Linear Systems: a Comparative Study

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Abstract— **The output tracking problem of periodic reference signals (of known period) for single-input single-output observable minimum phase uncertain linear time-invariant systems with unitary relative degree is considered. A continuous global iterative learning control via output error feedback is designed which guarantees closed loop boundedness and asymptotic output tracking, thus improving the $\mathcal{L}_2$ convergence achieved in [6]. Its closed loop performances are then compared with those obtained by using the adaptive learning control in [2] and the adaptive regulator in [5]: in particular the effects of finite memory implementation, measurement noise and uncertainties in the period of the output reference signal are explicitly taken into account in the simulations.**

I. INTRODUCTION

The problem of tracking an output reference signal for single-input single-output uncertain linear systems on the basis of the output tracking error only constitutes an important and challenging control problem: when reference signals are periodic (of known period), learning control techniques (see for instance [7] and [9]) may be succesfully applied to reduce the output tracking error. An adaptive iterative learning control is designed in [6] for relative degree one systems with known high gain sign: only $\mathcal{L}_2$ output tracking is guaranteed. On the other hand, the global adaptive learning control proposed in [2] for observable minimum phase systems of any known relative degree and known high gain sign, by relying on a Fourier series expansion of the uncertain input reference signal, is able to achieve exponential convergence of both output tracking and input estimation errors to a residual set (containing the origin) whose size can be arbitrarily reduced. The estimation of a possibly large number of Fourier coefficients may be however required leading to a high order dynamic control. An analogous problem may arise when the global adaptive regulator recently designed in [5] for minimum phase observable systems of any known relative degree and known high gain sign is used to track periodic reference signals generated by possibly uncertain linear exosystems (and thus of possibly uncertain period).

The continuous global iterative learning control designed in this paper guarantees, for observable minimum phase systems of relative degree one and known high gain sign, asymptotic output tracking (which improves the $\mathcal{L}_2$ output tracking in [6]) without requiring any resetting procedure: since infinite memory is required to store the control input exerted in the preceding trial, a tracking error is to be expected when a finite memory implementation is used. Nevertheless the simple structure of the controller and the small number of design parameters to be tuned regardless of the nature of the periodic output reference signal (provided that its period is known) are definite advantages. The property of asymptotic output tracking and the continuity of the designed learning control allows us to compare its performances with those of the adaptive learning control in [2] and the adaptive regulator in [5]: the effects of finite memory implementation, measurement noise and uncertainties in the period of the output reference signal are explicitly evaluated.

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II. PROBLEM STATEMENT AND PRELIMINARY COMPUTATIONS

Consider the single-input single-output observable minimum phase uncertain linear time-invariant system with relative degree $\rho = 1$ [$\zeta \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the output to be controlled]

$$\begin{aligned} \dot{\zeta} &= F\zeta + gu \\ y &= h\zeta \end{aligned} \tag{1}$$

with transfer function [$s \in \mathbb{C}$, $b_i \in \mathbb{R}^+$, $a_i \in \mathbb{R}$, $1 \le i \le n$]

$$W(s) = \frac{b_1 s^{n-1} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n} \tag{2}$$

under the following assumptions: i) the sign of the high frequency gain b_1 is assumed to be known (positive without loss of generality); ii) the zeroes of the polynomial $\pi(s) = b_1 s^{n-1} + \ldots + b_n$ all belong to $\mathbb{C}^-$; iii) the uncertain vectors $a = [a_1, \ldots, a_n]^T$ and $b = [b_1, \ldots, b_n]^T$ belong to the closed ball $\overline{\mathcal{B}_{(0)}(a_M)} \subset \mathbb{R}^n$ with center the origin and known radius a_M and to the known compact set $\mathcal{S}_b = \{\xi = [\xi_1, \ldots, \xi_n]^T \in \mathbb{R}^n : 0 < b_{i,m} \le \xi_i \le b_{i,M}, 1 \le i \le n\} \subset \mathbb{R}^n$, respectively. Let $y_r(t) \in \mathcal{C}^2$ be the periodic reference signal for the output y which is assumed to satisfy

$$\begin{aligned} y_r(t+T) &= y_r(t) \quad \forall\, t \ge -T \\ |y_r^{(i)}(t)| &\le M_{y,i} \quad \forall\, t \in [0,T), \quad i = 0, 1 \end{aligned}$$

in terms of the known positive reals T, $M_{y,0}$, $M_{y,1}$.
According to Theorems 7.1.1 (and subsequent Remark 7.1.2 and proof of Theorem 7.1.2) in [4] [pgs. 282-285], system (1)

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is transformable by a linear change of coordinates $[\eta^T, y]^T = Q\zeta$ ($Q \in \mathcal{M}(n, \mathbb{R})$) into

$$\begin{aligned}
\dot\eta &= \begin{bmatrix} -\frac{b_2}{b_1} & 1 & 0 & \cdots & 0 \\ -\frac{b_3}{b_1} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{b_{n-1}}{b_1} & 0 & 0 & \cdots & 1 \\ -\frac{b_n}{b_1} & 0 & 0 & \cdots & 0 \end{bmatrix} \eta + \begin{bmatrix} \frac{b_3}{b_1} - \frac{b_2^2}{b_1^2} \\ \frac{b_4}{b_1} - \frac{b_3 b_2}{b_1^2} \\ \vdots \\ \frac{b_n}{b_1} - \frac{b_{n-1} b_2}{b_1^2} \\ -\frac{b_n b_2}{b_1^2} \end{bmatrix} y \\
&\quad + \begin{bmatrix} -a_2 + \frac{a_1 b_2}{b_1} \\ -a_3 + \frac{a_1 b_3}{b_1} \\ \vdots \\ -a_{n-1} + \frac{a_1 b_{n-1}}{b_1} \\ -a_n + \frac{a_1 b_n}{b_1} \end{bmatrix} y \doteq \Gamma\eta + \beta y \qquad (3) \\
\dot y &= \eta_1 + \left(\frac{b_2}{b_1} - a_1\right) y + b_1 u \doteq \eta_1 + \gamma y + b_1 u.
\end{aligned}$$

Define the reference subsystem [$\eta_r = [\eta_{r,1}, \ldots, \eta_{r,n-1}]^T \in \mathbb{R}^{n-1}$]

$$\dot{\eta_r}(t) = \Gamma\eta_r(t) + \beta y_r(t) \qquad (4)$$

with the initial value [$I \in \mathcal{M}(n-1, \mathbb{R})$ is the identity matrix]

$$\eta_r(0) = -\left(\mathrm{e}^{\Gamma T} - I\right)^{-1} \int_0^T \mathrm{e}^{\Gamma(T-\tau)} \beta y_r(\tau) d\tau. \qquad (5)$$

The following lemmas will be instrumental in proving the main result of the next section.

Lemma 1: *The signal $\eta_r(t)$ is periodic of period T, that is*

$$\eta_r(t+T) = \eta_r(t) \quad \forall\, t \geq -T.$$

Proof: For any $t \geq -T$ we have

$$\begin{aligned}
\eta_r(t) &= \int_0^t \mathrm{e}^{\Gamma(t-\tau)} \beta y_r(\tau) d\tau + \mathrm{e}^{\Gamma t} \eta_r(0) \\
\eta_r(t+T) &= \int_T^{T+t} \mathrm{e}^{\Gamma(t+T-\tau)} \beta y_r(\tau) d\tau + \mathrm{e}^{\Gamma t} \eta_r(T)
\end{aligned}$$

so that

$$\eta_r(t+T) - \eta_r(t) = \mathrm{e}^{\Gamma t} \left[\eta_r(T) - \eta_r(0)\right] = 0$$

owing to the periodicity of $y_r(t)$. To conclude the proof, it suffices to observe that the matrix $\left(\mathrm{e}^{\Gamma T} - I\right)$ is invertible since, according to assumption ii), the spectrum $\sigma(\Gamma)$ of the matrix Γ is a subset of $\mathbb{C}^-$. □

Lemma 2: *The signal $\eta_r(t)$ satisfies, for any $t \geq -T$, the following inequality:*

$$\|\eta_r(t)\| \leq \mathrm{e}^{\|\Gamma\| T} \|\beta\| M_{y,0} T \left[1 + \left\|\left(\mathrm{e}^{\Gamma T} - I\right)^{-1}\right\| \mathrm{e}^{\|\Gamma\| T}\right]$$

with

$$\left\|\left(\mathrm{e}^{\Gamma T} - I\right)^{-1}\right\| \leq \frac{1}{1 - \|\mathrm{e}^{\Gamma T}\|}$$

when $\|\mathrm{e}^{\Gamma T}\| < 1$.

Proof: The thesis is a straightforward consequence of Lemma 1 and of the following facts [$t \in [0, T)$]:

$$\begin{aligned}
\|\eta_r(t)\| &\leq \int_0^t \|\mathrm{e}^{\Gamma(t-\tau)}\| \|\beta\| M_{y,0} d\tau \\
&\quad + \|\mathrm{e}^{\Gamma t}\| \|\eta_r(0)\| \\
-\left(\mathrm{e}^{\Gamma T} - I\right)^{-1} &= I - \mathrm{e}^{\Gamma T}\left(\mathrm{e}^{\Gamma T} - I\right)^{-1}.
\end{aligned}$$
□

On the basis of Lemma 2, a known bound of $\|\eta_r(t)\|$ can be computed according to the definition of Γ and β in (3) and assumption iii), while Lemma 1 guarantees that the input reference signal

$$u_r(t) = \frac{1}{b_1}\left[\dot{y_r}(t) - \gamma y_r(t) - \eta_{r,1}(t)\right], \quad t \geq -T, \qquad (6)$$

which achieves perfect tracking (see [3])) when $\eta(0) = \eta_r(0)$ and $y(0) = y_r(0)$, is periodic of period T, that is

$$u_r(t+T) = u_r(t), \quad \forall\, t \geq -T.$$

Owing to the uncertainties on vectors a and b, the periodic signal $u_r(t)$ given by (6) is uncertain and it is to be reconstructed. However, since

$$|u_r(t)| \leq \frac{1}{b_{1,m}}\left[M_{y,1} + |\gamma| M_{y,0} + \|\eta_r(t)\|\right] \leq M_u$$

a known bound M_u of $|u_r(t)|$ can be computed according to the definition of γ in (3), Lemma 2 and assumption iii).

III. Iterative Learning Control

The main result of this section, which is stated in the following theorem, provides a continuous global iterative learning control via output error feedback ($\tilde y = y - y_r$) which, without requiring any resetting procedure, guarantees boundedness of all closed loop signals and asymptotic output tracking of the reference $y_r(t)$.

Theorem: *Consider system (1) under the assumptions i)-iii). For any initial condition $\zeta(0) \in \mathbb{R}^n$, boundedness of all closed loop signals and the asymptotic property*

$$\lim_{t\to\infty} \left[y(t) - y_r(t)\right] = 0$$

are guaranteed by the continuous iterative learning control

$$\begin{aligned}
u(t) &= -k\tilde y(t) + \hat u_r(t) \\
\hat u_r(t) &= \hat u_r(t-T) - \mu\varphi(t)\tilde y(t) \qquad (7) \\
\hat u_r(q) &= 0 \quad \forall\, q \leq 0
\end{aligned}$$

in which $\varphi(t)$ is a class $\mathcal{C}^1$ increasing function for $t \in [0, T]$ with $\varphi(0) = 0$ and $\varphi(T) = 1$, defined, for $t > T$, as

$$\varphi(t) = \begin{cases} 1 & \text{if } \mathcal{C}_{*1} \text{ or } \mathcal{C}_{*2} \text{ or } \mathcal{C}_{*3} \\ \left(1 - \frac{\mathrm{sat}(\hat u_r(t-T)) - M_u}{\delta}\right) & \text{if } \mathcal{C}_{*4} \text{ or } \mathcal{C}_{*5} \end{cases}$$

$$\begin{aligned}
\mathcal{C}_{*1} &: |\hat u_r(t-T)| \leq M_u \\
\mathcal{C}_{*2} &: \hat u_r(t-T) > M_u \text{ and } \tilde y(t) > 0 \\
\mathcal{C}_{*3} &: \hat u_r(t-T) < -M_u \text{ and } \tilde y(t) < 0 \\
\mathcal{C}_{*4} &: \hat u_r(t-T) > M_u \text{ and } \tilde y(t) \leq 0 \\
\mathcal{C}_{*5} &: \hat u_r(t-T) < -M_u \text{ and } \tilde y(t) \geq 0
\end{aligned}$$

$\text{sat}(q)$ *is the saturation function*

$$\text{sat}(q) = \begin{cases} |q| & \text{if } |q| \leq M_u + \delta \\ M_u + \delta & \text{otherwise} \end{cases}$$

k is a suitable positive control parameter while μ, δ are arbitrary positive reals. Furthermore, if the resulting $\hat{u}_r(t)$ is uniformly continuous for all $t \geq 0$, then

$$\lim_{t\to\infty} [u_r(t) - \hat{u}_r(t)] = 0.$$

Proof: Define the tracking and estimation errors

$$\begin{aligned} \tilde{\eta}(t) &= \eta(t) - \eta_r(t) = [\tilde{\eta}_1(t), \ldots, \tilde{\eta}_{n-1}(t)]^T \\ \tilde{u}_r(t) &= u_r(t) - \hat{u}_r(t) \end{aligned}$$

so that by (3), (4), (6) and (7) we obtain $[\tilde{y}(t) = y(t) - y_r(t)]$

$$\begin{aligned} \dot{\tilde{y}}(t) &= \tilde{\eta}_1(t) + \gamma\tilde{y}(t) - b_1 k\tilde{y}(t) - b_1\tilde{u}_r(t) \\ \dot{\tilde{\eta}}(t) &= \Gamma\tilde{\eta}(t) + \beta\tilde{y}(t). \end{aligned} \quad (8)$$

Note that, according to the definition of $\varphi(t)$ (see [8]) and $\hat{u}_r(t)$ in (7), the control signal $u(t)$ in (7) is a continuous function.

Consider the quadratic function

$$V_\eta(\tilde{\eta}) = \tilde{\eta}^T P_\eta \tilde{\eta} \quad (9)$$

in which $P_\eta \in \mathcal{M}(n-1, \mathbb{R})$ is the positive definite symmetric solution of the Lyapunov equation

$$P_\eta \Gamma + \Gamma^T P_\eta = -I.$$

The time derivative of function V_η along the trajectories of the closed loop system satisfies

$$\begin{aligned} \dot{V}_\eta(t) &\leq -\|\tilde{\eta}(t)\|^2 + 2\|P_\eta\|\|\beta\|\|\tilde{\eta}(t)\||\tilde{y}(t)| \\ &\doteq -\|\tilde{\eta}(t)\|^2 + N_\eta\|\tilde{\eta}(t)\||\tilde{y}(t)|. \end{aligned} \quad (10)$$

Following the ideas in [1], we consider the function

$$V(t) = V_\eta(\tilde{\eta}(t)) + \frac{1}{2b_1}\tilde{y}^2(t) + \frac{1}{2\mu}\int_{t-T}^{t} \tilde{u}_r^2(\tau)d\tau \quad (11)$$

whose time derivative along the trajectories of the closed loop system satisfies

$$\begin{aligned} \dot{V}(t) \leq\; & -\|\tilde{\eta}(t)\|^2 + \tilde{N}_\eta\|\tilde{\eta}(t)\||\tilde{y}(t)| \\ & -\left(k - \frac{|\gamma|}{b_1}\right)\tilde{y}^2(t) - \tilde{u}_r(t)\tilde{y}(t) \\ & +\frac{1}{2\mu}\tilde{u}_r^2(t) - \frac{1}{2\mu}\tilde{u}_r^2(t-T) \\ =\; & -\|\tilde{\eta}(t)\|^2 + \tilde{N}_\eta\|\tilde{\eta}(t)\||\tilde{y}(t)| \\ & -\left(k - \frac{|\gamma|}{b_1}\right)\tilde{y}^2(t) - \tilde{u}_r(t)\tilde{y}(t) \\ & +\frac{1}{2\mu}\tilde{u}_r^2(t) - \frac{1}{2\mu}\left[\tilde{u}_r(t) - \mu\varphi(t)\tilde{y}(t)\right]^2 \\ \leq\; & -\frac{\|\tilde{\eta}(t)\|^2}{2} - (k - M_\eta)\tilde{y}^2(t) \\ & + [\varphi(t) - 1]\,\tilde{u}_r(t)\tilde{y}(t) \end{aligned} \quad (12)$$

with $\tilde{N}_\eta = N_\eta + \frac{1}{b_1}$ and M_η is a known positive bound of $\left(\frac{\tilde{N}_\eta^2}{2} + \frac{|\gamma|}{b_1}\right)$ according to assumption iii). Choose

$$\begin{aligned} k &= k_y + k_m \\ k_m &\geq M_\eta \end{aligned}$$

so that, according to (7), we obtain

$$\begin{aligned} \dot{V}(t) \leq\; & -\frac{\|\tilde{\eta}(t)\|^2}{2} - k_y\tilde{y}^2(t) \\ & + [\varphi(t) - 1]\,[u_r(t) - \hat{u}_r(t-T)]\,\tilde{y}(t) \\ & +\mu\,[\varphi(t) - 1]\,\varphi(t)\tilde{y}^2(t) \\ \leq\; & -\frac{\|\tilde{\eta}(t)\|^2}{2} - k_y\tilde{y}^2(t) \\ & + [\varphi(t) - 1]\,[u_r(t) - \hat{u}_r(t-T)]\,\tilde{y}(t). \end{aligned} \quad (13)$$

The definition of $\varphi(t)$ guarantees that for $t > T$

$$\dot{V}(t) \leq -\frac{\|\tilde{\eta}(t)\|^2}{2} - k_y\tilde{y}^2(t) \quad (14)$$

while for $t \in [0, T]$, from (13) we can write

$$\dot{V}(t) \leq -\frac{\|\tilde{\eta}(t)\|^2}{2} - k_y\tilde{y}^2(t) + \frac{M_u^2}{2k_y}. \quad (15)$$

Accordingly, $\tilde{y}(t)$ and $\tilde{\eta}(t)$ are bounded for all $t \geq 0$ so that (7) guarantees that $\hat{u}_r(t)$ is bounded for all $t \geq 0$ (and therefore $u(t)$ is bounded for all $t \geq 0$). Since, according to (8), $\dot{\tilde{y}}(t)$ and $\dot{\tilde{\eta}}(t)$ are bounded for all $t \geq 0$, $\tilde{y}^2(t)$ and $\|\tilde{\eta}(t)\|^2$ are uniformly continuous for all $t \geq T$ and therefore, according to (14), by Barbalat Lemma

$$\lim_{t\to\infty} \left[\tilde{y}^2(t) + \|\tilde{\eta}(t)\|^2\right] = 0 \quad (16)$$

which implies

$$\lim_{t\to\infty} [\tilde{y}(t)] = 0.$$

If the resulting $\hat{u}_r(t)$ is uniformly continuous for all $t \geq 0$, then, according to (8), $\dot{\tilde{y}}(t)$ is uniformly continuous for all $t \geq 0$ and therefore, by Barbalat Lemma,

$$\lim_{t\to\infty} \left[\dot{\tilde{y}}(t)\right] = 0$$

which, according to (8), implies

$$\lim_{t\to\infty} [u_r(t) - \hat{u}_r(t)] = 0.$$

Remark 1: The control algorithm (7) incorporates a data storage mechanism and formally requires infinite memory. In real applications, a finite-dimensional realization (for instance Padé realization) of the delay operator may be required. When the delay operator is approximated by a linear finite-dimensional system and $\varphi \equiv 1$, the control algorithm (7) reduces to a linear finite-dimensional system.

IV. Adaptive Learning Control and Adaptive Regulator

A. Adaptive Learning Control

The adaptive learning control law presented in [2] [c_1 is a suitable positive control parameter, c_2 is an arbitrary positive real, $p > 1$ is an odd integer, $i = 1, ..., \frac{(p-1)}{2}$]

$$\begin{aligned}
u(t) &= -c_1\tilde{y}(t) - \Phi^T(t)\hat{\theta}(t) \doteq -c_1\tilde{y}(t) + \hat{u}_r(t) \\
\dot{\hat{\theta}}(t) &= c_2\Phi(t)\tilde{y}(t) \\
\Phi(t) &= [\Phi_1(t), \ldots, \Phi_p(t)]^T \\
\Phi_1(t) &= 1 \\
\Phi_{2i}(t) &= \sqrt{2}\sin\left(\frac{2\pi i}{T}t\right) \\
\Phi_{2i+1}(t) &= \sqrt{2}\cos\left(\frac{2\pi i}{T}t\right)
\end{aligned} \tag{17}$$

does not require previously stored data and relies on a Fourier series expansion (with Fourier coefficients θ_i) of the uncertain input reference $u_r(t)$. Under persistency of excitation, the output tracking error is guaranteed to converge to a residual set (containing the origin) which can be arbitrarily reduced by increasing the number of the estimated Fourier coefficients $\hat{\theta}_i(t)$, while exponential output tracking along with uncertain input estimation may be achieved in the case of finite Fourier series expansion of $u_r(t)$.

B. Adaptive Regulator

The adaptive regulator designed in [5] [k_r, r_{Ω_2} are suitable positive control parameter, λ, ϵ_r are arbitrary positive reals, $\bar{m} \in \mathbb{N}$, $\bar{I}$ is the identity matrix in $\mathcal{M}(2\bar{m}+1, \mathbb{R})$, E_j is a vector of suitable dimension with all zero entries excepting for the j-th unitary element ($3 \le j \le 2\bar{m}+1$), D is a Hurwitz matrix, $\text{grad}(\cdot)$ is the gradient vector]

$$\begin{aligned}
u(t) &= -k_r\tilde{y}(t) - \hat{\chi}_1(t) - \nu(t)^T\hat{\vartheta}(t) \\
&\doteq -k_r\tilde{y}(t) + \hat{u}_r(t) \\
\dot{\hat{\chi}}(t) &= D\hat{\chi}(t) - \bar{d}u(t) \\
\dot{\xi}_i(t) &= D\xi_i(t) + [0, \bar{I}]E_{2i+1}u(t) \\
\nu_i(t) &= [1, 0, \ldots, 0]\xi_i(t), \quad 1 \le i \le \bar{m} \\
\dot{\hat{\vartheta}}(t) &= \lambda \text{Proj}[\nu(t)\tilde{y}(t), \hat{\vartheta}(t)] \\
\hat{\chi}(t) &= [\hat{\chi}_1(t), \ldots, \hat{\chi}_{2\bar{m}+1}(t)]^T \\
\nu(t) &= [\nu_1(t), \ldots, \nu_{\bar{m}}(t)]^T \\
D &= \begin{bmatrix} -d_2 & 1 & 0 & \cdots & 0 \\ -d_3 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -d_{2\bar{m}+1} & 0 & 0 & \cdots & 1 \\ -d_{2\bar{m}+2} & 0 & 0 & \cdots & 0 \end{bmatrix} \\
\bar{d} &= [d_2, \ldots, d_{2\bar{m}+2}]^T \\
\text{Proj}[\psi, \hat{\vartheta}] &= \psi \quad \text{if } p_r(\hat{\vartheta}) \le 0 \\
\text{Proj}[\psi, \hat{\vartheta}] &= \psi \quad \text{if } p_r(\hat{\vartheta}) > 0 \text{ and } \langle \text{grad}[p_r(\hat{\vartheta})], \psi \rangle \le 0 \\
\text{Proj}[\psi, \hat{\vartheta}] &= \Xi \quad \text{if } p_r(\hat{\vartheta}) > 0 \text{ and } \langle \text{grad}[p_r(\hat{\vartheta})], \psi \rangle > 0 \\
\Xi &= \left[I - \frac{p_r(\hat{\vartheta})\text{grad}[p_r(\hat{\vartheta})]\text{grad}[p_r(\hat{\vartheta})]^T}{\|\text{grad}[p_r(\hat{\vartheta})]\|^2}\right]\psi \\
p_r(\hat{\vartheta}) &= \frac{\|\text{grad}[p_r(\hat{\vartheta})]\|^2 - r_{\Omega_2}^2}{\epsilon_r^2 + 2\epsilon_r r_{\Omega_2}}
\end{aligned} \tag{18}$$

achieves output tracking of reference signals and/or rejection of disturbances generated by a linear uncertain exosystem: asymptotic regulation is guaranteed when the exosystem is overmodeled, while both exponential regulation and estimation of the uncertain excited frequencies are obtained when the exosystem is exactly modeled. Robustness with respect to unmodeled exosystem dynamics is however guaranteed: the output tracking error is exponentially reduced to a residual set (containing the origin) whose size decreases as the order of the unmodeled dynamics decreases.

Remark 2: The adaptive regulator (18) relies on an estimate $\hat{\vartheta}(t)$ of the uncertain exosystem parameters grouped in the vector ϑ; in the case of periodic references the control algorithm (18) shows a linear parameterization of the uncertain period (not required to be known by the controller (18)), so that when the exosystem is known and $\hat{\vartheta} \equiv \vartheta$ the controller (18) reduces to a linear one.

V. Simulation Results

We consider the single-input single-output observable minimum phase uncertain linear time-invariant system with known relative degree $\rho = 1$ (see [2])

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) + bu(t) + a_1 y(t) \\
\dot{x}_2(t) &= bu(t) + a_2 y(t) \\
y(t) &= x_1(t)
\end{aligned}$$

with zero initial conditions and uncertain parameters $a_1 = -2.5$, $a_2 = -0.5$, $b = 2$ belonging to the known compact sets $[-3, 3]$, $[-4, 4]$, $[1, 3]$, respectively. The control algorithms (7), (17) and (18) are tested with control parameters: $k = 40$, $\mu = 60$, $\delta = 1$, $M_u = 6$, $\varphi(t) = \frac{t^3}{T^3}$ ($0 \le t \le T$), $c_1 = 22.13$, $c_2 = 2$, $p = 7$, $k_r = 2.3$, $\lambda = 110$, $\bar{d} = [7, 21, 35, 35, 21, 7, 1]^T$, $r_{\Omega_2} = 2$, $\epsilon_r = 1$, $\bar{m} = 3$. A tenth order Padé realization of the delay operator is used for the implementation of the iterative learning control (7). The Bode diagrams of the closed loop system with the controllers (7) and (17) (which are linear with $\varphi \equiv 1$ and $\hat{\vartheta} \equiv \vartheta$) are reported in Fig. 1.

A. Case 1

The class $\mathcal{C}^2$ output reference signal (see Fig. 2(a)) [$p_0 = -0.5$, $p_1 = 1.0$, $\bar{p}_1 = 0.2$, $p_2 = -1.5$, $\bar{p}_2 = 1.7$, $p_3 = 2.3$, $\bar{p}_3 = -0.8$, $q_0 = -1.7$, $q_1 = -6.3$, $\bar{q}_1 = 0.2$, $q_2 = -1.5$, $\bar{q}_2 = 1.7$, $q_3 = 2.3$, $\bar{q}_3 = -0.8$]

$$y_r(t) = \begin{cases} p_0 + \sum_{i=1}^3 (p_i \sin(\omega_i t) + \bar{p}_i \sin(\omega_i t)) & \text{if } t \le 4T \\ q_0 + \sum_{i=1}^3 (q_i \sin(\varpi_i t) + \bar{q}_i \sin(\varpi_i t)) & \text{if } t > 4T \end{cases}$$

with $\omega_i = 2\pi T^{-1}$ and $\varpi_i = 2\pi(T + \Delta T)^{-1}$ ($i = 1, 2, 3$) is periodic of period $T = 20$ s (which is known to the controllers (7) and (17) and uncertain to the controller (18)) for $t \leq 80$ s and periodic of period $T + \Delta T = 18$ s (which is unknown to the controllers (7) and (17) and uncertain for the controller (18)) for $t > 80$ s. In this case, the uncertain input reference signal $u_r(t)$ is generated by a third order exosystem and admits a finite Fourier series expansion so that exponential output tracking and uncertain input estimation can be achieved by the controller (17) when the period T is known and by the controller (18) even in the case of uncertain period T. Figures 2(b)-2(d) show the output tracking error $\tilde{y}(t)$ while the uncertain input reference signal $u_r(t)$ along with the corresponding estimation errors are reported in Figs. 3(a)-3(d). While satisfactory performances are achieved by the control algorithms (7) and (17), only the adaptive regulator (18) is able to guarantee exponential output tracking despite period T perturbations. The estimates $\hat{\theta}_i(t)$ of the Fourier coefficients θ_i of the uncertain input reference signal $u_r(t)$ provided by the controller (17) are reported in Fig. 4(a): while parameter identification is obtained when the period T is known, estimation errors appear when period perturbation occurs. On the other hand, the adaptive regulator (18), which does not require the exosystem knowledge, achieves good estimation of the uncertain exosystem parameter vector ϑ (see Fig. 4(b)) even in the perturbed case.

B. Case 2

The class $\mathcal{C}^2$ (square wave-type) output reference signal (see Fig. 5(a)) is periodic of period $T = 20$ s (which is known to the controllers (7) and (17) and uncertain to the controller (18)). In this case, the adaptive learning control (17) (with $p = 7$) does not incorporate a sufficient number of Fourier coefficient estimates while the adaptive regulator (18) (with $\bar{m} = 3$) undermodels the exosystem, so that non-zero steady-state output tracking errors appear (see Figs. 5(c)-5(d)). On the other hand, the iterative learning control (7) is able to achieve satisfactory output tracking (see Fig. 5(b)). The good performances obtained by the iterative learning control (7) in comparison with the control algorithms (17) and (18) can be still guaranteed even when white noise (see Fig. 6(a)) affects the output tracking error measurement: as Figs. 6(b)-6(d) show, a steady-state output traking error of about 10% appears.

VI. Conclusions

A solution to the output tracking problem for the relative degree one system (1) has been presented: the continuous global iterative learning control (7) guarantees, without requiring any resetting procedure, asymptotic output tracking of periodic reference signals (of known period) along with closed loop boundedness. The effects of finite memory implementation, measurement noise and uncertainties in the period of the output reference signal have been evaluated by a comparative study with respect to the adaptive learning control in [2] and to the adaptive regulator in [5]. Satisfactory performances are obtained, which, along with the simple structure of the controller and the small number of design

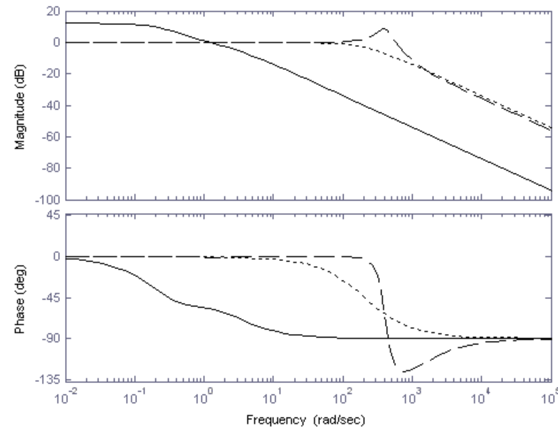

Fig. 1. Bode diagrams: uncontrolled system (solid); closed loop system with the iterative learning control (dash); closed loop system with the adaptive regulator (dot).

parameters to be tuned make the proposed solution suitable for applications.

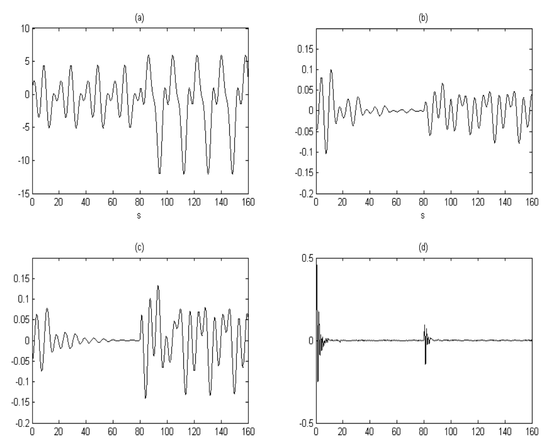

Fig. 2. (a) Output reference signal $y_r(t)$. Output tracking error $\tilde{y}(t)$: (b) iterative learning control; (c) adaptive learning control; (d) adaptive regulator.

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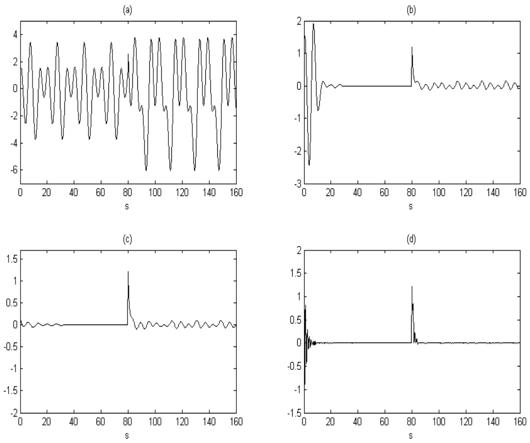

Fig. 3. (a) Uncertain open loop input reference $u_r(t)$. Estimation error $\tilde{u}_r(t) = u_r(t) - \hat{u}_r(t)$: (b) iterative learning control; (c) adaptive learning control; (d) adaptive regulator.

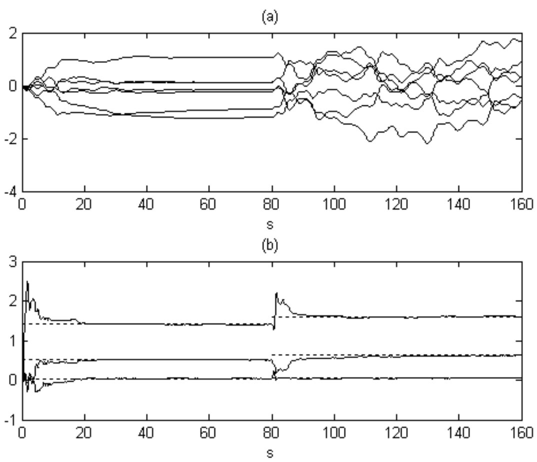

Fig. 4. (a) Estimates of the uncertain input Fourier coefficients (adaptive learning control); (b) Estimates (solid) of the uncertain exosystem parameters (dot) (adaptive regulator).

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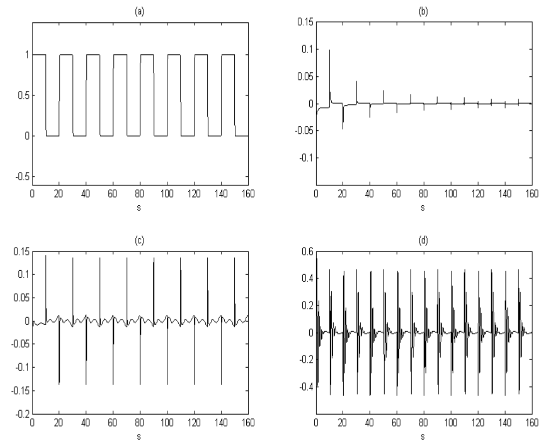

Fig. 5. (a) Output reference signal $y_r(t)$. Output tracking error $\tilde{y}(t)$: (b) iterative learning control; (c) adaptive learning control; (d) adaptive regulator.

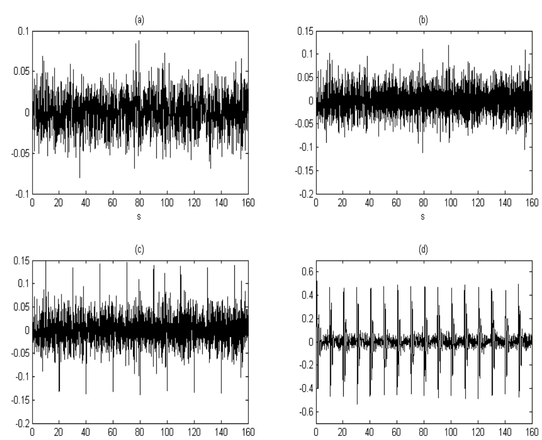

Fig. 6. (a) White noise affecting $\tilde{y}(t)$. Output tracking error $\tilde{y}(t)$: (b) iterative learning control; (c) adaptive learning control; (d) adaptive regulator.