

Control of Ironless Permanent Magnet Linear Synchronous Motor Using Fast Terminal Sliding Mode Control with Iterative Learning Control

Yiqiang Li, Yaobin Chen, and Huixing Zhou

Abstract—In this paper, a novel controller is proposed to improve the trajectory tracking performance of a direct drive ironless permanent magnet linear synchronous motor (ILPMLSM). The proposed control scheme combines fast terminal sliding mode control scheme with iterative learning control. Specifically, the fast terminal sliding mode controller which can predefine the finite reaching time is used as the primary controller to handle the effect of parametric uncertainties, unknown nonlinearities and external disturbances. A PD type iterative learning controller is employed as the secondary controller to eliminate the periodic tracking errors. Computer simulation results illustrate that the proposed combined controller can achieve better tracking performance and robustness compared with other control algorithms such as PID control, sliding mode control, and the iterative learning control.

Keywords: permanent magnet linear motor, motion control, force ripple, fast terminal sliding mode control, iterative learning control.

I. INTRODUCTION

DIRECT-drive permanent magnet linear synchronous motors (PMLSM) have many advantages such as transmission-free mechanical structure and achievable high force-to-inertia ratio. One of the major downsides of the PMLSM servo system is the force ripple caused by factors such as payload variation, unknown nonlinearities, magnetic flux harmonics, asymmetric phase windings, resistance variation, the end effect associated with the finite length of the mover, and disturbances. Although the proper motor design could effectively reduce the force ripple, they would not meet the high accuracy requirements. Thus, many control techniques have been developed to achieve the potential high performance [1].

Authors in [2, 3] presented an adaptive robust control (ARC) scheme for high speed and high accuracy motion control, where on-line parameter adaptation and certain robust control laws were used to reduce the effect of various

parameter uncertainties and handle the uncompensated uncertain nonlinearities. In [4] a non-linear PID controller has been proposed for the position control of a linear ultrasonic motor. In [5], a force ripple model of a linear motor was developed at first, and then a PID controller combined with adaptive feed forward control algorithm was designed to compensate the force ripple. According to the characteristic of periodic motion, in [6] an adaptive learning control algorithm was developed to improve the trajectory tracking performance of PMLSM. A sliding mode control together with the iterative learning control scheme was presented in [7] to enhance the trajectory tracking of linear ultrasonic motor. H_∞ optimal feedback control was applied in [8] to provide high dynamic stiffness to external disturbances. However, the adaptive technique based control algorithms may become too complicated to be implemented in case of different run phases, particularly when there exists force ripple. The PID based control methods cannot provide satisfactory control performance with system parameter variations. The accurate model is required in feed-forward and H_∞ control methods. The iterative learning control alone may not guarantee the robustness. Although the conventional linear type sliding mode control can provide asymptotic stability, the reaching time which is infinite theoretically is very difficult to meet the requirement of high speed motion.

In this paper, a novel controller is proposed by combining fast terminal sliding mode control (FTSMC) [9] with iterative learning control (ILC) for high performance trajectory tracking control of an ILPMLSM motor. We choose the FTSMC controller which can predefine the finite reaching time as the primary controller to handle the effect of parametric uncertainties, unknown nonlinearities and external disturbances. And then a PD-type ILC controller is employed as the secondary controller to eliminate the periodic tracking errors. Various simulation results demonstrate the effectiveness and robustness of the proposed control scheme.

The remainder of this paper is organized as follows. Section II presents the structure and dynamic model of ironless linear motor. The design and robust analysis of the fast terminal sliding mode controller are shown in Section III. Section IV presents the convergence analysis for the iterative learning control. The results of the computer simulations are given in Section V. Finally, conclusions are

Manuscript received September 22, 2008. This work was supported in part by the China Scholarship Council (CSC 2007103188).

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given in Section VI.

II. DYNAMIC MODEL OF ILPMLSM

The ILPMLSM motor, as depicted in Fig.1, consists of a face-to-face permanent magnet stator, a translator (or called mover) composed of a specific number of coils, and a Hall effect sensor mounted on the translator which being used to detect the polarity of the stator for electronic commutation. Similar to permanent magnet rotary motors, the thrust (named torque in rotary motor) is generated by the interaction between permanent magnetic field and travelling magnetic field, while the synchronous speed of the motor is the same as the speed of the travelling magnetic field [10].

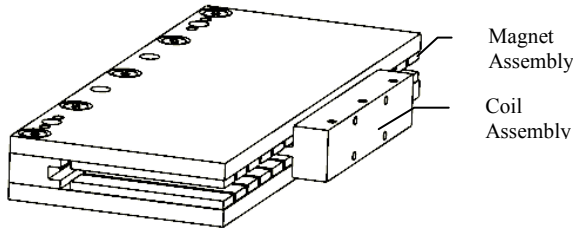


Fig. 1. An ILPMLSM physical model

In this work, we focus on the trajectory tracking motion of the ILPMLSM servo system. The dynamic model can be expressed by the following compact form,

$$\begin{aligned} \dot{x}(t) &= g(x) + B(x)u(t) \\ y &= Cx \end{aligned} \quad (1)$$

where $x = [x_1, x_2]^T$, $B(x) = [0, \frac{K_F}{R}]^T$, $C = [1, 0]$,

$$g(x) = \begin{pmatrix} x_2 \\ \frac{1}{M} \left[-\left(\frac{K_F K_E}{R} + B \right) x_2 + f_{fric}(x_2) + f_{dis}(t, x_1, x_2) \right] \end{pmatrix}.$$

here M is the total mass of the inertia load and the translator, defining the position velocity as the state variables $x = [x_1, x_2]^T$, B is the coefficient of the damping and viscous friction on the load, R is the armature resistance, K_E represents the electromotive force coefficient, K_{F0} is the average force constant, K_{Fr} is the coefficient of the end effect force, denoting $K_F = K_{F0} + K_{Fr}$. Let f_{fric} be the combination of the static friction and Coulomb friction, and f_{dis} represents the external disturbance and u is the control input.

In addition, the friction model can be written as:

$$f_{fric}(x_2) = -[f_c + (f_s - f_c)e^{-|x_2/\dot{x}_s|^\varepsilon}] \text{sgn}(x_2) \quad (2)$$

where f_c is the minimum level of the Coulomb friction, f_s represents the level of static friction, \dot{x}_s and ε are empirical parameters used to describe the Stribeck effect [2].

Furthermore, we define the trajectory tracking error $e(t)$

$$\text{as: } e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - x_{1d}(t) \\ x_2(t) - x_{2d}(t) \end{bmatrix} = x(t) - x_d(t)$$

(3)

where x_d is the desired state trajectory. Accordingly, we can obtain the following error dynamic equations.

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} e_2 \\ h(e) \end{pmatrix} + \begin{pmatrix} 0 \\ b(e) \end{pmatrix} u(t) \quad (4)$$

Where $h(e) = \frac{1}{M} \left[-\left(\frac{K_F K_E}{R} + B \right) x_2 + f_{fric}(x_2) + f_{dis}(t, x_1, x_2) \right] - \dot{x}_{2d}$,

$$b(e) = \frac{K_F}{R}.$$

III. FAST TERMINAL SLIDING MODE CONTROLLER DESIGN

The FTSMC control combines linear manifolds with nonlinear ones, where the reaching time can be dramatically shorten [11-13]. In this paper, we propose a control structure that combines the FTSMC (primary controller) with an iterative learning control (secondary controller), as depicted in Fig. 2. The detailed description of this control structure will be given in the following sections.

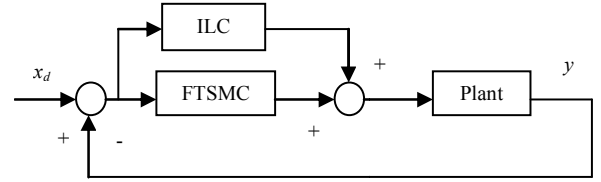


Fig. 2. Block diagram of the proposed control scheme.

A. Fast Terminal Sliding Mode Concept

The fast terminal sliding mode concept is described as

$$s = \dot{z} + \alpha z + \beta z^{q/p} = 0 \quad (5)$$

where z is a scalar variable, $\alpha, \beta > 0$ are constants and p, q ($p > q$) are odd positive integers.

In (5), the scalar z can be one of the state variables. The linear part $\dot{z} = -\alpha z$ has the faster convergence rate than that of the nonlinear counterpart when the state variable is far away from the zero. It works as the main attractor. When close to zero, the nonlinear part $\dot{z} = -\beta z^{q/p}$ speeding up the convergence rate acts as a terminal attractor. The following lemma shows the exact time of reaching the equilibrium while keeping the system stable.

Lemma 1. A scalar variable will be globally asymptotically stable once entering the fast terminal sliding manifolds; Furthermore, it will take the variable to reach its equilibrium in finite time.

Proof. First, we prove the stability. Let's select $V = \frac{1}{2} x_1^2$ as the Lyapunov function candidate, where x_1 is a scalar variable. Obviously, V is positive definite function, from Eq. (5) in which z replaced by x_1 , we have,

$$\dot{V} = x_1 \dot{x}_1 = x_1(-\alpha x_1 - \beta x_1^{q/p}) = -(\alpha x_1^2 + \beta x_1^{(q+p)/p}) \leq 0$$

If and only if $x_1 = 0$, then $\dot{V} = 0$. Therefore, the variable is globally asymptotically stable in the sense of Lyapunov theory.

We now show the equilibrium of the state can be reached in finite time. When the scalar variable stays on the fast terminal sliding hyperplane, given the initial state variable $x_1(0)$ by solving (5), the exact time to reach the equilibrium (or zero) is,

$$t_s = \frac{P}{\alpha(p-q)} (\ln(\alpha x_1(0)^{(p-q)/p} + \beta) - \ln \beta) \quad (6)$$

and the equilibrium is a terminal attractor. The proof is complete

As for the ILPMLSM motor system in (4), the recursive structure based on the FTSMC concept for second order systems similar to (5) can be derived as,

$$\begin{aligned} s_0 &= e_1 \\ s_1 &= \dot{s}_0 + \alpha_0 s_0 + \beta_0 s_0^{q_0/p_0} \end{aligned} \quad (7)$$

here s_0 is a scalar variable, $\alpha_0, \beta_0 > 0$ are constants and p_0, q_0 ($p_0 > q_0$) are odd positive integers. One can easily see if s_1 reaches zero, s_0 will reach zero subsequently according to the dynamical structure of the terminal attractor in (5).

B. Design of Fast Terminal Sliding Mode Controller

Theorem 1. For the system in (4), if we choose the following control law [11]

$$u(t) = -\frac{1}{b(e)} (h(e) + \alpha_0 \dot{s}_0 + \beta_0 \frac{d}{dt} (s_0^{q_0/p_0}) + \phi_1 + \gamma s_1^{q/p}) \quad (8)$$

with $s_0 = e_1$, then the system states will reach the sliding manifold $s_1 = 0$ in finite time t_{s1} , where

$$t_{s1} = \frac{P}{\phi(p-q)} (\ln(\phi e_1(0)^{(p-q)/p} + \gamma) - \ln \gamma) \quad (9)$$

with $\phi, \gamma > 0$, p, q ($p > q$) are odd positive integers. The system will follow the recursive structure (7) to converge to the system equilibrium in finite time t_s , where

$$\begin{aligned} t_s &= \frac{P}{\phi(p-q)} (\ln(\phi e_1(0)^{(p-q)/p} + \gamma) - \ln \gamma) + \\ &\frac{P_0}{\alpha_0(p_0 - q_0)} (\ln(\alpha_0 e_1(0)^{(p_0 - q_0)/p_0} + \beta_0) - \ln \beta_0) \end{aligned} \quad (10)$$

Proof. This theorem can be proved in the similar way as in [11].

We can obtain that $s_1 = 0$ will be reached in t_{s1} . According to Lemma 1 and recursive structure (7), we will have $s_0 = 0$ during the time defined by (10), subsequently, $e_2 = 0$ can be obtained. Eventually the system in (4) converges to the equilibrium. Therefore, the system output can exactly track the referenced trajectory.

It is noted that the proposed controller is a continuous system. Therefore, the chattering phenomenon which results in the system oscillation is eliminated. The singularity of the system, as discussed in [9], can be avoided by choosing

adequate values of p and q .

C. Robustness Analysis

We now investigate the robustness of the proposed global sliding mode control scheme. Let us choose the following second order nonlinear uncertain system derived from (1) as the analysis model.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + \Delta f(x) + b(x)u + d(x) \end{aligned} \quad (11)$$

where $f(x)$ and $b(x)$ are the nominal scalar fields on R^2 , $g(x) \neq 0$ and $u \in R^1$. $\Delta f(x)$ and $d(x)$ represent the system uncertainties and external disturbance respectively. Additionally, the error dynamic equations of the nominal part of (11) are the same as (4). Assume that $|\Delta f(x) + d(x)| \leq L$, we have the following results,

Theorem 2. For the system in (11), if we choose the control law in (8), then the system will reach the neighborhood Δ of the sliding manifold $s_1 = 0$ according to the terminal attractor $\dot{s}_1 = -\phi s_1 - \gamma s_1^{q/p}$ at least in finite time

$$t'_{s1} = \frac{P}{\phi(p-q)} (\ln(\phi e_1(0)^{(p-q)/p} + \eta) - \ln \eta) \quad (12)$$

where

$$\Delta = \{x : |s_1| \leq \left(\frac{L}{\gamma}\right)^{p/q}\} \quad (13)$$

$$\gamma' = \gamma - \frac{\Delta f + d}{s_1^{q/p}} \quad (14)$$

$$\gamma = \frac{L}{|s_1^{q/p}|} + \eta, \eta > 0$$

with $\phi, \gamma > 0$, p, q ($p > q$) are odd positive integers.

Proof. The theorem can be proved in the similar way as in [11].

According to Theorem 2, if we choose suitable γ and p/q with respect to bounded uncertainties and disturbances, the small enough neighborhood Δ of the sliding manifold s_1 which satisfies the requirement of high-performance applications can be achieved.

Even though the fast terminal sliding mode controller is robust against parameter uncertainties and external disturbances, for various high performance applications the tracking error is quite substantial. Moreover, for the ILPMLSM motor system the tracking error is periodic for periodic reference signal. In such cases the additional iterative learning controller, or secondary controller, can be introduced to further improve the tracking performance from cycle to cycle [14, 15].

IV. ITERATIVE LEARNING CONTROLLER DESIGN

In this work, the ILC method [16] was employed to eliminate as much of the periodic errors as possible. Specifically, the proposed secondary controller is based on a PD type iterative learning control algorithm.

A. ILC Controller Design

Consider the system in (4), the control law of PD type open-loop ILC is designed as

$$u_{k+1}(t) = u_k(t) + \lambda e_k(t) + \gamma \dot{e}_k(t), k = 0, 1, 2, \dots \quad (15)$$

where $u_k(t)$ and $e_k(t)$ are the k -th control input and tracking error, respectively. Eventually, the optimal control input $u^*(t)$ can be obtained by adjusting the input from the current trial ($u_k(t)$) to a new input ($u_{k+1}(t)$) by evaluating the tracking error $e_k(t) = y_d(t) - y_k(t)$ on the interval $t \in [0, T]$. And this adjustment is accomplished according to the above PD type open-loop ILC control algorithm.

B. Convergence Analysis

The key issue of the ILC is to guarantee its convergence [14, 17], that is, to make sure that the limit of the control input sequence $u_k(t)$ uniformly converges to the desired one $u_d(t)$, $\forall t \in [0, T]$. Consequently, $y_k(t)$ uniformly converges to the desired trajectory $y_d(t)$. Unfortunately, there exists no general condition to satisfy various systems using different types of ILC control algorithms, even though many researchers have addressed this problem. We follow the proof of the proposed control algorithm for the linear motor system by choosing the adequate design parameters such as λ and γ .

Consider the nonlinear system in (16) below based on (1).

$$\begin{aligned} \dot{x}(t) &= g(t, x) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (16)$$

Without loss of generality, we make the following assumptions for the proposed linear motor system:

- (1) Nonlinear function $g(t, x)$ is global Lipschitz continuous with respect to x , that is, there exists Lipschitz constant, $M > 0$ and the following expression holds
$$\|g(t, x_1) - g(t, x_2)\| \leq M \|x_1 - x_2\| \quad (17)$$
- (2) The resetting state error sequence $\{\delta x_k(0)\}_{k \geq 0}$ converges to zero for all iterations.
- (3) The desired control input $u_d(t)$ is unique, which follows the desired state and system output.
- (4) B and C are bounded for all $t \in [0, T]$.

Furthermore, we have the following lemma.

Lemma 2. Assume the sequence $\{b_k\}_{k \geq 0}$ converges to

zero, thus as $k \rightarrow \infty$, $\sum_{i=0}^k \rho^{k-i} b_i$ is limited to zero, where

$$0 < \rho < 1.$$

Proof. According to the above-mentioned condition, we have $\lim_{i \rightarrow \infty} b_i = 0$, $\lim_{i \rightarrow \infty} \rho^i = 0$. Let $M_1 = \max\{b_i, i = 0, 1, \dots\}$.

Thus, $\forall \varepsilon > 0, \exists N$, if $n > N$, then the following inequalities hold

$$|b_n| \leq \frac{\varepsilon(1-\rho)}{1-\rho^N}, \rho^n \leq \frac{\varepsilon}{M_1} \cdot \frac{1-\rho}{1-\rho^N}$$

We denote

$$\begin{aligned} I_{2n} &= \sum_{i=0}^{2n} \rho^{2n-i} b_i = \sum_{i=0}^n \rho^{2n-i} b_i + \sum_{i=n+1}^{2n} \rho^{2n-i} b_i \\ &= \rho^n \sum_{i=0}^n \rho^{n-i} b_i + \sum_{j=0}^{n-1} \rho^j b_{2n-j} \end{aligned}$$

Because

$$\begin{aligned} \left| \sum_{i=0}^n \rho^{n-i} b_i \right| &\leq \sum_{j=0}^n \left| \rho^j b_{n-j} \right| \leq M_1 \sum_{j=0}^n \rho^j = M_1 \frac{1-\rho^{n+1}}{1-\rho} \\ \left| \sum_{j=0}^{n-1} \rho^j b_{2n-j} \right| &\leq \sum_{j=0}^{n-1} \rho^j |b_{2n-j}| \leq \frac{\varepsilon(1-\rho)}{1-\rho^N} \sum_{j=0}^{n-1} \rho^j \\ &= \frac{\varepsilon(1-\rho)}{1-\rho^N} \cdot \frac{1-\rho^n}{1-\rho} = \varepsilon \cdot \frac{1-\rho^n}{1-\rho^N} < \varepsilon \end{aligned}$$

Therefore,

$$\begin{aligned} |I_{2n}| &\leq \rho^n \left| \sum_{i=0}^n \rho^{n-i} b_i \right| + \left| \sum_{j=0}^{n-1} \rho^j b_{2n-j} \right| < \rho^n M_1 \frac{1-\rho^{n+1}}{1-\rho} + \varepsilon \\ &< \frac{\varepsilon}{M_1} \cdot \frac{1-\rho}{1-\rho^N} \cdot \frac{1-\rho^{n+1}}{1-\rho} \cdot M_1 + \varepsilon < 2\varepsilon \end{aligned}$$

Noting that ε is an arbitrary real positive number, as a result, as $n \rightarrow \infty, I_{2n} \rightarrow 0$ is achieved. Proof is complete.

The convergence of the control algorithm is given in the following theorem.

Theorem 3. Given the nonlinear system in (16) and the proposed control law in (15), the sequences $\{\delta x_k(t)\}_{k \geq 0}$, $\{\delta y_k(0)\}_{k \geq 0}$, and $\{\delta u_k(0)\}_{k \geq 0}$ are said to converge to $x_d(t)$, $y_d(t)$, and $u_d(t)$, respectively, if the following condition is satisfied

$$\|I - \gamma CB\| < 1 \quad (18)$$

Proof. We define $\delta x_k(t) = x_d - x_k$, $\delta y_k(t) = y_d - y_k$,

$$\delta u_k(t) = u_d - u_k.$$

Let $f_1(t, x) = g(t, x_d) - g_1(t, x_d - x)$, $\forall x \in R^n$. We have,

$$\begin{aligned} \delta \dot{x}_k &= g_1(t, \delta x_k) + B \delta u_k \\ \delta \dot{y}_k &= C \delta x_k \end{aligned} \quad (19)$$

$$\delta u_{k+1} = \delta u_k - \lambda \delta y_k - \gamma \delta \dot{y}_k$$

From (19), we also have

$$\delta \ddot{y}_k = C g_1(t, \delta x_k) + CB \delta u_k \quad (20)$$

Then we can generate

$$\delta u_{k+1} = (I - \gamma CB) \delta u_k - \lambda C \delta x_k - \gamma C g_1(t, \delta x_k) \quad (21)$$

Accordingly,

$$\begin{aligned} \|\delta u_{k+1}\| &= \|(I - \gamma CB) \delta u_k - \lambda C \delta x_k - \gamma C g_1(t, \delta x_k)\| \\ &\leq \|(I - \gamma CB) \delta u_k\| + \|\lambda C \delta x_k\| + \|\gamma C g_1(t, \delta x_k)\| \\ &\leq \|I - \gamma CB\| \cdot \|\delta u_k\| + (\|\lambda C\| + M \|\gamma C\|) \cdot \|\delta x_k\| \\ &\leq \|I - \gamma CB\| \cdot [\|I - \gamma CB\| \cdot \|\delta u_{k-1}\| + \eta_{k-1}] + \eta_k \leq \dots \\ &\leq \|I - \gamma CB\|^{k+1} \cdot \|\delta u_0\| + \sum_{i=0}^k \|I - \gamma CB\|^{k-i} \eta_i \end{aligned} \quad (22)$$

where, $\eta_i = (\|\lambda C\| + M\|\gamma C\|) \cdot \|\delta x_i\|, i = 0, 1, \dots, k$.

In (22), if $\|I - \gamma CB\| < 1$, considering Assumptions (2), (3) and (4), and Lemma 2, as $k \rightarrow \infty$, then $\|\delta u_{k+1}\|$ is limited to zero. The proof is complete.

Thus theoretically, as $k \rightarrow \infty$, the control input variation is zero. Consequently, the output follows exactly the desired trajectory. If $\|I - \gamma CB\|$ is smaller, then the convergence rate is faster. Nevertheless, a choice of γ just satisfying the requirement that ensures reasonable fast convergence rate would suffice [18].

V. SIMULATION RESULTS

Computer simulations were performed based on the motor type of IL06-75 manufactured by Kollomorgen Company. The parameters are: $M=1.0\text{kg}$, $K_{F0}=42.8\text{N/A}$, $K_E=34.9\text{V/m/s}$, $R=11.7\Omega$, $B=0.5\text{N/m/s}$. The friction force for simulation just same as in [2], where $f_s=10$, $f_c=6$, $\xi=1$, and $\dot{x}_s = 0.001$.

The following case studies were performed: (1) the FTSMC only, that is, the FTSMC is activated while the ILC is kept off. The results were compared with that of a traditional PID controller; (2) combined controller, that is, the FTSMC controller is activated first and later the ILC controller is activated. The results were compared with that obtained using the FTSMC alone and the ILC alone.

The desired position trajectory is a sinusoidal curve with $x_d = 0.02 \sin(2\pi t)$. The FTSMC controller parameters were chosen as: $\alpha_0 = 500$, $\beta_0 = 2$, $p_0 = 9$, $q_0 = 5$, $p = 5$, $q = 3$, $\phi = 80$, $\eta = 1.5$, while the PID controller parameters are: $k_p = 1 \times 10^5$, $k_i = k_d = 10$. In case of no external disturbances, the tracking error is shown in Fig.3, we can see that the convergence rate of the FTSMC is much faster than that of PID controller and also the error amplitude of the FTSMC is smaller.

Fig.4 presents the results with no external disturbances. It is seen that the tracking performance of the proposed combined control scheme is better than that obtained by either control algorithms. In this comparative study, the control parameters for the FTSMC loop of the combined controller are the same as that in the FTSMC only scheme. The parameters for the ILC loop of the combined controller are: $\lambda = 9 \times 10^4$, $\gamma = 5.0$ with the iteration number of 10, the control parameters for the ILC only scheme are $\lambda = 9 \times 10^4$, $\gamma = 10$ with iteration number of 15. Furthermore, if there is external disturbance, as it is shown in Fig.5, the robustness of the combined controller is almost same as the FTSMC only control scheme. Apparently, these two algorithms outperformed the ILC only control scheme.

In case of higher frequency, for example, for a 5Hz desired sinusoidal trajectory as in Fig.6 the tracking error of the proposed control algorithm is also smaller than that of the other two algorithms while maintaining the reasonable robustness.

The simulation results for the above case studies are summarized in terms of several performance indices shown in Table I. The maximum absolute value of the tracking error during the last 0.5s, $e_F = \max_{2.5 \leq t \leq 3} \{e_1(t)\}$, is used as an index of final tracking accuracy. The rms value of the tracking error $\|e_{rms}\| = (1/T \int_0^T (e_1(t))^2 dt)^{1/2}$, is used to measure average tracking performance. It is seen that the overall tracking performance of the combined control scheme is better than the other three control methods.

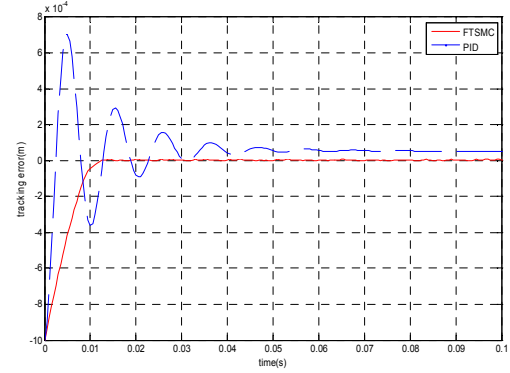


Fig. 3. Tracking errors in case of parametric uncertainties and nonlinearities ($0 < t < 0.1\text{s}$)

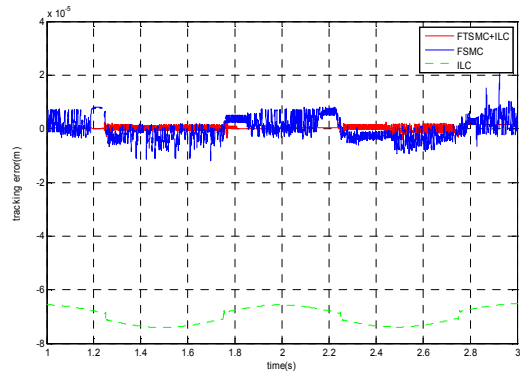


Fig. 4. Tracking errors in case of parametric uncertainties and nonlinearities ($1 < t < 3\text{s}$)

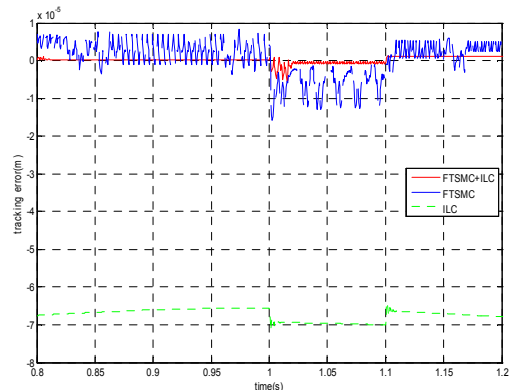


Fig. 5. Tracking errors in case of external disturbance

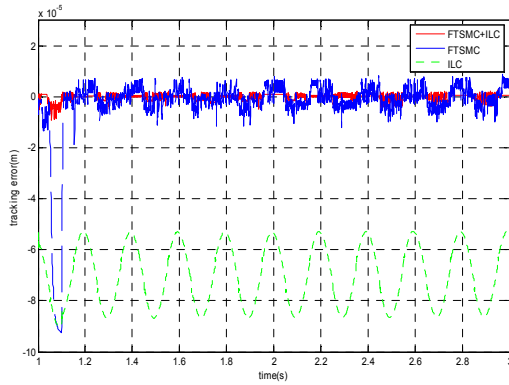


Fig. 6. Tracking errors with external disturbance for frequency 5HZ desired trajectory ($1 < t < 3s$)

TABLE I
SIMULATION RESULTS

	FTSMC/ILC	FTSMC	ILC	PID
e_F (μm)	7.8139	10.304	73.884	63.942
$\ e_{rms}\ $ (μm)	10.228	10.873	69.990	50.614

VI. CONCLUSIONS

A novel controller using fast terminal sliding mode control combined with iterative learning control has been developed for a nonlinear ironless permanent magnet linear synchronous motor systems. The FTSMC controller, as the primary controller, guarantees the system performance robustness and fast convergence rate. Furthermore, the finite reaching time can be determined. The secondary ILC controller is used to reduce the periodic tracking error, which enhances the performance of the linear motor with no need of exact machine model. The stability and convergence of the proposed control system were verified. The simulation results illustrated the effectiveness of the proposed scheme, nonlinearities and external disturbances. Furthermore, this control scheme can also be applied to other types of linear motors.

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