2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Predicting Numbers of Performance Failures in the Manufacture of Dynamic Systems

Roger Fales, *Member, ASME*

***Abstract*—In this work, a method for determining the reliability of dynamic systems is discussed. Using statistical information on system parameters, the goal is to determine the probability of a dynamic system achieving or not achieving frequency domain performance specifications such as low frequency tracking error, and bandwidth. An example system is considered with closed loop control. A performance specification is given and converted into a performance weight transfer function. The example system is found to have a 20% chance of not achieving the given performance specification. An example of a realistic higher order system model of an electro hydraulic valve with spring feedback and position measurement feedback is also considered. The spring rate and viscous friction are considered random variables with normal distributions. It was found that nearly 6% of valve systems would not achieve the given frequency domain performance requirement.**

I. INTRODUCTION

THE stability and performance of dynamic systems can define the success or failure of a manufactured system. The goal of this work is to predict the numbers of failures as measured by performance specifications and stability for dynamic systems that are mass produced. The variations in parameters such as spring rates and part geometry should serve as an input to this sort of analysis since these characteristics can be obtained or are already known by the manufacturer. For a long time, manufactures have predicted the number of assembly failures due to the interference fit between parts that experience tolerance stack up (Stoll, 1999). In this work, statistical data about parameters of the dynamic system will be used to predict the probability of failure due to not meeting frequency domain performance specifications and robust stability. A Mote Carlo method will be used to determine the probability that a system with random parameters will be stable or meet performance specifications. The Monte Carlo approach used in this work provides a convenient way to solve a problem that is a function of potentially numerous random variables (Rubinstein, 1981).

Other researchers are working on probabilistic control design which is similar in concept to methods presented here but does not provide for predictions of failure rates but seeks to design controls such that success would be most likely given the statistical distribution of plant dynamics (Crespo and Kenny, 2005). There is also active research in the area of Probability Stability where one can predict the probability of stability based on the statistical distribution of dynamic equation parameters but these methods tend to be very cumbersome for all but the simplest low order systems (Jovanovic, 2004).

Some research has focused on the stochastic nature of plant uncertainty (i.e. uncertain parameters) which can be used to determine the stochastic root locus and probability of instability (Stengel, 1991). In addition, this work has been extended to determining the probability of achieving performance requirements and to nonlinear systems (Wang and Stengel, 2002). Further research in the area of probability stability showed that uniform distribution is the "right" way to estimate the probability densities for sampled parameters (Barnish, 1997). A research field of recent popularity is stochastic H_1 in which deterministic and stochastic perturbations are considered in H_1 control problems (Hinrichsen, 1998). Stochastic H_1 control analysis is extended to H_2/H_1 problems with state dependent noise (Chen and Zhang, 2004) (Chen and Zhang, 2005).

The approach proposed here provides a way to evaluate new system designs for probability of reliable stability and performance. Often times, manufacturers do not use a systematic approach for analyzing robust stability and for predicting reliability and numbers of failures in multi component dynamic systems. However, manufactures may have access to data which could be used to describe parameter variations statistically. For example, dynamically important parameters such as mass, spring rate, etc., can be described using a probability density function. The parameters affect dynamic performance and stability in a way that can be seen by observing the frequency response of a transfer function model. The magnitude of the frequency response becomes random at each frequency if the parameters of the model are random. The random magnitude of the frequency response of generalized plant transfer functions can then be analyzed to determine if performance objectives and/or stability are achieved using methods commonly used in robust control applications where the transfer functions do not have random magnitudes (Skogestad, 1996).

The organization of the rest of this work is given as follows. Section II describes how probability of failure can be computed for performance requirements given in the frequency domain. Section III discusses how the analysis can be extended to robust stability. In Section IV, two examples are given for computing the probability of achieving performance goals. Conclusions and a description

Roger Fales is an Assistant Professor at the University of Missouri Mechanical and Aerospace Engineering Department (e-mail: falesr@ Missouri.edu).

978-1-4244-4524-0/09/$25.00 ©2009 AACC

of the nomenclature are given in Sections V and VI respectively.

II. A METHOD FOR DETERMINING THE PROBABILITY OF ACHIEVING FREQUENCY RESPONSE PERFORMANCE CRITERION

For any given dynamic system, the system is either stable or unstable depending on the operating point if it is nonlinear. Here, we consider linear time invariant systems modeled using single input single output transfer functions. If the system has uncertain parameters, then stability and performance depends on those parameters. For a transfer function (or transfer function matrix), the probability that the system frequency response magnitude will take on any value can be determined. The frequency dependent gain of a system can be used to determine if frequency dependent performance specifications are met or if stability is achieved. For performance analysis, the goal is to select a performance transfer function and then determine what the maximum probability that the magnitude of the performance transfer function will exceed a given performance specification at any frequency.

Each numerical parameter in a system which is replicated many times may be modeled as a randomly generated number p_i with a probability density function associated with it $f_i(\theta)$, where θ is a dummy variable. The magnitude of the frequency response of a transfer function which is a function of the random parameters, $|G_p(j\omega)|$, then is also a random number (for each frequency) which depends on the random parameters. Let the probability density function for the transfer function magnitude at a frequency, ω, be $f_{Gp\omega}(\theta)$ which can be estimated or found analytically at each frequency given the random parameters. Any transfer function constructed from $G_p(s)$ such as a closed loop transfer function would then also have a random frequency response which would depend on the random parameters.

Performance goals in the frequency domain may consist of upper or lower limits on the magnitude of the frequency response. Let the upper and lower limits at each frequency be a(ω) and b(ω) respectively. Therefore the probability that the magnitude of a transfer function with random parameters would lie within the limits is

$$P_\omega(b(\omega) < \|G_p(j\omega)\|_\infty < a(\omega)) = \int_{b(\omega)}^{a(\omega)} f_{G_p\omega}(\theta)d\theta. \quad (1)$$

The probability that the magnitude of $G(j\omega)$ would be within the limits for all frequencies is

$$P\left(\boldsymbol{b} < \|G_p(j\omega)\|_\infty < \boldsymbol{a}\right) = \max_\omega \int_{b(\omega)}^{a(\omega)} f_{G_p\omega}(\theta)d\theta. \quad (2)$$

This result may be used to determine the probability that the H-infinity norm of a transfer function is within a defined range between frequency dependent bounds $a(\omega)$ and $b(\omega)$ at each frequency. Often one of the bounds will be set to infinity or negative infinity since it is only necessary to determine the probability of a system either exceeding a performance requirement or not achieving a performance requirement.

III. ROBUST STABILITY ANALYSIS

This concept can also be used in stability analysis by employing the small gain theorem. The small gain theorem states that for a stable loop transfer function matrix, $L(s)$, the closed loop system is stable if

$$\|L(j\omega)\| < 1 \ \forall\ \omega$$

(Skogestad, 1996). The norm indicated can be any norm satisfying $\|AB\| \leq \|A\| \cdot \|B\|$. Note that the small gain theorem is a conservative way to determine the stability of a closed loop control system. The lower bound of the probability of closed loop stability can be found using the small gain theorem,

$$P(\|L(j\omega)\| < 1) \geq \max_\omega \int_{b(\omega)}^{a(\omega)} f_{L\omega}(\theta)d\theta.$$

IV. APPLICATION EXAMPLES

There are many example applications where the probability of failure due to poor dynamic performance or instability could be computed. Good candidates for the analysis presented in this work include dynamic systems with performance requirements defined in the frequency domain and parameter values that can be described statistically with probability density functions. In this work, two examples are presented. The first example is of a second order under damped system with random parameters and a proportional feedback control system. The second example is of a two stage electrohydraulic flow control valve with an uncertain spring constant and friction parameter. Both examples include a feedback control system designed to control the output such that a given performance requirement in the frequency domain is achieved.

A. Second order system with P control

As an example, consider a typical control system (Figure 1) with a model of the plant $G(s)$. The goal of this example is to determine what percentage of replications of the system would fail to achieve a performance objective if the system is mass produced with randomly varying parameters. The plant model is given as

$$G(s) = \frac{\omega_n^2}{s2 + 2\zeta\omega_n s + \omega_n^2}$$

where the parameters for natural frequency, ω_n, and damping ratio, ζ, are normally distributed random variables each with standard deviation of 0.5 and nominal values, 6.28 rad/sec and 0.50 respectively.

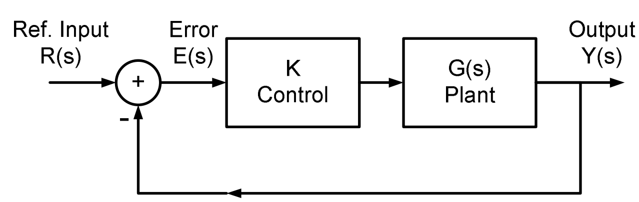

Figure 1 Control system diagram

A closed loop transfer function relating the input to the error is given as $S(s)=1/(1+G(s)K)$ (sensitivity transfer function). The performance objective is given as follows: $|S(j\omega)| < 1/|w(j\omega)|$ for all frequencies. The transfer function, $w(s)$, is a performance objective weighting function the inverse of which can be chosen to be any frequency response desired. In this example, the a typical performance weighting function is given as

$$w(s) = \frac{s/M + \omega_b}{s + A\omega_b} \tag{3}$$

where M=6.0 (maximum high frequency error), ω_b=5.2 rad/sec (0.83 Hz) (desired bandwidth), and A=0.10 (maximum low frequency error) (Skogestad, 1996).

Let $G_P(s) = w(s)S(s)$ so that the performance objective is achieved if $|G_P(j\omega)| \leq 1$ for all frequencies and all plant perturbations. Ultimately, we want to know the probability that the magnitude will not be greater than one for all frequencies using Eq. 2, $P\left(-\infty < \|G_p(j\omega)\|_\infty \leq \mathbf{1}\right)$.

The next step is to determine for each frequency the probability that the magnitude of $G_P(j\omega)$ will take on any particular magnitude – determine an estimate of the probability density function. In this case, this is accomplished through Mote Carlo analysis, randomly generating values of natural frequency, ω_n, and damping ratio, ζ. The analysis continues by dividing the magnitudes of $G_P(j\omega)$ (which is a function of random variables ω_n, and ζ) into bins and then determining estimates of the probability that the magnitude of $G_P(j\omega)$ would fall into each bin. In the Figure 2, the probability for $G_P(j\omega)$ to take on magnitudes over a frequency range of interest is plotted. Note that where there is extremely low probability of $G_P(j\omega)$ having a certain magnitude, there is no data since there are a finite number of bins and samples (100 bins per frequency and 1000 random parameter samples in this example) used in the calculations. The threshold locations where the magnitudes of $G_P(j\omega)$ are greater than one and thus the performance requirement is violated are indicated the asterisk (*) symbol for each frequency in Figure 2.

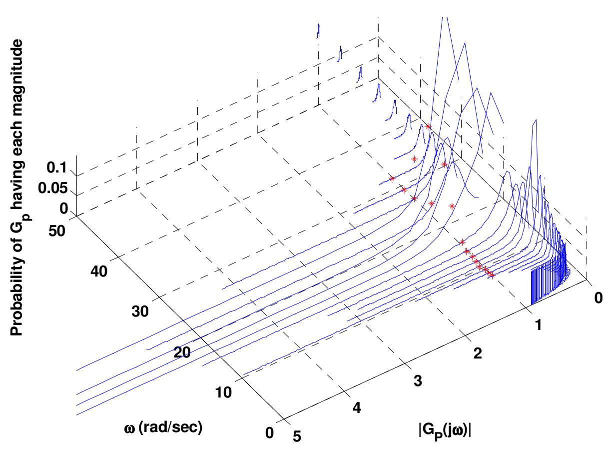

Figure 2 Probability that a performance objective, $G_P(j\omega)$, will have a range of magnitudes for each frequency

The probability of failure is determined by the area under the curve in Figure 2 where $|G_P(j\omega)|$ is greater than one for each frequency. The resulting probabilities of performance failures are given in Figure 3 for each frequency. This is a powerful result in that the likelihood of failure to achieve a performance objective can now be calculated by considering the maximum value over all frequencies of the probability of failure in Figure 3. The resolution of the results could be improved by taking a finer grid of frequencies and a finer grid of bins for the performance magnitude.

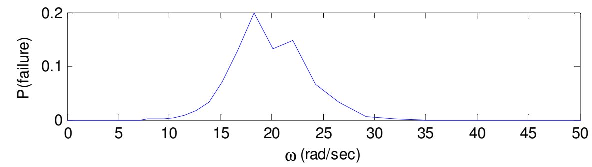

Figure 3 Probability of performance failure at each frequency: maximum is 0.20 indicating 20% failure.

B. Two Stage Electro Hydraulic Valve Example

The metering poppet valve is used to control flow in hydraulic control systems. The metering poppet valve in this example is unique and was designed to achieve a specific dynamic performance requirement without use of electronic feedback control (Muller, 2008). Muller noted that variations in the parameters have a great effect on the dynamic performance of the system. Feedback control can be used to improve robustness to variations in parameters. The metering poppet valve in this example will be used in a control system where the position of the valve flow metering element (the main poppet) will be controlled using a PID controller. It is desired to control the valve position with 1 % low frequency tracking error, a bandwidth of 10 Hz, and maximum high frequency tracking error of 200%. The metering poppet valve design is intended to be manufactured in mass production. Therefore it is important to determine how many replications of the design will not achieve the performance requirements.

The following gives a brief description of the operation of the metering poppet valve design presented in this work. With respect to Figure 4, the valve is in the closed position with high pressure connected to the inlet port while low pressure is connected to the outlet port. The only two pathways from high to low pressure are sealed by poppet seats and therefore the valve maintains a very low leakage flow when closed. It is assumed that the pilot poppet is pressure balanced by the passage shown within the pilot poppet while being subjected to the pilot solenoid force, the feedback spring's force, viscous damping, and flow forces. In order to raise the main poppet off its seat, a pulse width modulated signal (PWM) signal supplies current to the pilot actuator solenoid which pushes the pilot poppet off its seat and allows fluid to exit the pilot control volume through its outlet orifice (2) restriction. Once the pilot poppet opens, the control volume inlet orifice 1 is effectively smaller than its outlet orifice creating a net outflow which allows the main poppet to lift off its seat. This opens the main metering orifice (3) restriction directly between supply and load, allowing flow to be metered. The upward movement of the main poppet will push the feedback spring and in turn push the pilot poppet back towards its seat until an equilibrium position is reached where the main poppet is open yet no longer moving. In order to close the main poppet, the actuator current is turned off allowing the feedback spring to push the pilot poppet back to its seat. The control volume outlet orifice is now closed while high pressure fluid from the inlet orifice fills the control volume and pushes the main poppet closed.

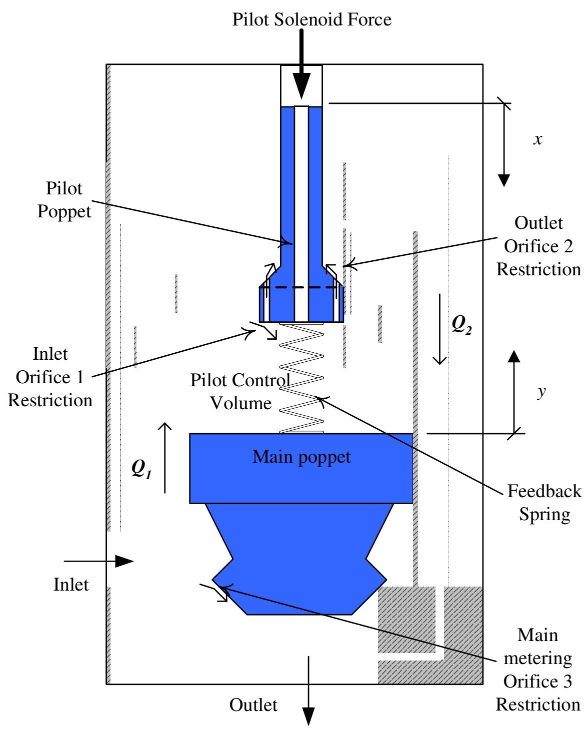

Figure 4 Metering Poppet Valve Diagram

The linearization of the model and the design for the valve are discussed in previous work (Muller and Fales, 2008). The parameters for the valve model are given in Nomenclature

Table 1. The linearized model equations are given as follows:

$$M\ddot{y} = -b_y\dot{y} - k(y_n + x_n) - kfq_3 y_n - kfc_3(P_s - P_L) - P_c A_c + P_s A_s + P_L A_L \quad (4)$$

$$\dot{P}_c = \frac{\beta}{V_{CO}}\begin{Bmatrix} A_C\dot{y} + kq_1 x_n - kq_2 x_n - kc_1 P_C - \\ kc_2(P_C - P_L) + kc_1 P_s \end{Bmatrix} \quad (5)$$

$$m\ddot{x} = -B_x\dot{x} - k(y_n + x_n) - kfq_2 x_n + kfq_1 x_n - kfc_1 P_C - kfc_2(P_C - P_L) + f + kfc_1 P_s \quad (6)$$

The system is 5th order, making it more complicated than the previous second order system example (the frequency response of the plant is given in Figure 5). The same procedures can be used to analyze this system as in the previous example however. As in all mass produced products, parameters vary from one repetition to another of the same manufacturing processes. All parameters in the model could be considered random variables. In this example, the analysis of uncertain parameters will be limited to two parameters to limit the complexity of the calculations.

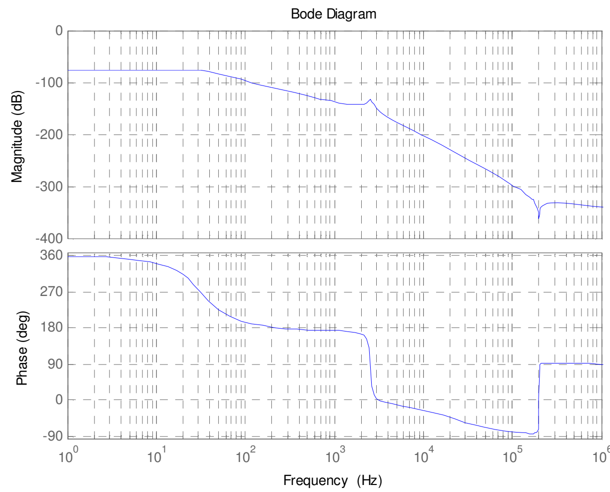

Figure 5 Frequency response of nominal plant model G(s)

Therefore, in this example, the spring rate, *k*, and viscous friction, b_x, coefficients are chosen to have normal random distributions with variance equal to 10% of their nominal values.

A standard PID control, $K_{PID}(s)$ was design with a block diagram similar to Figure 1 replacing K with $K_{PID}(s)$. In this case the plant input is solenoid force, *f*, and the output was main poppet valve position, *y*. The controller reference input is the desired main poppet valve position. Equation 3 was used again to form a performance weight transfer function, *w(s)*, with *M*=2, *A*=0.01, and ω_b=62.8 rad/sec. Again the performance transfer function is given as the weighted sensitivity transfer function for the closed loop control system, $G_p(s) = Sw(s)$, where $S = 1/(1 + G(s)K(s))$. With the PID control applied to the nominal plant, the nominal performance can be tested by examining the frequency response of the weighted sensitivity transfer function. This performance result given in Figure 6 shows that the required performance is achieved for the nominal plant parameters since the magnitude is less than 0 dB.

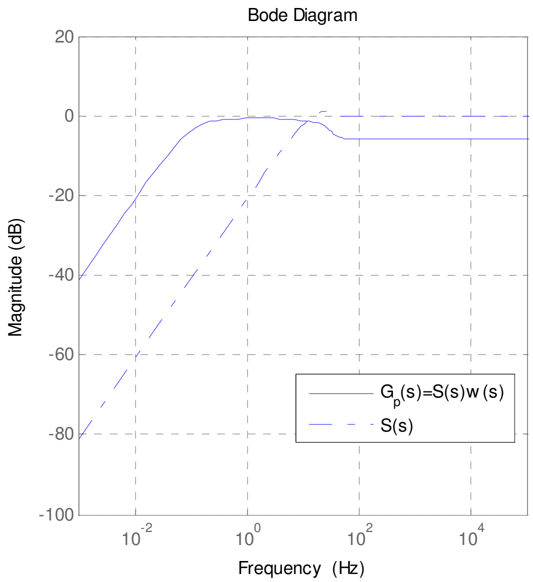

Figure 6 Nominal performance for the metering poppet valve with control

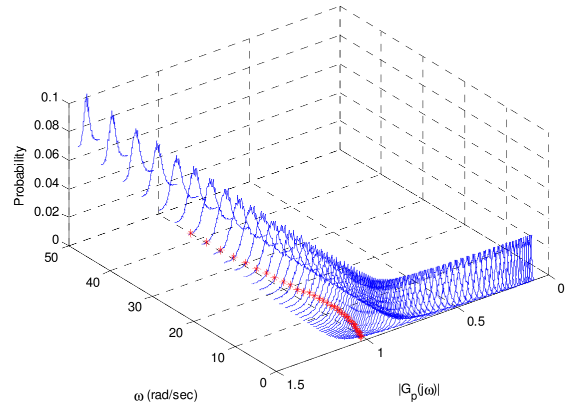

Figure 7 Probability that the performance objective, $G_P(j\omega)$, for the metering poppet valve control system will have a range of magnitudes for each frequency

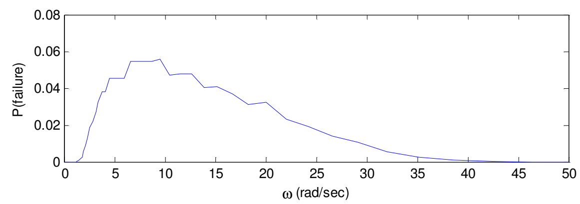

Figure 8 Probability of performance failure for the metering poppet valve – maximum is 0.0556

The same procedure was used to analyze this metering poppet valve example as was used to analyze the previous second order system example. In this case the probability of failure is due to the randomness of the spring constant and viscous damping. The results are shown in Figure 7 and Figure 8 which are comparable to Figure 2 and Figure 3. As can be seen in Figure 7, there is range starting at low frequencies and extending past 40 rad/sec where there is significant probability that the performance specification will not be achieved since the probability distribution of the magnitude of $G_p(j\omega)$ extends past 1 (the '*' symbol is used in the figure to indicate where the probability distribution extends past the magnitude of one. The results in Figure 7 show that the maximum probability of failure is at least 5.56%. This means that if the system were manufactured with randomly distributed spring and friction parameters as described, then 5.56% of manufactured valves would not have the desired performance.

V. Conclusions and Future Work

In this work, a concept for determining the number of failures in terms of dynamic performance is described for dynamic systems. In general, the method can be used to predict the probability of a system achieving a performance objective described by a function. It was found that for a particular example of a second order under damped system, the method predicted that 20 percent of replicates of the system would not achieve the given frequency domain performance requirement. A second example showed that for a 5th order metering poppet valve control system model, 5.56% of manufactured units would not have performance that achieves the stated specification. It should be noted that stability is required for all cases in the analysis. Robust stability is assumed for the examples that are presented but should be proven to complete the analysis. The examples used random simulations of the distribution of parameters to calculate estimates for the integrals required to determine the number or failures.

Future work will focus on analytically determining the integrals using the model to determine the distributions of the performance magnitudes at each frequency given the parameter distribution functions. Also, the techniques presented here should be compared to methods in similar work by other researchers to determine what differences there may be in the methods (noted in the introduction) for determining probability of performance. Future work should also include testing the concept using data for a system that is made of real components which exist in the market. This would require obtaining data on the variability in the parameters of these components. This data would need to include a model of the random distribution of the parameters. Theoretical models of the random distribution of the parameters of manufactured components could also be used.

VI. Nomenclature

Table 1 Metering Poppet Valve Model Parameters

Symbol	Description	Units
A_C	Area of main poppet exposed to control pressure	[m^2]
A_L	Area of main poppet exposed to load pressure	[m^2]
A_S	Area of main poppet exposed to supply pressure	[m^2]
b_X	Damping coefficient for the pilot poppet	[N/m/s]
b_Y	Damping coefficient for the main poppet	[N/m/s]
C_d	Orifice discharge coefficient	
f	Actuator input force	[N]
h_2	Slope of orifice 1 area vs. position curve	[m^2/m]
h_2	Slope of orifice 2 area vs. position curve	[m^2/m]
h_3	Slope of orifice 3 area vs. position curve	[m^2/m]
k	Feedback spring coefficient	[N/m]
$kc_{(1-4)}$	Pressure flow coefficient for orifices 1 – 4	[m^3/s/Pa]
$kfc_{(1-3)}$	Pressure flow force coefficient for orifices 1 - 3	[N/Pa]
$kq_{(1-3)}$	Flow gain for orifices 1 – 3	[m^3/s/m]
M	Mass of the main poppet	[kg]
m	Mass of the pilot poppet	[kg]
o	All o subscripts represent nominal conditions	
P_C	Control volume pressure	[Pa]
P_L	Load volume pressure	[Pa]
P_S	Fixed supply pressure	[Pa]
P_T	Fixed tank pressure	[Pa]
$Q_{(1-4)}$	Flow rate across orifices 1 - 4	[m^3/s]
θ	Jet angle for flow force	[rad]
V_C	Volume of the control volume above the main poppet	[m^3]
x	Position of the pilot poppet referenced from closed position (positive is down in Fig. 1)	[m]
x_n	Pilot poppet position referenced from the nominal opening	[m]
y	Position of the main poppet referenced from closed position (positive is up in Fig. 1)	[m]
y_n	Main poppet position referenced from the nominal opening	[m]
β	Fluid bulk modulus	[Pa]
ρ	Fluid density	[kg/m^3]

References

[1] Crespo, L., S. Kenny. 2005. “Reliability-Based Control Design for Uncertain Systems”. AIAA Guidance, Navigation, and Control Conference and Exhibit, 15-18 Aug. 2005, San Francisco, CA, United States.

[2] Doyle, J., B. A. Francis, and A. R. Tannebaum, 1992. Feedback Control Theory, Macmillan Publishing Company, New York.

[3] Franklin, G. F., J. D. Powell, A. Emami-Naeini, 2007. Feedback Control of Dynamic Systems 5th ed. Pearson Prentice Hall, Upper Saddle River, New Jersey.

[4] Jovanovic, Z., B. Dankovic. 2004. “On the Probability Stability of Discrete-Time Control Systems.” FACTA UNIVERSITATIS. SERIES: ELECTRONICS AND ENERGETICS. vol. 17, April 2004, 11-20.

[5] Stoll, Henry W., 1999, Product Design Methods and Practices, Marcel Dekker, New York.

[6] Skogestad, S., and Postlethwaite, I., 1996. Multivariable Feedback Control: Analysis and Design, John Wiley and Sons, New York.

[7] Muller, M., and R. Fales. 2008. “Design and Analysis of a Two-Stage Poppet Valve For Flow Control.” International Journal of Fluid Power. Vol. 9, No. 1, Pages 17-26. Mar. 2008.

[8] B.S. Chen, W. Zhang. \Stochastic H2/H infinity control with state-dependent noise", IEEE Trans. Automatic Control, 49(1), pp.45-57, 2004.

[9] W. Zhang, B. S. Chen, C. S. Tseng. \Robust H infnity filtering for nonlinear stochastic systems", IEEE Trans. Signal Processing, 53(2), pp. 289-298, 2005.

[10] R.F. Stengel and L.R. Ray, "Stochastic Robustness of Linear Time-Invariant Control Systems," IEEE Trans. on Automatic Control, Vol. 36, No. 1, 1991.

[11] B.R. Barmish and C.M. Lagoa, "The Uniform Distribution: A Rigorous Justication for Its Use in Robustness Analysis," Math. of Control, Signals, and Systems, Vol. 10, pp. 203-222, 1997.

[12] Rubinstein, R. Y. 1981. Simulation and the Monte Carlo Method . New York: John Wiley & Sons.

[13] Muller, M., and R. Fales. 2008. “Design and Analysis of a Two-Stage Poppet Valve for Flow Control.” International Journal of Fluid Power. Vol. 9, No. 1, Pages 17-26. Mar. 2008.

[14] Wang, Q. and Robert F. Stengel, 2002. “Robust control of nonlinear systems with parametric uncertainty”, Automatica, Volume 38, Issue 9, September 2002, Pages 1591-1599.