2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009



# Positional Consensus in Multi-Agent Systems using a Broadcast Control Mechanism

Kaushik Das and Debasish Ghose

*Abstract*— **In this paper a strategy for controlling a group of agents to achieve positional consensus is presented. The proposed technique is based on the constraint that every agents must be given the same control input through a broadcast communication mechanism. Although the control command is computed using state information in a global framework, the control input is implemented by the agents in a local coordinate frame. We propose a novel linear programming formulation that is computationally less intensive than earlier proposed methods. Moreover, we introduce a random perturbation input in the control command that helps us to achieve perfect consensus even for a large number of agents, which was not possible with the existing strategy in the literature. Moreover, we extend the method to achieve positional consensus at a pre-specified location. The effectiveness of the approach is illustrated through simulation results.**

## I. Introduction

The principle of using multiple agents is motivated by the idea that instead of using a highly sophisticated and expensive robots, it may be advantageous in certain situations to use a group of small, simple, and relatively cheap robot. The group of agents can be used to accomplish various tasks in different environment such as tactical operations, exploratory space missions, remote monitoring with mobile sensor networks, avoidance of collision and over-crowding in automated air traffic control, cleanups of toxic spills, fire fighting and cooperative search with unmanned air vehicles.

One of the problems that is of paramount importance in multi-agent systems is that of achieving consensus, that is, achieving identical values for some specified subset of the states of the agents. For instance, the agents may try to converge to the same direction of movement [1] after some time or they might want to converge to a point. Both are problems in achieving consensus. If we have a centralized system with perfect information then achieving consensus is a trivial matter, since the central controller can instruct each agent suitably to reach a common consensus point. However, if the communication system has a constraint on the number of messages that it can communicate, then one may opt for a broadcast protocol where the central controller will communicate simple and identical instructions to all the agents through a broadcast mechanism. We further impose the additional constraint that each agent can interpret the control command only in its local coordinate frame or local state space. Only the central controller has access to the global states of the system. Some of these constraint are common to other problems of a similar nature (for instance, see [2], [3], [4], [5]).

K. Das is a Research Scholar and D. Ghose is a Professor in the Guidance, Control, and Decision Systems Laboratory in the Department of Aerospace Engineering, at the Indian Institute of Science, Bangalore 560012, India kaushikdas+dghose@aero.iisc.ernet.in

**This project is supported by an AOARD/AFOSR grant.**

This problem was motivated by a recent paper by Bretl [6] where a control strategy for a group of micro-robots is developed to perform a useful task even when every robot receives the same control signal. The paper considers point robots with simple kinematics. It was shown that when there are only two agents, there exists a broadcast control command (that is, both agents receive identical instructions from the central controller) using which both agents can meet at the same location at the same time, for almost all initial conditions. However, if the number of agents is more than two, then the best that the agents can achieve is to come close to each other within a certain distance (measured by the radius of the smallest disc that contains all the agents positions), which is a function of the initial conditions. Bretl [6] formulates this problem as an optimization problem that minimizes the radius of the disc, and proposes a solution using the second order cone programming (SOCP) technique [8]. However, using this strategy the agents cannot be made to converge to a point. Once the solution of the SOCP is implemented, no further improvement is possible. Bretl's paper was in turn motivated by an interesting paper by Donald et al. [7] on the development of untethered, steerable micro-robots, where every robot receives the same power and control signal through an underlying electrical grid.

Our paper makes several specific contributions. The first is to propose a strategy that uses the basic Bretl's model with an additional randomization feature that allows large number of agents to achieve positional consensus or point convergence on repeated application of the algorithm without compromising the broadcast constraint on the control command. The second contribution is that our method can be extended to the case where the agents can be made to converge to any pre-specified point. The third contribution is to propose an optimization problem for this task that is based on a linear programming formulation. This allows standard and easily available software to be used for obtaining the solution. Moreover, this formulation also retains the property that the number of decision variables, whose values are to be communicated to agents, remains unchanged even when the number of agents increases. Finally, we also propose some interesting properties related to the positional formation of agents when the LP based strategy is applied iteratively.

It is worth noting that the randomization feature in the algorithm has some similarity with the random perturbation used in Viscek's model [1]. Vicsek et al. [1] propose a simple

978-1-4244-4524-0/09/$25.00 ©2009 AACC 

but compelling discrete time model of $n$ autonomous agents (points or particles) all moving in the plane with constant speeds but with different headings. Each agent updates its heading using a local rule based on the average of the headings of its neighbors plus some random perturbation.

The paper is organized as follows: In Section II we consider two agents and show that it is possible to move two agents to a common location using identical control. In Section III we formulate a linear programming problem for minimizing the proximity between agents by using identical broadcast control. In Section IV we have discussed some results on the formation of the agents after the linear programming solution is implemented. In section V we introduce the notion of iterative solution of the problem by repeated use of the LP algorithm and show that introducing a random perturbation in the broadcast mechanism leads to point convergence of the agents by repeated application of the LP technique. In Section VI we present a modification of the algorithm to ensure that the swarm of agents converge to a pre-specified point. In section VII we show several simulation results that illustrate the salient features of the proposed algorithm. Section VIII concludes the paper with a discussion of possible future directions of research.

## II. Formulation and Solution for Two Agents

We will first pose the problem in a general framework and then address the two agents case to clarify many of the assumptions and concepts discussed in the previous section.

Assume that $n$ agents are located on an obstacle-free plane. We assume that the central controller has access to the global state of the system which, in this case, consists of the position ($x_i \in \mathbb{R}^2$) and orientation ($\theta_i \in (-\pi, \pi]$) of the agents, $i = 1, \ldots, n$. The central controller computes a common local control for the agents and broadcasts it to the agents for implementation. The local control is in the form of a tuple $(\theta, d)$, which is interpreted by each agent in its local frame of reference. Here, $\theta$ refers to the angle by which each agent changes its orientation, and $d$ is a scalar that refers to the distance by which each agent moves after effecting the orientation change. Note that, the broadcast mechanism $(\theta, d)$ is the same for all the agents. Also, the local frame of reference for each agent is centered at the agent's location and its reference axis is oriented along its current orientation. As an illustration see Fig. 1, where agents are shown located initially at $x_{i0}$ with initial orientation $\theta_{i0}$ in the global reference frame. If the control command broadcast to all the agents is $(\theta, d)$, then the agents implement it in their local coordinate frame by each of them changing their orientation by the same angle $\theta$ and advancing by the same distance $d$ to reach the final destination $x_{if}$. Even in this figure it can be seen that by doing this the agents have come closer to each other. Our objective is to determine a $(\theta, d)$ such that the agents can achieve the closest proximity with each other.

*Theorem 1:* For two agents, for all initial conditions of the agents except when $\theta_{10} = \theta_{20}$, there exists a control $(\theta, d)$ using which point convergence can be achieved.

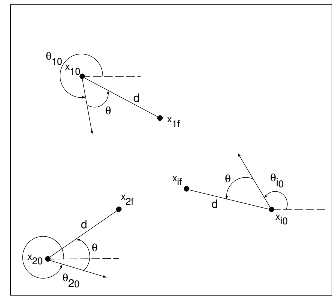


Fig. 1. Basic configuration

Note that this result is also available in Bretl [6] and is stated here for completion. The above theorem shows that it is possible to use a broadcast control command to make two agents meet at the same location simultaneously for almost all initial conditions. However, the solution is also unique and hence the location of the meeting point cannot be chosen arbitrarily. One can also interpret this result by noting that the final meeting point is on the Voronoi edge (equidistant line) between the two initial positions of the agents. It can be shown that only one unique point on the Voronoi edge satisfies the requirement that the orientation change angle is the same for both the agents (see Fig. 2). The point $p$ moves on the equidistant line from $-\infty$ to $+\infty$ and the corresponding orientation angle change $\theta$ is plotted for the two agents. The intersection of the two curves is the unique control command point. It can be seen that when the number

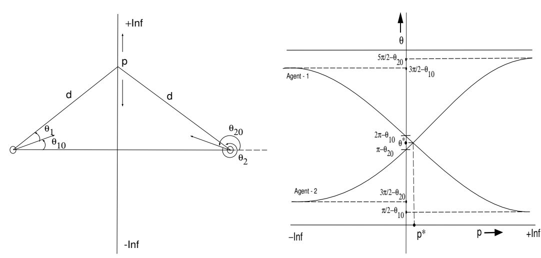


Fig. 2. Voronoi interpretation

of agents is more than two, each pair gives rise to a different unique meeting point. Thus, there does not exist a common control command to be broadcast so that all the agents meet at a point. In the absence of such a command, the best that can be done is to determine a $(\theta, d)$ which brings the agents in closest proximity with each other. Note that in this case $(\theta, d)$ may not be unique.

In the next two sections we will propose solutions to overcome both the drawbacks without compromising the broadcast based control mechanism.

## III. A Linear Programming Formulation

Let the initial position and initial orientation of the $n$ agents be $x_{i0} = (p_{i1}\ p_{i2}) \in \mathbb{R}^2$ and $\theta_i \in (-\pi, \pi]$, respectively, for all $i \in \{1, ..., n\}$. As before, we define the control command to be broadcast as $(\theta, d)$. We define our performance measure as the half length, denoted by $r > 0$, of the side of a square oriented along the global coordinate frame, and containing all the final positions of the agents. Let this square be centered at $z = (z_1, z_2) \in \mathbb{R}^2$.

Assuming that all the agents execute the command $(\theta, d)$, their final positions, given by $x_{if} = [q_{i1} \;\; q_{i2}] \in \mathbb{R}^2$ will be,

$$x_{if} \quad = \quad x_{i0} + R(\theta_{i0})R(\theta)\begin{bmatrix} d \\ 0 \end{bmatrix} \tag{1}$$

That is,

$$\begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} = \begin{bmatrix} p_{i1} \\ p_{i2} \end{bmatrix} + \begin{bmatrix} \cos\theta_{i0} & -\sin\theta_{i0} \\ \sin\theta_{i0} & \cos\theta_{i0} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2}$$

where, $u_1 = d\cos\theta$ and $u_2 = d\sin\theta$ are the control variables that replace $(\theta, d)$. Note that Eqn. (2) are linear equations. Now, we formulate the linear programming problem as,

Minimize $r$

Subject to

$$-r \leq p_{i1} + u_1\cos\theta_{i0} - u_2\sin\theta_{i0} - z_1 \leq r \tag{3}$$
$$-r \leq p_{i2} + u_1\sin\theta_{i0} + u_2\cos\theta_{i0} - z_2 \leq r \tag{4}$$
$$i = 1, \dots, n.$$
$$r \geq 0 \tag{5}$$

The above is a linear programming problem with the decision vector as $(r, z_1, z_2, u_1, u_2)$. Note that the decision vector remains same irrespective of the number of agents. Only the number of inequality constraint increases with the number of agents. Also, note that $z_1$,$z_2$,$u_1$ and $u_2$ are free variables and can take both positive or negative values.

## IV. Some Results on the Formation of Agents

After executing the LP the distance between $i$ and $j$ agent along $x$-axis and $y$-axis will be

$$\begin{aligned} d_{x_{ij}} &= (p_{i1} - p_{j1}) + d(\cos(\theta_i + \theta) - \cos(\theta_j + \theta)) \\ &= (p_{i1} - p_{j1}) + dC \end{aligned} \tag{6}$$
$$\begin{aligned} d_{y_{ij}} &= (p_{i1} - p_{j1}) + d(\sin(\theta_i + \theta) - \sin(\theta_j + \theta)) \\ &= (p_{i1} - p_{j1}) + dS \end{aligned} \tag{7}$$

where, $C = (\cos(\theta_i + \theta) - \cos(\theta_j + \theta))$, $S = (\sin(\theta_i + \theta) - \sin(\theta_j + \theta))$ and $i, j \in \{1 \dots n\}$. Thus the resulting distance along $x$-axis and $y$-axis are dictated by the value of $C$ and $S$.

After executing the LP, the new formation of the agent obtained is of interest. Below, we investigate some properties of the formation. For this let us define the span of the formation along $X$ and $Y$ axis as follows: Let $(x_i, y_i)$ be the position of the agents, where $i \in I = \{1, \dots, n\}$. Then define $x_{max} = \max\{x_i\}_{i \in I}$, $x_{min} = \min\{x_i\}_{i \in I}$, $y_{max} = \max\{y_i\}_{i \in I}$ and $y_{min} = \min\{y_i\}_{i \in I}$. Then the span of the formation along the $X$ and $Y$ axis are given by,

$$S_x = x_{max} - x_{min}$$
$$S_y = y_{max} - y_{min}$$

The formation is said to be square if $S_x = S_y$ and rectangular otherwise. Essentially, the spans are the length of the sides of the minimal rectangle that contains the position of all the agents. Note that the LP problem attempts to minimize the quantity $r = \max\{S_x, S_y\}$.

We first consider a very special case with three agents. Let us assume that minimal formation by three agents is a rectangle and not necessarily a square where $S_x = S_y$. Then, there are four way in which this can be occur. This is shown in Figure 3.

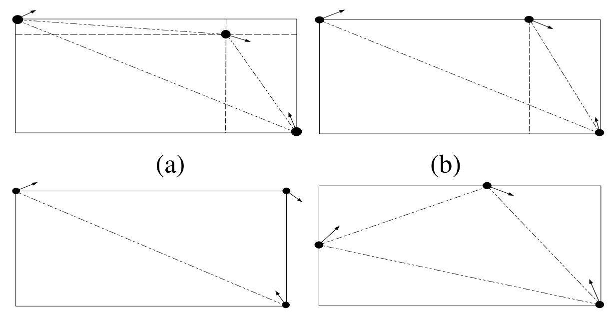

(a) (b) (c) (d)

Fig. 3. Rectangle formation by three agents (a) Two agents at corner, one in the interior (b) Two agents at corner, one on edge (c) All the agents at corner (d) One agent at a corner and two agents on edges

Before we prove some general results on the formation of the agents after the LP is executed we will state a lemma that will be useful to prove the main results.

*Lemma 1:* If $C = \cos(\theta_i + \theta) - \cos(\theta_j + \theta)$ then there exists a $\theta$ such that $C < 0$ where $\theta_i, \theta_j \in (-\pi, \pi]$ and $\theta_i \neq \theta_j$.

*Proof:* Let, $\theta = (\pi - (\frac{\theta_i - \theta_j}{2}) - \Delta\phi)$. After replacing $\theta$ in $C = \cos(\theta_i + \theta) - \cos(\theta_j + \theta)$ we will get $C = -2\sin(\frac{\theta_i - \theta_j}{2})\sin(\Delta\phi)$. It is clear from the expression that we can make $C < 0$ by choosing $\Delta\phi$ properly. ■

*Theorem 2:* After executing the LP, square is the optimal formation for three agents.

*Proof:* We will prove this by contradiction. Suppose after executing the LP, the optimal formation is a rectangle. Let the position and orientation of the three agents be $x_{i0} = (p_{i01}, p_{i02}) \in \mathbb{R}^2$ and $\theta_{i0} \in (-\pi, \pi]$, respectively, for all $i \in \{1, 2, 3\}$. Let agents $k$ and $l$ be on the left edge and right edge of the rectangle. Without loss of generality, we can assume $S_{x0} > S_{y0}$ and $\Delta d < \frac{1}{4}\min\{(S_{x0} - S_{y0}), | p_{io1} - p_{jo1} |\}$, where $i, j \in \{1, 2, 3\}$. Let us define a broadcast control command $(\theta, \Delta d)$ such that $\Delta d > 0$ is very small. After broadcasting $(\Delta d, \theta)$, for some $\theta$, the new positions will be

$$p_{i11} = p_{i01} + \Delta d\cos(\theta_{i0} + \theta)$$
$$p_{i12} = p_{i02} + \Delta d\sin(\theta_{i0} + \theta)$$

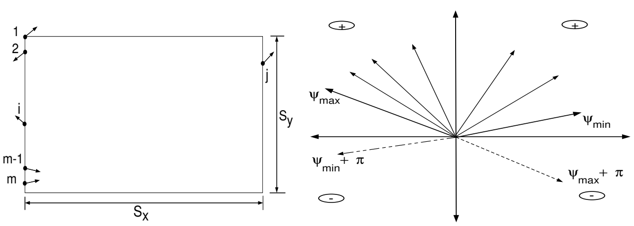
The new dimensions of the rectangle are given by $S_{x1}$ and $S_{y1}$, where $S_{x1} = p_{k11} - p_{l11}$. As $S_{x0} > S_{y0}$ and $\Delta d < \frac{1}{4}\min\{(S_{x0} - S_{y0}), | p_{io1} - p_{jo1} |\}$ then $S_{y1} < S_{x1}$.

$$\begin{aligned} S_{x1} &= S_{x0} + \Delta d\{\cos(\theta_k + \theta) - cos(\theta_l + \theta)\} \\ &= S_{x0} + \Delta dC \end{aligned}$$

According to Lemma 1, there always exists $\theta$ such that $C < 0$. As $C < 0$ then $S_{x1} < S_{x0}$. This implies that there exist a broadcast command $(\Delta d, \theta)$ such that the maximum length of the sides of the rectangle can be further reduced. This implies that the optimal solution of the LP problem for three agents can not yield a rectangle. It has to be a square.

This proof is valid for Fig. 3(a), 3(b) and 3(d). The proof for Fig. 3(c) will be taken care of by the next Theorem. ■

*Theorem 3:* When the initial conditions are such that only one agent is on one edge of the longer span and $m$ agents ($m > 1$) are on the other edge of the larger span, then the LP solution leads to a square formation.




(a) (b)

Fig. 4. An illustration for the proof of Theorem 3.

*Proof:* Consider Fig. 4 where $S_x > S_y$. The $X$ distance between agent $j$ and the other $m$ number of agent $S_x = p_{j1} - p_{i1}$, where $i \in \{1, \dots m\}$. Let $\Delta d < \frac{1}{4}(S_x - S_y)$. After broadcasting $(\Delta d, \theta)$, the new dimensions of the rectangle are $S_x^{'}$ and $S_y^{'}$. Since $\Delta d < \frac{1}{4}(S_x - S_y)$, we have $S_y^{'} < S_x^{'}$. We can write

$$\begin{aligned} S_x^{'} &= p_{j1} - p_{i1} + \Delta d(\cos(\theta_j - \theta) - \cos(\theta_i - \theta)) \\ &= S_x + \Delta d R \sin(\Phi_i + \theta) \end{aligned} \tag{8}$$

where $\Phi_i = \left(\frac{\theta_j + \theta_i}{2}\right)$ and $R$ is a positive quantity. For a particular $\theta_j$ we will get a set of angles $\{\Phi_i\}$ for $m$ agents. Let $\Phi_{max} = \max\{\Phi_i\}$ and $\Phi_{min} = \min\{\Phi_i\}$. Note that $\theta_i$ is fixed here. Let $\theta_{j1}$ contribute to $\Phi_{max}$ and $\theta_{j2}$ contribute to $\Phi_{min}$. Then $\Phi_{max} = \frac{\theta_{j1} + \theta_i}{2}$ and $\Phi_{min} = \frac{\theta_{j2} + \theta_i}{2}$. So $\Phi_{max} - \Phi_{min} = \frac{\theta_{j1} - \theta_{j2}}{2} \leq \pi$. We will get $S_x^{'} < S_x$, when $\sin(\Phi_i + \theta) < 0$. The set of angles $\{\Phi_i\}$ will be within a bounded envelope of $(0, \pi)$ as $\Phi_{max} - \Phi_{min} \in (0, \pi)$. Then there always exist an angle $\theta$ such that all the envelope will come in the lower two quadrants such that all $\sin(\Phi_i + \theta) < 0$. This implies that there $S_x^{'} < S_x$. Thus the LP solution cannot yield a rectangular formation since there will be another formation which can be achieved by broadcasting and which will have a smaller value of $\max\{S_x, S_y\}$. ■

However, there may not exist a solution when the number of agents on the opposite edges are more than two. As an example we can show that four agents with position and orientation of $((4.004, 3.9128), 0^o)$, $((4.015, 2.8264), 5^o)$,$((0.0024, 0.9302), -1^o)$ and $((0.0049, 1.9007), 0.7^o)$ will be move to $((5, 4), 5^o$, $((5, 3), 10^o)$, $((1, 1), 4^o)$, $((1, 2), 5.7^o)$. Although $r$ decreases from 2.0064 to 2 but $S_x \neq S_y$. The reason behind this can be explained as follows:

Let agent $j$ and $k$ be on one edge and a set of agent $\{i\}$ on the opposite edge where $i \in \{1, \dots, m\}$. For agent $j$ there will be a set of angles $\{\Phi_{ij}\}$ and for agent $k$ there will be a set of angles $\{\Phi_{ik}\}$. For agent $j$, $\Phi_{max_j} = \max\{\Phi_{ij}\}$ and $\Phi_{min_j} = \min\{\Phi_{ij}\}$ and for agent $k$, $\Phi_{max_k} = \max\{\Phi_{ik}\}$ and $\Phi_{min_k} = \min\{\Phi_{ik}\}$. The range of $\Phi_{max_j} - \Phi_{min_j}$ and $\Phi_{max_k} - \Phi_{min_k}$ will both be $(0, \pi)$. Now, the range of $\max\{\{\Phi_{ij}\} \cup \{\Phi_{ij}\}\} - \min\{\{\Phi_{ij}\} \cup \{\Phi_{ij}\}\}$ will be greater than $(0, \pi)$. In which case there does not exist any common $\theta$ such that the $X$ or $Y$ axis distance will be reduced. They will remain at the same position after executing the LP.

*Theorem 4:* If the solution of the LP problem yields a square formation then the number of agents on the boundary of the square is more than two.

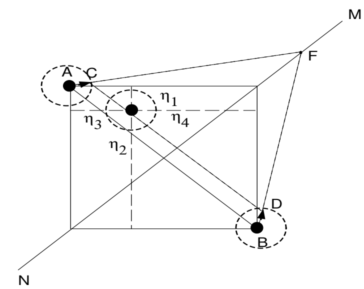



Fig. 5. An illustration for the proof of Theorem 4.

*Proof:* We will prove this by contradiction.Let, the number of agent on the square are two and they are located at diagonally opposite corners. The other agents are the interior of the square. For the sake of simplicity we will consider only three agents. This is given in Fig. 5,where $MN$ is the Voronoi edge between agents located at $A$ and $B$. According to Theorem 1, there always exists $(\theta, d)$ such that they can meet at an unique point on the Voronoi edge ($MN$). Let the unique meeting point be $F$. Let us define a very small positive quantity $\Delta d$ such that $\Delta d < min\frac{1}{4}\{\eta_1, \eta_2, \eta_3, \eta_4\}$. After broadcasting $(\theta, \Delta d)$ the agents will move from their positions. The new position of agents $A$ and $B$ are $C$ and $D$, respectively. The interior agent will remain in the interior of the square. $CD$ is the new diagonal of the square. We can show that $\triangle ABF \sim \triangle CDF$ and so $CD \| AB$ and $CD < AB$. This implies that further improvement of the square is possible. This is a contradiction. ■

## V. ACHIEVING PERFECT CONSENSUS

The solution of the linear programming (LP) problem will yield control instructions which can be broadcast to all the agents. The agents will move to a new position or within a new square region of smaller area. It can be shown that no further improvement of the performance (reduction in $r$) can be achieved by repeated use of the algorithm. In other words, repeated application of the LP algorithm with the new final positions will not reduce the value of $r$ any further.

Suppose we represent the LP algorithm as an operation $L$ on the initial conditions that yields the solution as,

$$L(x_{i0}, \theta_{i0} | i = 1, \dots, n) = (u_1^*, u_2^*, r*, x_{if}, \theta_{if}) \tag{9}$$

then,

$$L(x_{if}, \theta_{if} | i = 1, \ldots, n) = (0, 0, r*, x_{if}, \theta_{if}) \quad (10)$$

That is, there will be no further change in the performance measure $r$. In other words, $(x_{if}, \theta_{if})$ is a stationary point so far as the LP algorithm is concerned.

We can generalize this process by assuming that each step in the iteration is denoted by the index $k$, with the first step in the iteration as $k = 1$. We call this the *unperturbed case* as the solution of the LP is directly implemented by the agents without any perturbation to the solution.

*Theorem 5:* In the unperturbed case, for $k \geq 2$, $u^*_{1,k} = u^*_{2,k} = 0$ and $x_{i,k+1} = x_{i,k}$; $\theta_{i,k+1} = \theta_{i,k}$. This means that repeated use of the LP solution on subsequent positions will not reduce $r$.

*Proof:* We will prove this by contradiction. Suppose for a given initial condition $(x^1_{i0}, \theta^1_{i0})$ we have $r = r_0$ as the measure of proximity of the agents. Applying the LP algorithm we obtain $u^{1*}_1, u^{1*}_2$, $r_1$, and the final positions as $(x^1_{if}, \theta^1_{if})$. Now, considering the initial conditions as $(x^2_{i0}, \theta^2_{i0}) = (x^1_{if}, \theta^1_{if})$ and applying the LP algorithm let us assume that we get $u^{2*}_1 \neq 0, u^{2*}_2 \neq 0$, $r_2 < r_1$, and the final positions as $(x^2_{if}, \theta^2_{if})$. Then, let us define $\hat{u}_i = u^{1*}_i + u^{2*}_i$, $i = 1, 2$. It can be shown that if in the first step $\hat{u}_i$ is used it would yield a $r = r_2 < r_1$, which implies that $r_1$ was not a solution to the LP. This is a contradiction. ■

Now, consider a *perturbed case* where, the agents receive a broadcast command containing the LP solution and a command to randomly perturb the final orientation angle after the LP solution has been implemented. This process is shown in Figure 6, where, the orientation angle, after implementing the LP solution is perturbed by each agent as follows,

$$\hat{\theta}_{i,k+1} = \theta_{i,k+1} + \nu_{i,k+1} \quad (11)$$

where, the perturbation angle $\nu_{i,k+1}$ is given by $\nu_{i,k+1} = \eta_i \beta$. $\eta_i$ is a random number generated by each agent independently and $\beta$ is a scaling angle which is common to all the agents. The scaling angle can be set manually and $\eta$ can be generated through various distributions. Here, we consider both normal distribution and uniform distribution.

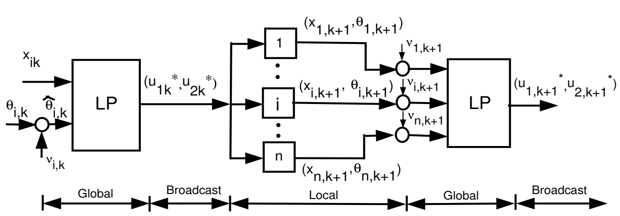



Fig. 6. The perturbed case

## VI. Achieving Positional Consensus at Desired Point

In the previous section, we consider the problem of positional consensus, but did not have control over at which the agents can meet. Suppose we have a pre-specified meeting point then we can achieve this by slightly modifying the previous formulation. In this modified form we define the meeting point as the center of the agent formation and is denoted by $(z_1, z_2)$. Now the input to the LP are the initial positions, initial orientation and the meeting point. We can formulate the modified linear programming problem as,

Minimize $r$

Subject to

$$-r \leq p_{i1} + u_1 \cos\theta_{i0} - u_2 \sin\theta_{i0} - z_1 \leq r \quad (12)$$

$$-r \leq p_{i2} + u_1 \sin\theta_{i0} + u_2 \cos\theta_{i0} - z_2 \leq r \quad (13)$$

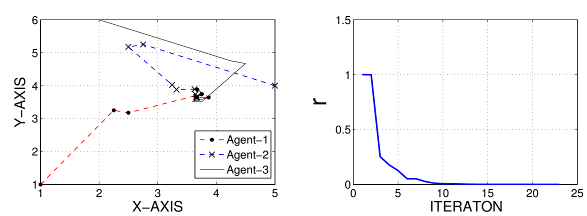
$$i = 1, \ldots, n.$$

$$r \geq 0 \quad (14)$$

The above is a linear programming problem with the decision vector as $(r, u_1, u_2)$. Note that the number of decision variables has reduced over the previous formula. In the next section we will give simulations results.

## VII. Simulation Results

In the first set of simulations we start with three agents. Initially we consider $((1,1), 45^o)$, $((5,4), 135^o)$, and $((2,6), -45^o)$ as the initial position and orientation angle of the three agents. Using the perturbation technique, with normal distribution for $\eta$ and a scaling angle of $\beta = 120^o$, the agents converge to a point after a few iterations (see Fig. 7). The variation in the $x$ and $y$ coordinates of the three agents against the number of iterations are also shown. The convergence criterion for terminating the simulation was when the value of $r$ became less that $2 \times 10^{-4}$. We continue




(a) (b)

Fig. 7. Consensus with three agents (a) Trajectory of the agents (b) Reduction in $r$ with iteration

with our study with three agents. In Fig. 8 we plot the average number of iterations needed, and the average length of path traveled by each agent, to achieve convergence, as a function of the scaling angle $\beta$ for the two cases when the random number $\eta$ is generated by a uniform distribution or by a normal distribution. These results are given by averaging over 200 trials. We also plot the standard deviations. From these results we can conclude that using larger scaling angle reduces $r$ faster than when the scaling angles are small. Also, using normal distribution gives better results than uniform distribution.

Next, we consider larger number of agents (10 and 15). We use normal distribution and scaling angle of $180^o$ for the perturbation. The convergence criterion is also relaxed

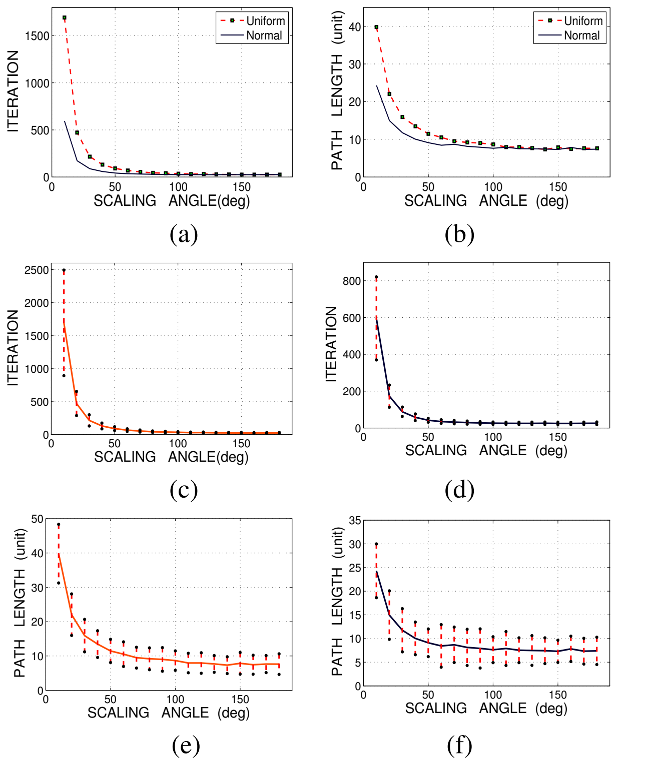



Fig. 8. Convergence results (a) Average number of iterations (b) Average length of path (c) Standard deviations for number of iterations: Uniform distribution (d) Standard deviations for number of iterations: Normal distribution (e) Standard deviations for path length: Uniform distribution (f) Standard deviations for path length: Normal distribution

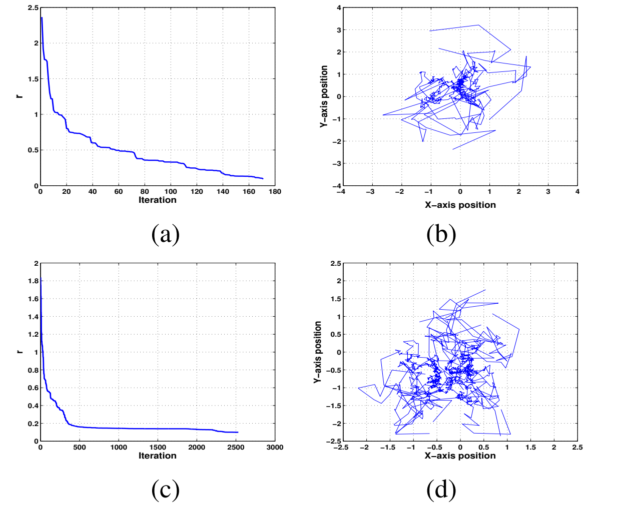



Fig. 9. Convergence results (a) $r$ vs number of iterations for 10 agents (b) Trajectory for 10 agents (c) $r$ vs number of iterations for 15 agents (d) Trajectory for 15 agents

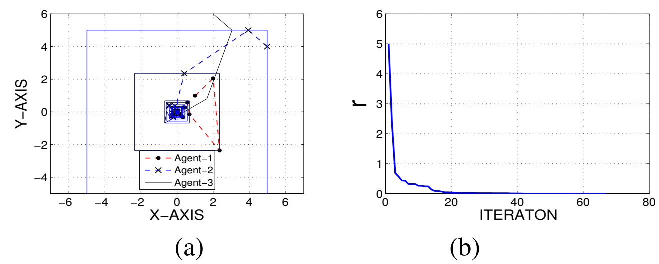



Fig. 10. Consensus with three agents at specified point (a) Trajectory of the agents (b) Reduction in $r$ with iteration

to $r \leq 0.1$. The results are shown in Figure 9. These results show that the computation time and the number of iterations rises with the number of agents. This is expected since the complexity of the algorithm is same as that of the LP algorithm.

To demonstrate the result that the agents can be meet any pre-specified point, we consider same set of initial position and orientation angle( $((1, 1), 45^o)$, $((5, 4), 135^o)$, and $((2, 6), -45^o)$ ) of the three agents. The meeting point we set as origin $(0, 0)$. The result is illustrated in Fig. 10.

## VIII. Conclusions

In this paper we considered the problem of controlling a group of agents to converge at a location using only broadcast control input (identical control) for all the agents. The results shows that it is possible for a group of agents to meet at a location by sending them a sequence of simple and exactly identical instruction. The location point can be pre-specified. We were able to show that introducing a perturbation in the broadcast command helped to achieve point convergence which was not possible earlier. We also proposed a novel linear programming based solution approach that is computationally less intensive than the SOCP technique proposed in the literature. There are several opportunities for future work in this direction. It seems possible to extend this work to consider process noise, sensor uncertainty, and the presence of obstacles in the environment. Moreover, the algorithm can most probably shown to be robust to failures in terms of packet loss or failure of agents.

## References


[1] T. Vicsek, A. Czirok, E. B. Jacob, I. Cohen and O. Schochet, Novel type of phase transistions in a system of self-driven particles , *Phys. Rev. Lett.*, vol. 75, 1995, pp 1226-1229.

[2] G. Antonelli, and S. Chiaverini, Kinematic control of platoons of autonomous vehicles, *IEEE Trans. Robotics and Automation*, vol. 22(6), 2006, pp 1285-1292.

[3] B. V. Gervasi and G. Prencipe, Coordination without communication: The case of the flocking problem, *Discrete Applied Mathematics*, vol. 143, 2004, pp 203-223.

[4] M. Cieliebak, P. Flocchini, G. Prencipe and N. Santoro, Solving the robots gathering problem , *In Proc. 30th Int. Colloq. on Automata, Language and Programming*, 2003, pp 1181-1196.

[5] I. Suzuki and M. Yamashita, Distributed anonymous mobile robots , *SIAM J. on Computing* , vol. 28, 1999, pp 1347-1363.

[6] T. Bretl, "Control of Many Agents Using Few Instructions", *in Third Robotics Science and System Conference* , Atlanta, 2007.

[7] B.R. Donald, C.G. Levey, C.D. Mcgray, I. Paprotny and D. Rus, An untethered,electrostatic, globally controllable MEMS micro-robot, *J. Microelectromech. Syst*, vol. 15(1), 2006, pp 1-15.

[8] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[9] H. Ando, Y. Oasa and I. Suzuki, Distributed Memoryless Point Convergence Algorithm for Mobule Robots with Limited Visibility, *IEEE Tran. of Robotics and Automation*, vol. 15(5), 1999, pp 818-828.

[10] W. Rudin, *Principles of Mathematicla Analysis*, McGRAW-HILL Book Company, 1976.