

# Assessment of linguistic dynamic cause-and-effect rules with delays

Ming Su and R. Russell Rhinehart

**Abstract**—A data-driven procedure is used to find linguistic rules that describe a dynamic process. In order to select valid rules, the concept of *trip* is proposed to reveal a rule status in a Truth Space Diagram (TSD). Based upon the *trip*, a normalized metric is proposed to assess a rule, which then makes the comparison possible among rules with the same antecedent but conflicting consequents. In addition, a novel rule structure is proposed to include linguistic delays. The procedure is evaluated.

## I. INTRODUCTION

Applications of linguistic rules appear in chemical process industry operation instructions and manuals, where knowledge is rendered via linguistic levels such as High, Low, Fast or Slow to be intuitively understandable. Generally, such knowledge requires no background in a particular modeling field (for instance, time series, linear systems, differential equations, etc.). The human understanding of the fundamentals is the primary source of the linguistic knowledge. However, expert knowledge is often limited to idealized experimental conditions; and, the empirical knowledge is not updated often enough to acknowledge process evolution. On the other hand, autonomous learning of linguistic rules[1] appears to be an efficient alternative to obtain knowledge directly from data.

There are generically two major approaches to categorize linguistic rules. Local approaches learn each rule individually. In [2], the criteria of *confidence factor* and *interestingness* was used to assess each rule. By contrast, global approaches tend to learn a set of rules in a parallel fashion. To assess validity of rule sets, *fidelity* [3, 4] and *complexity* of rule sets [4] were proposed. Local and global approaches do not exclude each other. In [5], a local approach selects rules to initialize a fuzzy system. Redundant rules are then merged and rules are refined in a global approach to simplify models and improve accuracy. However, the reviewed techniques are not directly applicable if process dynamics are considered. In dynamic systems, the data are no longer independent to each other in time. Dynamics and correlations among the data should be involved for rule assessment.

Lags, time delays, especially linguistically stated delays

are not usually addressed in fuzzy systems, although delay is a major concern in system identification and is often seen or prescribed by experts in a rule such as:

*IF the fluid is flowing rapidly THEN a downstream sensor can detect fluid composition change after a short delay.*

A number of research articles analyze fuzzy systems with a constant delay [6, 7] or an artificially assigned time-varying delay [8, 9]. In this paper, a new rule structure is proposed to include linguistic delays in consequents. Accordingly, the rules with linguistic delays are evaluated and assessed by a data-driven procedure. Basically, the techniques proposed in this paper tend to simulate experts that could investigate the data generated from dynamic processes and prescribe linguistic rules with a delay.

## II. METHODOLOGY

### A. Rule Structures

With any empirical modeling approach, users need determine the kind of variables (input, output-feedback, prediction-feedback, and model error feedback) as well as their dynamic orders in rules. This task in this paper is referred as rule structure determination. The rules could adopt structure directly from linear models and result in different types of rules such as finite impulse response (FIR), autoregressive with exogenous (ARX), output error (OE), and Box-Jenkins-like rules. Among them, ARX and OE are favorable choices in nonlinear systems. In general, further complication in structures would compound the already existing nonlinearities without worthwhile compensation in model qualities in terms of predictions errors and computational cost [10]. An ARX-like linguistic rule (for instance, the  $i^{\text{th}}$  rule) is defined in Rule (1)

$$\begin{aligned} \text{IF } \{ & u(t-d(t)) \text{ is } A_i \text{ AND} \cdots \text{ AND } u(t-d(t)-n_u) \text{ is } B_i \\ & \text{AND } y(t-1) \text{ is } C_i \text{ AND} \cdots \text{ AND } y(t-n_y) \text{ is } D_i \} \\ \text{THEN } & \{ y(t) \text{ is } E_i \text{ AND } d(t) \text{ is } F_i \} \end{aligned} \quad (1)$$

where, the time-dependent delay  $d(t)$  is added, which is specified in the consequent part by level  $F_i$  and used to determine the time lags between the input,  $u$  and output,  $y$ .

The dynamic orders are  $n_u$  and  $n_y$  for  $u(t)$  and  $y(t)$  respectively, which need to be specified, too. In nonlinear system identification, there are a few techniques could be used for this purpose. A Lipschitz index [11] is introduced to identify a comprehensive coverage of influential inputs while

This work was supported in part by the Edward E. and Helen Turner Bartlett Foundation.

M. Su is with the School of Chemical Engineering, Oklahoma State University, Stillwater, OK 74078-5021 USA (ming.su@okstate.edu).

R. Russell Rhinehart is with the School of Chemical Engineering, Oklahoma State University, Stillwater, OK 74078-5021 USA (rrr@okstate.edu).

the index becomes steady. A false nearest neighbors based approach that finds the embedding dimensions of a nonlinear time series is implemented to determine model orders of nonlinear input/output systems in [12]. On the other hand, practical experience could also suggest some otherwise simple *ad hoc* choices. In this paper,  $n_u$  and  $n_y$  are set to 0 and 1 respectively, which represent a (1, 0) ARX-like or a first-order plus time delay rule as shown in Rule (2). The (1, 0) choice is a common choice in chemical processes.

$$\begin{aligned} &\text{IF } \{u(t-d(t)) \text{ is } A_i \text{ AND } y(t-1) \text{ is } B_i\} \\ &\text{THEN } \{y(t) \text{ is } E_i \text{ AND } d(t) \text{ is } F_i\} \end{aligned} \quad (2)$$

In addition to the rule structures, the number and shapes of fuzzy sets for  $u(t)$  and  $y(t)$  need to be initialized as well, which determine the space partitions in  $u$  and  $y$ . Unsupervised learning approaches are probable choices if rules are to be used for classification [10]. However, heuristics in unsupervised learning suggests nothing about nonlinearities that is important in modeling tasks. This work hence starts with a uniform partition, which is not only simple but also a reasonable initialization if users know nothing about the underlying nonlinearity between  $u$  and  $y$ .

### B. Truth Analysis

Truth analysis quantifies the truth of antecedent (TA) and consequent (TC) in a rule using observed data. To compute TA, users need to know the membership function values of input variables to their fuzzy sets. The fuzzy set  $A_i$  for  $u(t)$  is described by a Gaussian membership function with the center at  $c_i$  and the length of  $\sigma_i$ . The membership function value of  $u(t)$  to  $A_i$  is then defined by

$$\mu_u^{A_i}(u(t)) = \exp\left(\frac{-(u(t)-c_i)^2}{2\sigma_i^2}\right) \quad (3)$$

If the **AND** conjunction is interpreted as a product operator, the TA of Rule (2) is then defined by

$$TA_i(t) = \mu_u^{A_i}(u(t-d(t))) \times \mu_y^{B_i}(y(t-1)) \quad (4)$$

where  $d(t)$  is unknown and not measured. In this work, however, the range of  $d(t)$  is assumed to be available. Given the minimum and maximum  $d(t)$  are integers  $d_0$  and  $d_n$  respectively, an average truth of antecedent ( $\overline{TA}$ ) is then evaluated over all possible delays

$$\overline{TA}_i(t) = \frac{\sum_{k=0}^n TA_i(t)|_{d(t)=d_k} \times \mu_d^{F_i}(d_k)}{\sum_{k=0}^n \mu_d^{F_i}(d_k)} \quad (5)$$

Following the similar approach, the average truth of consequent ( $\overline{TC}$ ) is defined by

$$\overline{TC}_i(t) = \frac{\sum_{k=0}^n TC_i(t)|_{d(t)=d_k} \times \mu_d^{F_i}(d_k)}{\sum_{k=0}^n \mu_d^{F_i}(d_k)} \quad (6)$$

where  $TC = \mu_y^{E_i}(y(t))$  in Rule (2) since it has only one output.

### C. Rule Assessments

TA and TC are able to tell how good a rule is. A Truth Space Diagram (TSD) [13] that is basically a plot of TA and TC with four disjointed quadrants, as shown in Figure 1, is used as a qualitative tool for this purpose.

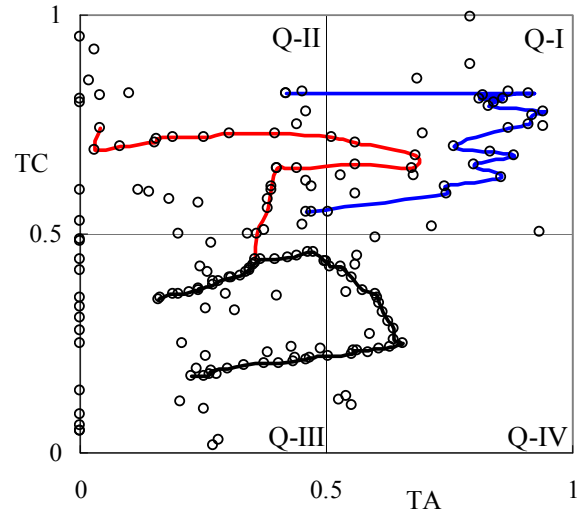


Fig 1. TSD with four quadrants split at  $\beta = 0.5$  and illustration of a validating trip in Q-I and an invalidating trip in Q-IV

Generally, there is no restriction on how to split a TSD into 4 quadrants. Users are free to choose preferred truth threshold ( $\beta$ ), which is 0.5 in the illustrated example. At each sampling time one can calculate TA and TC. The TSD then reveals the paths of points as the process evolves in time and reveals validity of an individual rule. In this paper, completeness and parsimoniousness are assumed on rules structures. If the data expresses a rule, the rule will have a path within Q-I. Therefore, markers in Q-I validate a rule. Markers in Q-IV invalidate it. Markers in Q-II and Q-III are not directly helpful for rule assessment due to low TA. Logically, it is not possible to validate or invalidate a hypothesis if its proposition is not met. A summary is given in [14] to enlist other possibilities of occurrence of TA-TC in each quadrant. For instance, the markers in Q-II might be due to antecedent over-specification of an otherwise good rule.

Random noise or spurious events could place a marker in quadrants. In this paper, a *trip* is a segment of a path of markers into a particular quadrant. In this work,  $p$  is the

minimum number of sequential markers required to define a *trip*. Validating trips ( $TR_I$ ) and invalidating trips ( $TR_{IV}$ ) are related to the quality of a rule. A validating trip is defined as a *trip* into Q-I that provides empirical evidence that the rule is consistent with data. An invalidating trip is defined as a trip into Q-IV, which reveals the inconsistency of a rule with data.

With *trips* being defined, a simple metric is proposed to assess a rule so that comparison could be made in rules with same antecedent but conflicting consequents. The metric is based on following two quantities

$$\alpha_I = \sum_{j=1}^{l_I} L(Tr_I^j) \quad (7)$$

$$\alpha_{IV} = \sum_{j=1}^{l_{IV}} L(Tr_{IV}^j)$$

where,  $L$  is the length of a *trip* (number of sequential markers along a trip) and should be greater than or equal to  $p$ .  $l_I$  and  $l_{IV}$  are the numbers of validating trips in Q-I and invalidating trips in Q-IV. Accordingly,  $\alpha_I$  and  $\alpha_{IV}$  are the total number of sequential markers in all validating and invalidating trips. Based on  $\alpha_I$  and  $\alpha_{IV}$ , a normalized metric is then defined as below to assess a rule

$$q = \frac{\alpha_I - \alpha_{IV}}{\alpha_I + \alpha_{IV}} \quad (8)$$

The  $q$ -metric reveals the consistency of a rule with observed process behavior and is normalized from 0 to 1.

### III. RULES SELECTION AND VALIDATING

Rules of a fuzzy system could be prescribed by experts. The experts' knowledge has to be thorough so that the prescribed rules are able to describe the process behavior completely. Often, such a comprehensive prescription is not available. Alternatively, data-driven procedures could be taken to find all rules automatically. Heuristics based stochastic schemes are proposed for searching efficiency [1]. In this work, in order to obtain the optimal results, an exhaustive search scheme is used instead. The exhaustive search covers all antecedents and finds each antecedent the best consequent that has the highest  $q$ -metric (8). The  $q$ -metric would select only and all valid rules expressed in the data.

The validation of the selected rules could be conducted by comparing against experts' knowledge, flow sequences as well as material or energy balance constraints, etc. The validation could also be conducted by comparing the predictions of the fuzzy system consisting of selected rules against the output measurements. In order to predict using the selected rules, Rule (2) is decoupled into two rules as below

$$\text{IF } \{u(t-d(t)) \text{ is } A_i \text{ AND } y(t-1) \text{ is } B_i\} \\ \text{THEN } \{y(t) \text{ is } E_i\} \quad (9)$$

$$\text{IF } \{u(t-d(t)) \text{ is } A_i \text{ AND } y(t-1) \text{ is } B_i\} \\ \text{THEN } \{d(t) \text{ is } F_i\} \quad (10)$$

where the fuzzy system with Rule (9) computes  $y(t)$  and Rule (10) computes  $d(t)$ . Unfortunately, calculating  $d(t)$  is inapplicable since the  $d(t)$  is required to determine  $u(t-d(t))$  in order to compute itself. This dilemma could be resolved by having the  $d(t)$  computation related to  $y(t-1)$  only, which results in following rules defined in Rule (11)

$$\text{IF } \{y(t-1) \text{ is } B_i\} \text{ THEN } \{d(t) \text{ is } F_i\} \quad (11)$$

The simplification not only makes the computation possible but also is justifiable for transportation delays. For instance, if the input is a flowrate, the transportation delay  $d(t)$  is related to the past input from  $t-d(t)$  to  $t$  and the relation between  $u(t-d(t))$  and  $d(t)$  is insignificant. Therefore, in Rule (10), the only variable that influences  $d(t)$  significantly is  $y(t-1)$ . Since  $y(t-1)$  is somehow related to the delayed input,  $d(t)$  and  $y(t-1)$  are indirectly related.

The computation starts with the time delay with the defuzzification in [15] as below

$$\hat{d}(t) = \frac{\sum_{i=1}^r TA_i^d(t) w_i^d V_i^d C_i^d}{\sum_{i=1}^r TA_i^d(t) w_i^d V_i^d} \quad (12)$$

where  $\hat{d}(t)$  is the predicted time delay of  $d(t)$  and  $r$  is the number of rules.  $TA_i^d(t)$  is simply  $\mu_{y^{B_i}}(y(t-1))$  since  $y(t-1)$  is the only variable in the antecedent.  $w_i^d$  is the weight associated with the  $i^{\text{th}}$  rule and represents contribution of the rule to the fuzzy system. Generally, more certain a rule is, the weight is higher.  $C_i^d$  and  $V_i^d$  are centroid and volume of the fuzzy set  $F_i$  and are defined by

$$C_i^d = \frac{\int_{d_0}^{d_n} x \times \mu_{F_i^d}(x) dx}{\int_{d_0}^{d_n} \mu_{F_i^d}(x) dx} \quad (13)$$

$$V_i^d = \int_{d_0}^{d_n} x \times \mu_{F_i^d}(x) dx \quad (14)$$

$V_i^d$  generally represents the uncertainty [15] of a rule and therefore relates to rule weights  $w_i^d$  inversely. The exact best relation between  $V_i^d$  and  $w_i^d$  is case dependent. In this work,

$w_i^d$  is  $(V_i^d)^{-0.9}$  and the relation is found after several trials of different exponents between -2 and 0.

The  $d(t)$  estimate is used to evaluate TA in the  $y(t)$  fuzzy system as below

$$TA_i^y(t) = \mu_u^{A_i} (u(t - \hat{d}(t))) \times \mu_y^{B_i} (y(t-1)) \quad (15)$$

with  $TA_i^y(t)$  evaluated, defuzzification is applied again to obtain to the output prediction

$$\hat{y}(t) = \frac{\sum_{i=1}^r TA_i^y(t) w_i^y V_i^y C_i^y}{\sum_{i=1}^r TA_i^y(t) w_i^y V_i^y} \quad (16)$$

The performance index is then defined by a normalized squared error (*nse*)

$$nse = \sum_{t=1}^N \frac{(y(t) - \hat{y}(t))^2}{(y(t) - \bar{y})^2} \quad (17)$$

where  $\hat{y}(t)$  is the output prediction and  $\bar{y}$  is the time average of  $y(t)$ .

#### IV. CASE STUDY

In this section, the linguistic rules are selected and validated for a fluid mixing process.

Figure 2 illustrates a fluid mixing process, where fluid with different temperatures ( $T1$  and  $T2$ ) and flowrates ( $F1$  and  $F2$ ) are mixed. In this process the  $T3$  response to any of the 4 inputs is nonlinear. The response delay of  $T3$  to a process change is dependent on the  $F1+F2$  history. This process has both nonlinear gain and dynamics, yet they are simple and well-understood, making it an ideal testing case. In this paper, a simplified situation is considered instead, where  $T1$ ,  $T2$  and  $F2$  are constants and only the dynamics of  $F1$  and  $T3$  are considered.

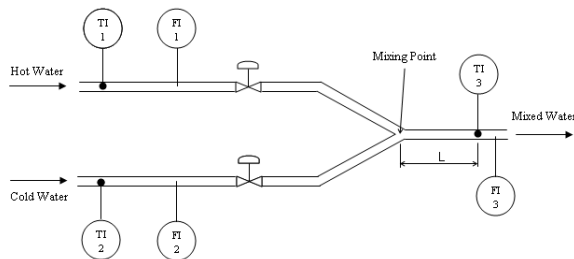


Fig 2. The fluid mixing process

Figure 3 presents time-series data of input,  $F1$  ( $u$ ), and output,  $T3$  ( $y$ ), data. A random signal is added to  $u(t)$  to simulate flowrate measurement noise.

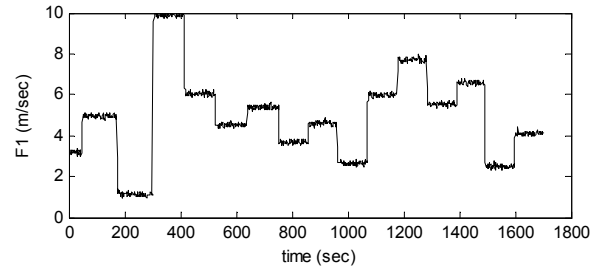


Fig 3.a. The input signal  $F1$ (m/sec)

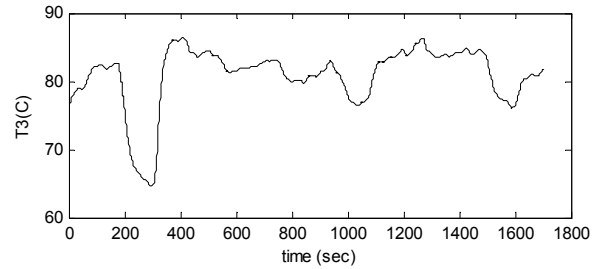


Fig 3.b. The output response of  $T3$  (C)

In this case, delays are assumed to be integer multiples of sampling interval between 0 and 20. Each variable is assigned three fuzzy membership functions labeled as High (H), Medium (M) and Low (L). The resulting specification of membership functions for  $u$  and  $y$  are listed in Table I, where “ $c$ ” and  $\sigma$  represent the center and width of Gaussian membership functions.

	$c$	$\sigma$
$u$ (L)	0.94	1.9433
$u$ (M)	5.52	1.9433
$u$ (H)	10.09	1.9433
$y$ (L)	64.59	4.64
$y$ (M)	75.51	4.64
$y$ (H)	86.43	4.64
$d$ (L)	1.0	4.03
$d$ (M)	10.5	4.03
$d$ (H)	20.0	4.03

With the truth threshold of  $0.5^2$  and the minimum number of sequential markers of 10, the following 8 rules in Table II are selected.

Based upon the process characteristics, the output,  $T3$ , should continuously change, which implies that adjacent  $T3$  measurements differ infinitesimally. This fundamental dynamical restriction is clearly observed in the selected rules, where each rule has a pair of identical fuzzy levels for  $y(t-1)$  in antecedents and  $y(t)$  in consequents.

There could be 9 total rules in Table II; however, all rules are reported except the one with Medium  $u(t-d(t))$  and Low  $y(t-1)$ . The reason is that there is not sufficient data in the experimental sequence of Figure 3 to express that antecedent.

TABLE II.  
SELECT 8 RULES WITH  $\beta$  OF 0.25 AND  $p$  OF 10

Rule	$u(t-d(t))$	$y(t-1)$	$y(t)$	$d(t)$
1	L	L	L	H
2	L	M	M	L
3	L	H	H	L
4	M	M	M	L
5	M	H	H	H
6	H	L	L	L
7	H	M	M	L
8	H	H	H	H

A fuzzy system with the 8 selected rules is used for prediction as shown in Figure 4, where prediction is dashed line of  $yhat(t)$ . The normalized squared error ( $nse$ ) is  $0.0093 \text{ } ^\circ\text{C}^2$ .

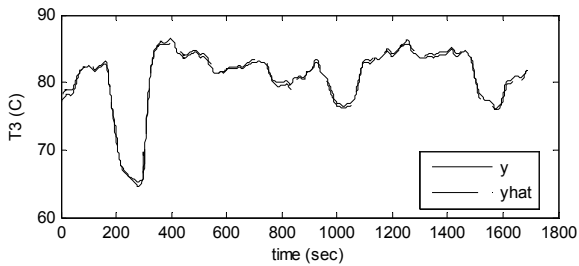


Fig.4 Output prediction  $yhat$  vs. output measurement  $y$  using selected 8 rules

In order to explore the validity of the selected rules to validate the proposed technique, the following comparison is made by replacing some of the 8 selected rules with unselected rules. For instance, if High  $y(t)$  is replaced by Medium  $y(t)$  in the consequent of the first rule, the resulting  $nse$  is then  $0.2483 \text{ } ^\circ\text{C}^2$ . As observed in Figure 5, the replacement results in a significant process-model mismatch between 200 and 400 seconds.

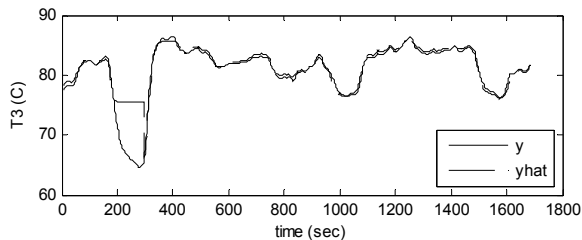


Fig.5 Output prediction  $yhat$  vs. output measurement  $y$  with the replacement of the Rule 1 by an unselected one

## V. DISCUSSION AND PERSPECTIVES

As mentioned in Section IV, two threshold numbers are chosen in order to count trips, the truth threshold  $\beta$  to split a TSD into 4 quadrants and the minimum number of sequential markers,  $p$ , to define a *trip*. It is desired to have the algorithm to be robust to the choice of them.

The initial  $\beta$  might be easy to set if users have any preference for truth value, like  $0.5^2$  used in this paper. The initial choice of  $p$  should be based on the number of samples to reject spurious events. For the fluid mixing process, the following experimental study is conducted over a number of choices of  $p$ , where it is observed that a wide range of choices of  $p$  from 5 to 15 are acceptable with  $nse$  at  $0.0093 \text{ } ^\circ\text{C}^2$ . It could conclude that the proposed technique is robust to the choice of  $p$  as long as it is not any small or large extreme numbers. An underestimated  $p$  might cause uncertainty in counting trips and produce unqualified trips, especially when there is presence of process noise. Several random markers (less than the desired trip length) might “jump” into the Q-I to be recognized as a validating trip due to noise. An inappropriately large  $p$  makes it very difficult to find a trip, which makes the algorithm unable to assess rules or result in statistically insignificant selection with far less number of trips.

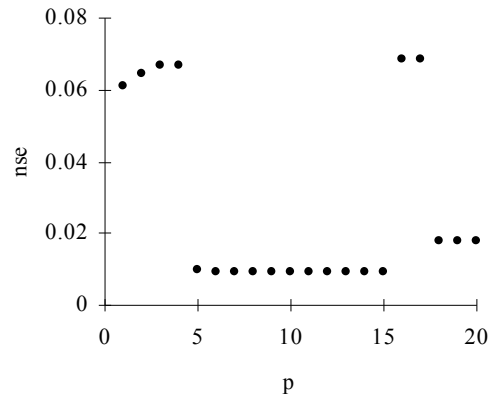


Fig. 6 Influence of the minimum number of sequential markers,  $p$ , on the prediction ability of the resulting fuzzy system

Figure 7 shows the experimental investigation of influence of  $\beta$  on resulting fuzzy systems given  $p$  at 10 as a reasonable value suggested by Figure 6.

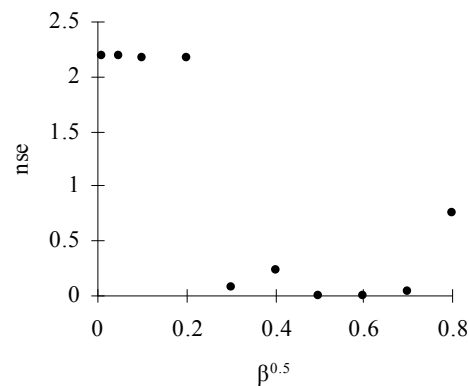


Fig. 7 Influence of the truth threshold,  $\beta$ , on the prediction ability of the resulting fuzzy system

As for the  $\beta$ , it seems that one does not have as many choices as for  $p$ . As observed in Figure 7, there are only two

acceptable  $\beta$ ,  $0.5^2$  and  $0.6^2$  with *nse* at 0.0093 and 0.0094 respectively. Fortunately, the  $\beta$  choice of  $0.5^2$  or  $0.6^2$  meets our intuition on truth. As a matter of fact, inappropriately small  $\beta$  would behave as poorly as too large  $\beta$  since both make no distinction on validating and invalidating trips. A too small  $\beta$  labels all trips as validating ones while an inappropriately large  $\beta$  admit no validating trips at all.

In addition to the above mentioned choices, other choices have to be made based on users' preference. The proposed technique has no restriction on choice of membership functions, although Gaussian functions are preferred in this work. There is neither restriction on types of logical operators for **AND** conjunction. A good summary of possible choices is found in [16], where every technique could be applied. In this paper, the choice of product operator is due to its well-known characteristics for better numerical accuracy [15] as well as the differentiability of the resulting fuzzy system, which could be further exploited to optimize model parameters. Instead of uniform partition of input space, other heuristics could certainly be applied if users prefer.

In this work, a simple rule structure, (1,0) ARX is applied. As mentioned in Section II, other types of rule structures (OE, ARMA and Box-Jenkins) could also be used. However, users should be cautious in using complex rule structures. The exponential growth of computational cost would easily make a complex choice inapplicable. For most cases, especially chemical processes, the preference is set to ARX or OE structure.

## VI. CONCLUSION

The use of the Truth Space Diagram and a  $q$ -metric to select valid rules is demonstrated in a case study of a fluid mixing process. The proposed procedure is able to discover valid rules that produce useful predictive models when combined as a rule set. In addition, the technique is robust to the algorithm configurations.

## REFERENCES

[1] O. Cordon, F. Herrera, F. Gomide, F. Hoffmann, and L. Magdalena, "Ten years of genetic fuzzy systems: current framework and new trends," in IFSA World Congress and 20th NAFIPS International Conference, 2001. Joint 9th. vol. 3 Vancouver, Canada, 2001, pp. 1241-1246.

[2] A. Ghosh and B. Nathb, "Multi-objective rule mining using genetic algorithms," *Information Sciences*, vol. 163, pp. 123-133, Jun 2004.

[3] P.P. Angelov, "An evolutionary approach to fuzzy rule-based model synthesis using indices for rules," *Fuzzy Sets and Systems*, vol. 137, pp. 325-338, Aug 2003.

[4] L. Baron, S. Achiche, and M. Balazinski, "Fuzzy decision support system knowledge base generation using a genetic algorithm," *International Journal of Approximating Reasoning*, vol. 28, pp. 125-148, 2001.

[5] Y. Jin, "Fuzzy Modeling of High-Dimensional Systems: Complexity Reduction and Interpretability Improvement," *IEEE Transactions on Fuzzy Systems*, vol. 8, pp. 212-221, Apr 2000.

[6] C.L. Chen, G. Feng, and X.P. Guan, "Delay-Dependent Stability Analysis and Controller Synthesis for Discrete-Time T-S Fuzzy Systems with Time Delays," *IEEE Transactions on Fuzzy Systems*, vol. 13, pp. 630-643, Oct 2005.

[7] H.N. Wu, "Delay-dependent stability analysis and stabilization for discrete-time fuzzy systems with state delay: a fuzzy Lyapunov-Krasovskii functional approach," *IEEE Transactions on Systems, Man and Cybernetics, Part B: Cybernetics*, vol. 36, pp. 954-962, Aug 2006.

[8] H.J. Lee, J.B. Park, and Y.H. Joo, "Design of fuzzy-model-based controller for time-varying input-delayed TS fuzzy systems," in *IEEE International Symposium on Industrial Electronics, 2001. Proceedings. ISIE 2001. vol. 3, Jun 2001*, pp. 1833-1838.

[9] Z. Zuo and Y. Wang, "Robust Stability Criteria of Uncertain Fuzzy Systems with Time-varying Delays," in *2005 IEEE International Conference on Systems, Man and Cybernetics. vol. 2, Oct 2005*, pp. 1303-1307.

[10] O. Nelles, *Nonlinear System Identification: From Classical Approaches to Neural Networks and Fuzzy Models*. New York: Springer, 2001.

[11] X. He and H. Sada, "A new method for identifying orders of input-output models for nonlinear dynamic systems," in *American Control Conference, San Francisco, USA, 1993*, pp. 2520-2524.

[12] C. Rhodes and M. Morari, "Determining the model order of nonlinear input/output systems directly from data," in *American Control Conference, Seattle, USA, 1995*, pp. 2190-2194.

[13] N. Sharma and R.R. Rhinehart, "Autonomous creation of process cause and effect relationships: metrics for evaluation of the goodness of linguistic rules," in *American Control Conference, 2004*, pp. 4511-4516.

[14] G. Arora, "On the use of a Truth-Space diagram for assessing linguistic rules", in *Department of Chemical Engineering. Master: Oklahoma State University, 2007*.

[15] B. Kosko, *Fuzzy Engineering*. New York: Prentice Hall, 1996.

[16] E. S. Lee and Q. Zhu, *Fuzzy And Evidence Reasoning*. New York: Physic-Verlag, 1995.