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Variable Structure Based Switching Adaptive Control for a Class of Unknown Switched Linear Systems

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Abstract— **In this paper, we discuss the model reference adaptive control problem of switched linear systems with unknown parameters. Since we do not have information about the system parameters nor the possible abrupt changes of parameters, conventional integral adaptive law do not function well for switched systems. We propose a variable structure based adaptive controller to the switched system and show error convergence and signal boundedness for a class of switching signals by multiple Lyapunov function theory. A sufficient condition for stability and error convergence of the switched system with unknown parameters is given. To improve the chattering phenomenon, we propose a switching adaptive controller that switches between the leakage type and the VS adaptive controller by hysteresis switching algorithm. Simulation results are provided to support the analysis.**

I. INTRODUCTION

A switched system consists of a family of continuous subsystems and the switching signal which chooses the active subsystem at switching time instants. Switched systems may occur in real world, for example, when operating environment suddenly varies or parameter variations, etc. Stability analysis and stabilization of switched systems have attracted attentions in recent years. There are increasing applications that require more thorough switched system theory in order to seek better solutions. So far, there are various approaches and results for such problems, such as those presented in papers [1], [2], [3], [4], and the reference book [5]. For linear switched systems, if the switching signal is known or pre-specified, the stability analysis can be carried out by direct manipulation with transition matrix during each switching duration, e.g., [6] and [7]. For nonlinear switched systems, Lyapunov theory is still useful and important tools for stability analysis of such systems include one with multiple Lyapunov function (MLF)[8], another with common Lyapunov function, and the last with switched Lyapunov function [9].

In this paper, we discuss the model reference adaptive control (MRAC) problem of switched systems with unknown parameters. Given a reference model, we want to design the adaptive controller such that the output of the switched plant system can track the output of the reference model. Although switched systems can be deemed as the class of time-varying parametric systems, the parameter variations are even abrupt in nature. Conventional adaptive control techniques claimed to be able to cope with this class of systems usually assume that the parameter's variations should be slow, smooth, or parameterizable. However, switched systems inevitably have abrupt changes in plant parameters and thus conventional adaptive control techniques can hardly handle such situations so as to meet the stabilization purpose.

There two approaches to discuss this problem. One is robust adaptive control and the other is supervisory switching control. Robust adaptive control approach aimed to design the controller that can tolerate parameter jumps while maintain the desired property. For example, [10], [11] and [12]. In this approach, detection of plant switches is not necessary. Supervisory switching control approach switches the controllers between a family of pre-specified adaptive controllers to improve the performance or robustness. The works [13], [14], [15] can be classified into this appraoch. Though supervisory control approach claims good performance for many situations, analysis of this approach to switched systems is not presented. One of the main problem is switched system analysis. In [15], a general methodology called multiple model adaptive control (MMAC) is proposed to deal with linear time invariant systems. The controller is capable of adapting rapidly to an unknown environment. This idea has received great attention and is shown to be applicable to many applications. Despite that good performance of applying this approach to the switched linear time invariant (LTI) systems has been observed via simulations, rigorous stability analysis of MMAC applied to switched systems remain missing in that paper. The problem we considered involves theories of adaptive control, switching control, and stability of switched systems. We utilize the output feedback variable structure(VS) adaptive controller to model reference control problem for a class of switched systems. Though the performance of VS adaptive controller to parameter variations has been shown to be good by simulations [10], there is no stability results for MRAC of switched systems in existing results. In this paper, we show that if the switching signals satisfy some conditions, the tracking error will converge to zero rather than being small in the mean square sense. Stability properties are analyzed by MLF and some simulations are provided to validate the proposed controller.

The remainder of this paper is organized as follows. Preliminaries and problem formulation are given in section II. Variable structure based adaptive control of switched systems are proposed in section III. are guaranteed under suitable assumptions on switching signals using the VS

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adaptive controller. In section IV, some simulation results are presented and the conclusions are given in section V.

II. Preliminaries and Problem Statement

A. Preliminaries

Switched systems are a class of hybrid systems that consisting of a family of continuous subsystems and a switching rule that governs the switching among subsystems. Consider the switched system

$$\dot{x} = f_{\sigma(t)}(x), \quad \sigma(t) \in \mathbb{P} \triangleq \{1, 2, \ldots, P\} \tag{1}$$

which consists of subsystems $f_i(x)$, $i = 1, 2, ...P$, and the piecewise constant switching signal $\sigma : [0, \infty) \to \mathbb{P}$. Solutions of switched systems are in the sense of *Carathéodory*, that is, they are those which satisfy the integral equation

$$x(t) = x(t_0) + \int_{t_0}^{t} f_{\sigma(t)}(\tau) d\tau$$

The switched systems we consider in this paper have an identity reset map which means that the instantaneous jumps of the continuous state are identity. Thus the state trajectory is continuous everywhere. For a given single switching signal, the switched system can be viewed as a special case of the time-varying system. Generally, study of switched systems associates a family of admissible switching signals $\mathcal{S}$ and discusses the properties of solutions to the switched system.

There are many results of stability theories of switched systems in recent years. Multiple Lyapunov function (MLF) is a useful tool to study stability of switched systems.

MLF Theorem [8]: For switched system (1), if (i) for each subsystem f_i, there exists a Lyapunov function V_i and $\dot{V}_i \leq 0$ ($\dot{V}_i < 0$), $\forall i \in \mathbb{P}$, along the solution, and (ii) for every pair of switching time instants $T_m < T_r$ such that $\sigma(T_m) = \sigma(T_r) = i \in \mathbb{P}$ with $\sigma(t) \neq i$ for $T_m < t < T_r$,

$$V_i(T_r) - V_i(T_m) \leq 0 (< 0). \tag{2}$$

Then the switched system is stable (asymptotically stable).

Condition (ii) means that the values of V_i at the beginning of each time interval with $\sigma(t) = i$ should be non-increasing. We will use this idea to discuss stability results of adaptive control with switched systems.

B. Problem statement

Consider the SISO switched system

$$y_p = W_{p\sigma(t)}(s)u = k_{p\sigma(t)} \frac{Z_{p\sigma(t)}(s)}{R_{p\sigma(t)}(s)} u \tag{3}$$

where $\sigma : [0, \infty) \to \mathbb{P}$ is the switching signal that governs the switching sequence of the switched system. The transfer functions $W_{pi}(s)$, $i \in \mathbb{P}$, are strictly proper and parameters of them are all unknown. The reference model is given by

$$y_m = W_m(s)r = k_m \frac{Z_m(s)}{R_m(s)} r \tag{4}$$

where r is the reference input and y_m is the reference output. Only the input and output can be measured. The control purpose is to design the output feedback control u such that the output of switched plant tracks the reference output as good as possible, or, make the output error $e_1 = y_p - y_m$ as small as possible.

We denote the switching signal as

$$\sigma : \{(T_1, \sigma(T_1)), \ldots, (T_r, \sigma(T_r)), \ldots\}$$

where $\sigma(T_1), \sigma(T_2), \ldots \in \mathbb{P}$ are indices of active subsystems and $T_1, T_2, \ldots$ are time instants at which the system is switching. Throughout this paper, the switching signal is assumed to be right continuous, that is, $\lim_{t \to T^+} \sigma(t) = \sigma(T)$. The switching signals are *nonzeno*, which means that the number of switchings will be finite in any finite time interval, and the switching durations $\tau_r \triangleq T_{r+1} - T_r > 0$ for all positive integers r. Moreover, the switching signals will not stop switching after finite switches.

In this MRAC problem of switched systems, we do not have the information about the time instants at which the switchings occurred nor the knowledge of the active subsystem. The following assumptions for MRAC are made [16]: (A1) For all the transfer functions W_{pi}, $i \in \mathbb{P}$, $R_{pi}(s)$ is of order n and $Z_{pi}(s)$ is of order $n-1$. This means, W_{pi} has relative degree $n^* := 1$. (A2) The reference model has the same relative degree n^* as the plant $W_{pi}(s)$ (A3) All the plants and the reference model are completely controllable and observable. (A4) W_{pi} is minimum phase for all $i \in \mathbb{P}$. (A5) The signs of k_{pi} and k_m are *all positive*.

III. Switching Adaptive Control of Linear Switched Systems

A. MRAC of switched systems using integral adaptive law

Suppose that the transfer function of reference model $W_m(s)$ is strictly positive real (SPR), and for $i \in \mathbb{P}$, the state space representation of each plant transfer function

$$y_p = W_{pi}(s)u = k_{pi} \frac{Z_{pi}}{R_{pi}}(s)u \tag{5}$$

is

$$\begin{aligned} \dot{x}_p &= A_{pi}x_p + b_{pi}u, \\ y_p &= h^T x_p \end{aligned} \tag{6}$$

Note that in (6), we assume the output channel is fixed, (for example, the observer canonical form). It should be noticed that for the switched system (3), if the controller canonical from is considered, there will be output "jumps" at switching time instants due to parameter switches. As mentioned in previous, in this paper we consider the case that the reset map is identity and thus the realizations of the plant transfer functions are restrictive to the from of (6).

For each subsystem with index $i \in \mathbb{P}$, we know that there exists $\theta_i^* = [k_i^*, \theta_{1i}^{*T}, \theta_{0i}^*, \theta_{2i}^{*T}]^T \in \mathbb{R}^{2n}$ such that when $u = \theta_i^{*T} \omega$ with

$$\begin{aligned} \dot{\omega}_1 &= \Lambda \omega_1 + lu \\ \dot{\omega}_2 &= \Lambda \omega_2 + ly_p \end{aligned} \tag{7}$$

where $\Lambda \in \mathbb{R}^{(n-1)\times(n-1)}$, $\det(sI-\Lambda) = \lambda(s) = s^{n-1} + \lambda_{n-2}s^{n-2} + .. + \lambda_1 s + \lambda_0$ is a designed monic Hurwitz polynomial that contains $Z_m(s)$ as a factor, ω_1, $\omega_2 \in \mathbb{R}^{n-1}$, $l = [1,0,...0]^T \in \mathbb{R}^{n-1}$, and $\omega = [r, \omega_1^T, y_p, \omega_2^T]^T \in \mathbb{R}^{2n}$, then $y_p = W_{pi}(s)u = W_m(s)r$. Since parameters of the plants are unknown, θ_i^* are unknown. Use θ as the estimation of θ_i^*, and $\theta = [k, \theta_1^T, \theta_0, \theta_2^T]^T$. Then with this certainty equivalence principle controller

$$u = \theta^T \omega, \tag{8}$$

and if the there is no switching, we have

$$y_p = W_m(s)(r + \frac{1}{k_i^*}(\tilde{\theta}_i^T \omega))$$

where $\tilde{\theta}_i = \theta - \theta_i^*$, $i \in \mathbb{P}$. Now consider the switched plant case with the switching signal $\sigma(t)$. Let $e_1 = y_p - y_m$, then use $u = \theta^T \omega$ will lead to the error equation

$$e_1 = \frac{1}{k_\sigma^*} W_m(s)(\tilde{\theta}_\sigma^T \omega) \tag{9}$$

Let $\mathbf{x}_p = [x_p^T, \omega_1^T, \omega_2^T]^T \in \mathbb{R}^{3n-2}$, then the state space representation of the closed-loop systems would be

$$\begin{aligned} \dot{\mathbf{x}}_p &= \begin{bmatrix} A_{p\sigma} + b_{p\sigma}\theta_{0\sigma}^* h^T & b_{p\sigma}\theta_{1\sigma}^{*T} & b_{p\sigma}\theta_{2\sigma}^{*T} \\ l\theta_{0\sigma}^* h^T & \Lambda + l\theta_{1\sigma}^{*T} & l\theta_{2\sigma}^{*T} \\ lh^T & 0 & \Lambda \end{bmatrix} \mathbf{x}_p \\ &+ \begin{bmatrix} b_{p\sigma} \\ l \\ 0 \end{bmatrix} k_\sigma^* r + \begin{bmatrix} b_{p\sigma} \\ l \\ 0 \end{bmatrix} (\theta - \theta_\sigma^*)^T \omega \\ y_p &= [h^T,\ 0,\ 0]\mathbf{x}_p, \end{aligned} \tag{10}$$

or,

$$\begin{aligned} \dot{\mathbf{x}}_p &= A_{m\sigma}\mathbf{x}_p + B_{m\sigma} r + B_{p\sigma}(\tilde{\theta}_\sigma^T \omega) \\ y_p &= C^T \mathbf{x}_p \end{aligned} \tag{11}$$

where

$$C^T(sI - A_{mi})^{-1} B_{mi} = W_m(s), \quad \forall i \in \mathbb{P}. \tag{12}$$

Note that for all $i \in \mathbb{P}$, $B_{mi} = k_i^* B_{pi}$, and $k_i^* = \frac{k_m}{k_{pi}}$. From (12), the reference model can be realized by the nonminimal state space representation

$$\begin{aligned} \dot{\mathbf{x}}_m &= A_{m\sigma}\mathbf{x}_m + B_{m\sigma} r \\ y_m &= C^T \mathbf{x}_m \end{aligned} \tag{13}$$

and if we define $\mathbf{e} = \mathbf{x}_p - \mathbf{x}_m$, then the error equation of (9) can be realized as

$$\begin{aligned} \dot{\mathbf{e}} &= A_{m\sigma}\mathbf{e} + B_{p\sigma}(\tilde{\theta}_\sigma^T \omega) \\ e_1 &= C^T \mathbf{e} \end{aligned} \tag{14}$$

We should note that system (14) is a switched system and switches between stable systems may lead to an unstable system. Since $W_m(s)$ is SPR, from MKY lemma (see [16]) we know that for $i \in \mathbb{P}$, given $Q_i > 0$, there exists $P_i = P_i^T > 0$ such that

$$A_{mi}^T P_i + P_i A_{mi} = -Q_i < 0 \tag{15}$$

and

$$P_i B_{mi} = C, \quad \text{or,} \quad P_i B_{pi} = \frac{1}{k_i^*} C = \frac{k_{pi}}{k_m} C \tag{16}$$

If we employ the gradient adaptive law

$$\dot{\theta} = -\mathrm{sgn}(k_{p\sigma}) e_1 \omega = -e_1 \omega, \tag{17}$$

then for each subsystem, the multiple Lyapunov function can be defined as

$$V_i = \mathbf{e}^T P_i \mathbf{e} + \tilde{\theta}_i^T \tilde{\theta}_i, \quad i = \{1, 2, \ldots, P\}$$

and

$$\dot{V}_i = -\mathbf{e}^T Q_i \mathbf{e} \le 0,$$

for each subsystem. This means that if the plant is non-switching, signal boundedness and error tracking can be derived by Lyapunov theorem and Barbalat's lemma. However, switches between stable systems may lead to unstable. According to the MLF theorem, the decreasing condition of V_i at switching time instants (condition (ii) of MLF theorem) is not guaranteed and thus stability of the switched system can not be concluded. The stability problem is mainly caused by the parameter variations. It is possible that for $T_m < T_r$ such that $\sigma(T_m) = \sigma(T_r) = i \in \mathbb{P}$ with $\sigma(t) \ne i$ for $T_m < t < T_r$,

$$\tilde{\theta}(T_r) = \theta - \theta_{\sigma(T_r)}^* > \tilde{\theta}(T_m) \tag{18}$$

which may lead to $V_i(T_r) - V_i(T_m) > 0$. That is, overall, the energy function V may be increasing.

Remark 1. In [12], a robust adaptive controller is proposed for the linear time varying (LTV) systems with jump parameters. Robust adaptive law with projection is employed to cope with the time varying and jump parameters. There is no strategy to handle the switched parameters except the projection adaptive law to bound the parameter estimation. The tracking error is shown to be small in the mean square sense. In [11], the authors proposed a leakage type robust adaptive controller for switched systems. The problem is formulated as a continuous system with impulse and step change in input at the switching time instants. Thus stability analysis of switched systems are avoided.

B. VS-based adaptive controller

From analysis above we find that transient performance of the adaptive system is crucial to the stability of switched systems. Since during each non-switching time interval, all subsystems are stable and the reason that may cause unbounded signals is the transient response. Motivated by [10], [17], and [18], we propose a variable structure based adaptive controller with robust adaptive law for the switched system. Consider the control

$$u = \theta^T \omega + u_{vs} \tag{19}$$

where

$$u_{vs} = -\mathrm{sgn}(e_1)(\beta_1 \|\omega\|) \tag{20}$$

Here $\|\omega\| = (\omega^T \omega)^{\frac{1}{2}}$ and β_1 is a constant that satisfies $\beta_1 \ge \max_{i \in \mathbb{P}} \|\theta_i^*\|$. The adaptive law is designed with a leakage term by

$$\dot{\theta} = -(e_1 \omega + \gamma \theta) \tag{21}$$

Then the error equation of the switched system with adaptive controller (19) is

$$\begin{aligned}\dot{\mathbf{e}} &= A_{m\sigma}\mathbf{e}+B_{p\sigma}(\tilde{\theta}_\sigma^T\omega+u_{vs})\\ e_1 &= C^T\mathbf{e}\end{aligned} \tag{22}$$

Define the multiple Lyapunov function

$$V_i=\mathbf{e}^TP_i\mathbf{e}+\frac{1}{k_i^*}\theta^T\theta,\quad i=\{1,2,...,P\}.$$

Then the time derivative of V_i along the ith subsystem is

$$\begin{aligned}\dot{V}_i &= -\mathbf{e}^TQ_i\mathbf{e}+\mathbf{e}^TP_i(B_{pi}(\tilde{\theta}_i^T\omega+u_{vs}))-\frac{1}{k_i^*}\theta^T(e_1\omega+\gamma\theta)\\ &= -\mathbf{e}^TQ_i\mathbf{e}+\frac{1}{k_i^*}e_1[(\theta-\theta_i^*)^T\omega-\mathrm{sgn}(e_1)(\beta_1\|\omega\|)]\\ &\quad -\frac{1}{k^*}\theta^T(e_1\omega+\gamma\theta)\\ &\le -\mathbf{e}^TQ_i\mathbf{e}-\frac{1}{k_i^*}|e_1|(\beta_1\|\omega\|-|\theta_i^{*T}\omega|)-\frac{\gamma}{k_i^*}\theta^T\theta\\ &\le -\mathbf{e}^TQ_i\mathbf{e}-\frac{\gamma}{k_i^*}\theta^T\theta<0\end{aligned} \tag{23}$$

Thus the output tracking error e_1 will decrease at least exponentially during non-switching time intervals. If we can show that condition (ii) of MLF theorem holds, that is,

$$V_i(T_r)\le V_i(T_m),$$

where $T_m<T_r$ such that $\sigma(T_m)=\sigma(T_r)=i\in\mathbb{P}$ with $\sigma(t)\neq i$ for $T_m<t<T_r$, then we can conclude that $e_1\to 0$ as $t\to\infty$ by MLF stability Theorem. For all $i,\ j\in\mathbb{P}$, there exist two positive constants M_1 and m_1 such that

$$M_1\ge\frac{\max_{i\in\mathbb{P}}\lambda_{\max}(P_i)}{\min_{j\in\mathbb{P}}\lambda_{\min}(P_j)}, \tag{24}$$

and

$$0<m_1\le\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)}, \tag{25}$$

where $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ stand for the maximum and minimum of eigenvalues of matrix A. Existence of M_1 and m_1 is guaranteed by the positiveness of P_i and Q_i. Define $m_2=\min\{m_1,\ \gamma\}$. From (23), we have

$$\dot{V}_i\le -m_2V_i \tag{26}$$

Define

$$M_2=\max\{M_1,\ \max_{i\in\mathbb{P}}\frac{1}{k_i^*}\}. \tag{27}$$

Then

$$V_j(t)\le M_2V_i(t),\quad \forall i,j\in\mathbb{P} \tag{28}$$

If the switching signal switches slower than the convergence rate, we can show that condition (ii) of MLF theorem will be satisfied and thus all signal are bounded and $e_1\to 0$ as $t\to\infty$. From the analysis of dwell time switching system and average dwell time switching system discussed in [20], we derive the sufficient condition for stability of the VS-based MRAC of switched linear systems.

Theorem 1. (Dwell time switching): If the switching signal $\sigma(t)$ has dwell time $\tau_d\ge\frac{\ln M_2}{m_2}$, that is, time intervals between switchings are always greater than τ_d, then the relative degree 1 switched system (3) with variable structure adaptive control (19), (20 and (21) is stable and $e_1\to 0$ as $t\to\infty$.

Proof. The system is shown to be exponentially convergent during the non-switching time interval from (26). Thus if condition (ii) of MLF Theorem is satisfied, we can conclude the switched system stability from multiple Lyapunov function. Suppose that the switching signal has dwell time τ_d. Without loss of generality, assume that at switching instant T_i, $\sigma(T_i)=i$ and at time T_{i+1}, $\sigma(T_{i+1})=j$, and at time T_{i+2}, $\sigma(T_{i+2})=i$. From (26) and (28), we know that $\forall\tau\in[T_i,T_{i+1})$,

$$V_i(T_i+\tau)\le e^{-m_2\tau}V_i(T_i) \tag{29}$$

and for all $i,\ j\in\mathbb{P}$,

$$\begin{aligned}V_j(T_{i+1}) &= \mathbf{e}^TP_j\mathbf{e}\le M_2\mathbf{e}^TP_i\mathbf{e}\\ &\le M_2V_i(T_{i+1})\end{aligned} \tag{30}$$

Thus at T_{i+2},

$$\begin{aligned}V_i(T_{i+2}) &\le M_2V_j(T_{i+2})\le M_2e^{-m_2\tau_d}V_j(T_{i+1})\\ &\le M_2^2e^{-m_2\tau_d}V_i(T_{i+1})\\ &\le M_2^2e^{-2m_2\tau_d}V_i(T_i)\end{aligned} \tag{31}$$

Now we can check condition (ii) of the MLF Theorem by

$$V_i(T_{i+2})-V_i(T_i)\le(M_2^2e^{-2m_2\tau_d}-1)V_i(T_i)$$

If $M_2^2e^{-2m_2\tau_d}<1$, the conditions of Theorem 1 are satisfied. Hence if

$$\tau_d>\frac{\ln M_2}{m_2}, \tag{32}$$

the switched system preserves stability and the tracking error e_1 converges to zero as $t\to\infty$. □

The dwell time assumption can be relaxed to average dwell time [20]. Let $N_\sigma(t,t_0)$ denote the number of switchings of the switching signal σ during the time interval (t_0,t). We call the switching signal σ has average dwell time τ_a if given any time interval (t_0,t), it satisfies

$$N_\sigma(t,t_0)-N_0\le\frac{t-t_0}{\tau_a} \tag{33}$$

where N_0 is a given positive constant and τ_a is called the average dwell time.

Theorem 2. (Average dwell time switching): If the switching signal σ has average dwell time $\tau_a>\frac{\ln M_2}{m_2}$, then the switched system (3) with variable structure adaptive control (19), (20 and (21) is stable and $e_1\to 0$ as $t\to\infty$.

Proof. The proof is achieved by showing that when the average dwell time satisfies the bound, we can derive that $\forall T>0$,

$$V_{\sigma(T^-)}(T)\le e^{\mu_1}e^{-\mu_2T}V_{\sigma(0)}(0) \tag{34}$$

where $\mu_1,\ \mu_2$ are positive constants. Thus when $T\to\infty$, $V_{\sigma(T^-)}(T)$ converges to 0.

For any arbitrary $T>0$, the switching time instants during the time interval $(0,T)$ are $T_1,\ T_2,...T_{N_\sigma(T,0)}$. For

$i \leq N_{\sigma(T,0)} - 1$,

$$\begin{aligned} e^{m_2 T_{i+1}} V_{\sigma(T_{i+1})}(T_{i+1}) &\leq e^{m_2 T_{i+1}} M_2 V_{\sigma(T_i)}(T_{i+1}) \\ &= e^{m_2 T_{i+1}} M_2 V_{\sigma(T_{i+1}^-)}(T_{i+1}^-) \\ &\leq M_2 e^{m_2 T_i} V_{\sigma(T_i)}(T_i). \end{aligned} \quad (35)$$

Note that the first inequality is implied by (30) and the last inequality of (35) is implied by (29). Apply (35) from $t = 0$ to $t = T^-$, that is, from $i = 0$ to $i = N_{\sigma(T,0)} - 1$, then we have

$$e^{m_2 T} V_{\sigma(T^-)}(T) \leq M_2^{N_{\sigma}(T,0)} V_{\sigma(0)}(0).$$

And thus

$$\begin{aligned} V_{\sigma(T^-)}(T) &\leq e^{\left(-m_2 T + (N_0 + \frac{T}{\tau_a}) \ln M_2\right)} V_{\sigma(0)}(0) \\ &= e^{(N_0 \ln M_2)} e^{\left(\frac{\ln M_2}{\tau_a} - m_2\right) T} V_{\sigma(0)}(0) \end{aligned} \quad (36)$$

Hence if $\tau_a > \frac{\ln M_2}{m_2}$, then (34) is satisfied and thus $V_{\sigma(T^-)} \to 0$ as $T \to \infty$. This completes the proof. □

Remark 2: The VS based adaptive control of unknown switched linear systems guarantees that output error converges to zero as time goes to infinity under infinite times of switchings. This result is not shown in literatures that discussed the issue about control of unknown switched systems. Most of the existing results concluded that the error converges to a small residue set. For example, [11], [12].

Remark 3: In [19], the control problem of jump parameter systems is considered. The authors proposed a switching algorithm and a family of pre-specified controllers to stabilize the switched system and the error is converged to zero. However, this approach requires the condition that all the subsystems in the switched plant must be members of a "known" family of models. If the parameters of the switched plant are unknown, this approach is not applicable.

Remark 4: Since parameters are unknown, A_{mi} and thus P_i are unknown, $\forall i \in \mathbb{P}$, which means that M_1 and m_1 are not available. Though we can arbitrarily choose Q_i, the value of P_i is in turn determined and m_1 and M_1 are effected by P_i. Thus the value of dwell time τ_d (τ_a) is not exactly known. However, from above analysis we can still know that the system stability will not be destroyed unless the switching is exponentially fast.

C. VS based hysteresis switching adaptive control

The VS based adaptive controller (19) has chattering problem, especially in switched system case. Thus we modified the controller to a hysteresis switching adaptive controller which is

$$u = \theta^T \omega + (1 - \alpha(t)) u_{vs} \quad (37)$$

where the controlled switching signal $\alpha(t)$ is governed by the hysteresis switching algorithm

$$\begin{aligned} \alpha(t) &= \phi(\alpha(t^-), e_1(t)) \\ &= \begin{cases} 0, & \text{if } \alpha(t^-) = 0 \text{ and } |e_1(t)| > \varepsilon, \\ & \text{or if } \alpha(t^-) = 1 \text{ and } |e_1(t)| > \varepsilon + \delta, \\ 1, & \text{if } \alpha(t^-) = 0 \text{ and } |e_1(t)| \leq \varepsilon, \\ & \text{or if } \alpha(t^-) = 1 \text{ and } |e_1(t)| \leq \varepsilon + \delta. \end{cases} \end{aligned} \quad (38)$$

where ε and δ are positive constants that can be assigned. The variable δ called "hysteresis constant" is used to prevent fast switchings of $\alpha(t)$ in the case that $|e_1(t)|$ oscillates around ε caused by parameter switches. Our idea is that when the output error is large, we utilize the exponential convergence property of VS based controller to improve the transient response. When the error is small, we turn off the VS part of the controller and thus the chattering of control input can be alleviated. It is shown in simulation results that control input of the switching VS based controller is smaller and has fewer sign changes than the VS based adaptive controller. However, the output error only converged to a small residue set depends on ε and δ.

IV. SIMULATION RESULTS

In this section, we give some simulation results by the following example. For simplicity, consider the case that $\mathbb{P} = \{1, 2\}$. Let

$$W_{p1} = 2\frac{s+2}{s^2 - 3s + 2}, \quad W_{p2} = 1\frac{s+1}{s^2 - 4s + 4}, \quad (39)$$

and

$$W_m = \frac{1}{s+3}, \quad (40)$$

Then $\theta_1^* = [\frac{1}{2}, 0, 6, -4,]^T$ and $\theta_2^* = [1, 1, 16, -9]^T$. The reference input $r(t) = 2\sin 5.9t$ and the filter is chosen as $\lambda(s) = (s+2)$. Given a persistently switching signal as in Fig. 1(c). Tracking error of the switched system using integral adaptive law is oscillating as shown in Fig. 1(a) and value of estimation parameters is given in Fig. 1(b). Under the same switching signal as given in Fig. 1(c), variable structure adaptive controller is applied. By definition of (15), (24), (25), (26) and (27), we find that the value of dwell time can be chosen as $M_2 = 800$, and $m_2 = 0.05$ where $Q_1 = Q_2 = I_{4\times4}$. The value of γ in (21) is set to be 0.2 and β_1 in (20) is 20. Performance of the proposed VS adaptive controller and VS hysteresis switching adaptive controller is shown in Fig. 2. Here we choose $\varepsilon = 0.02$ and $\delta = 0.08$. In Fig. 2, we can see that the tracking error of VS based adaptive controller is converged to zero with some chattering which is due to fast switching of the sign function. However, the output error using hysteresis switching adaptive control did not converge to zero due to the nature of leakage term in adaptive law and the effect of the switched system. Both the two controllers used in switched system have very short time period of bursts at switching time instants. The control input of VS adaptive controller has obvious chattering phenomenon. We can see in Fig. 3 that the control input u of hysteresis switching adaptive controller is smaller and there are fewer sign changes in control gain.

In the simulation results, we can see that the error tracking of switched systems is preserved under dwell time switching signals. Note that in our example, the computed dwell time $\ln(800)/0.05 \approx 133.69$ is very conservative. However, the simulations show that even the time intervals between switchings are shorter than the dwell time we computed, the error can still converge. This shows that the conditions in Theorem 1 and 2 we derived are only sufficient, not necessary.

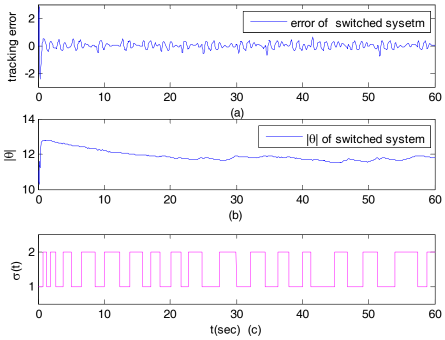

Fig. 1. Tracking error of MRAC of switched systems

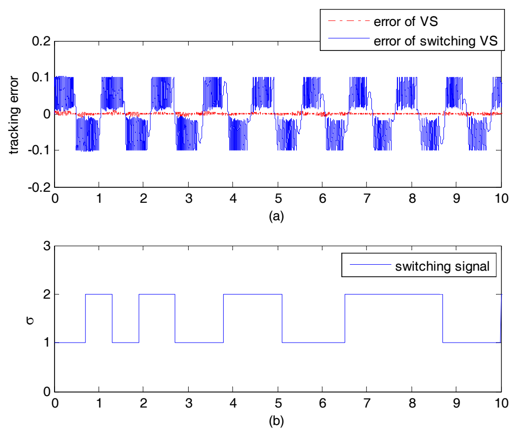

Fig. 2. Tracking error of switched systems using VS-adaptive control

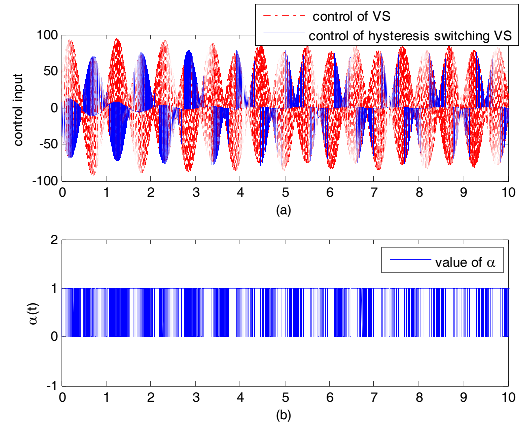

Fig. 3. Control input of the VS and switching VS adaptive controller

V. CONCLUSIONS

In this paper, we extend the result of model reference adaptive control problem to switched systems. Using the proposed output feedback variable structure based adaptive controller, we derive a sufficient condition for zero error convergence and signal boundedness of switched systems by multiple Lyapunov function theory. We also propose a hysteresis switching adaptive controller and show that chattering phenomenon can be reduced by simulation results.

Many issues can be further studied for MRAC of switched systems. For systems with higher relative degree, the analysis is more involved. It is assumed in this paper that the sign of high frequency gains of all the subsystems are all the same. If not, we have to detect the change of the signs. The stability analysis for switched systems with switching controller is more involved. These problems are under investigation.

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