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AQM Algorithm Based on Kelly's Scheme Using Sliding Mode Control

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Abstract—**This paper deals with the congestion control problem for queues in TCP/IP networks. In order to improve the congestion control performance for queues, based on the utility optimization source model proposed by Kelly, the linear and terminal sliding active queue management (AQM) algorithms are designed. Especially in the terminal sliding AQM algorithm, a special nonlinear terminal sliding surface is proposed in order to force queue length in router to reach the desired value in finite time. The upper bound of the time is also obtained. Simulation results demonstrate that the proposed sliding mode AQM controllers can obviously improve the performance of congestion control for queue length in routers.**

I. Introduction

An explosive growth in the Internet has resulted in the traffic congestion characterized by packet losses and delays, which has severely prevented the development of the Internet. Active Queue Management (AQM), as a class of packet dropping/marking mechanism in the router queue, has been recently proposed in order to convey congestion notification early enough to the senders, so that the senders are able to reduce the transmission rates before the queue overflows and any sustained packet loss occurs [1]. There are three typical kinds of AQM algorithms: One is heuristic algorithms, such as RED (Random Early Detection) [2], BLUE [3]; One is the utility function optimal model based on economics, like REM (Random Exponential Marking) [4], AVQ (Adaptive Virtual Queue) [5]; The other one is based on the sourcing and queuing dynamic model, as PI [6] and VRC (Virtual Rate Control) [7]. The advantage of the latter two algorithms is that the design of controller is based on explicit model, so the stability analysis and parameters modulation can be given theoretically.

This paper focus on the source algorithm which Kelly [8] proposed based on economic utility function through an optimization framework. And the link algorithm uses the sliding mode control (SMC) method to design the AQM controller, which is a new and effective method to analyze the performance of congestion control for the Internet. It is well known that sliding mode control (SMC) is an effective method of robustness control, and sliding mode control systems possess strong robustness against parameter perturbations and external disturbances [9], which is very suitable for time varying network system. In recent years, many studies have been focus on the domain [10]-[12]. This paper proposes two sliding mode controllers, one is plain SMC denoted as PSMC-AQM, and gives out the global stability analysis based on Lyapunov stability theory. The other one is terminal SMC [13], [14] denoted as TSMC-AQM, the nonlinear terminal sliding surface guarantees the finite reaching time to the sliding surface from initial states and the finite reaching time to the equilibrium point. So the converging time is limited, and the speed of the sliding mode control system is enhanced.

So in this paper we propose AQM algorithms based on Kelly's scheme by using sliding mode control. The structure of the paper are as follows: in section II we analyze the Kelly's method, in section III and IV, we design the linear and terminal sliding mode AQM controller respectively to study the convergence of the queue, the conclusion is given in the last section.

II. Kelly's Scheme

In this section we briefly describe the rate allocation problem in the Kelly's optimization framework. The framework is composing of two parts, the users and the network. For the users, they hope maximize their interest, which is the utility minus bandwidth cost. And the network is constructed by a series of link with fixed capacity which is shared by the users.

Consider a network with a set L of resources and a set I of users. Let C_l denote the finite capacity of resource $l \in L$. Each user $i \in I$ has a fixed route r_i, which is a set of resources traversed by user i's packets. $x_i(t)$ is the sending rate of source. We define a zero-one matrix A, where $A_{i,l} = 1$ if $l \in r_i$ and $A_{i,l} = 0$ otherwise. When its rate is x_i, user i receives utility $U_i(x_i)$. We take the view that the utility functions of the users are used to select the desired rate allocation among the users. The utility $U_i(x_i)$ is an increasing,

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strictly concave and continuously differentiable function of x_i over the range $x_i \geq 0$. Under this setting, the rate allocation problem of interest can be formulated as the following optimization problem [15]:

$$SYSTEM(U,A,C) \quad \max_{x_i \geq 0} \sum_{i \in I} U_i(x_i) \qquad \text{s.t. } A^T x \leq C \tag{1}$$

where $C = (C_l, l \in L)$ .The first constraint is the capacity constraint which states that the sum of the rates of all users utilizing resource should not exceed its capacity C_l.

Each user i adjusts its rate according to the following differential equation.

$$\frac{d}{dt} x_i(t) = k_i \left(\omega_i - x_i(t) \sum_{l \in r_i} p_l \left(\sum_{i \in I} x_i(t) \right) \right) \tag{2}$$

where k_i and ω_i are positive constants, k_i is the gain parameter ω_i shows the users' willingness to pay per unit time. $p_l(\cdot)$ is an increasing function of the aggregate rate of the users going through it, and it can also be seen as the packet loss function that similar to ECN possibility function [16].

The simplified dynamic model is

$$\dot{r}(t) = k(\omega - r(t)p(t)) \tag{3}$$

where $r(t)$ is the user's sending rate at time t, $p(t)$ is the marker probability of ECN, k and ω are the corresponding parameters.

Assume the network model is single user and single link. The dynamic buffer length at bottleneck is that

$$\dot{q}(t) = r(t) - C \tag{4}$$

where $q(t)$ is the instantaneous queue length in buffer, C is link capacity.

Let $x_1(t) = q(t) - q_d$, $x_2(t) = r(t) - C$, (3) and (4) can be described in a state space form:

$$\dot{x}_1(t) = x_2(t) \tag{5}$$

$$\dot{x}_2(t) = k\left(\omega - (x_2(t) + C)\right)p(t) \tag{6}$$

where q_d is the reference queue length.

Our control objective is that through design of the marker probability $p(t)$ with $0 \leq p(t) \leq 1$, regulate the queue length at a desired value and obtain higher link utilization, low packet loss rate and small queue fluctuations.

III. DESIGN OF PLAIN SLIDING MODE CONTROL ALGORITHM

According to the system model (5) and (6) in last section, a control theory-based approach shown in figure 1 is used to establish the AQM algorithm. The queue length $q(t)$ is the state variable and the marker probability $p(t)$ is the control variable. Through a feedback dynamic to regulate $p(t)$ and let the queue length at congested routers trace the reference value q_d. Then the system can maintain high link utilization and low delay.

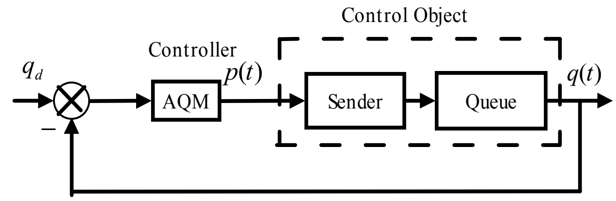

Fig. 1 TCP/AQM control system block diagram

There are usually two steps in the procedure of a sliding mode controller design. One is the sliding surface design and in this step you should design a sliding surface on which the states of the system can keep stabilization. The other is the sliding controller with which the system can converge to the sliding surface in finite time and keep sliding along it.

A. Sliding Surface Design

First choose a sliding surface as conventional

$$S(t) = cx_1(t) + x_2(t) \tag{7}$$

The objective of sliding mode control is to make the state slide to origin along the sliding surface in a finite time. That means the error of queue length is zero, and the sending rate and link capacity are totally matching.

When arrive at the sliding surface, $S(t) = 0$, so

$$cx_1(t) + x_2(t) = 0 \tag{8}$$

Substituting (8) into (5), we can obtain the sliding mode dynamics as follows

$$\dot{x}_1(t) = -cx_1(t) \tag{9}$$

$$x_1(t) = x_1(t_0)e^{-c(t-t_0)} \tag{10}$$

where t_0 means the initial time. So the system motion on the sliding surface (7) can converge to the origin point in finite time if $c > 0$.

B. Sliding Mode Controller Design

Let $S(t) = 0$, we can get the equivalent control law

$$p_{eq}(t) = \frac{cx_2 + k\omega}{k(x_2 + C)} = \frac{c(r(t) - C)}{kr(t)} + \frac{\omega}{r(t)} \tag{11}$$

Apparently, this controller can make the system (5) (6) stable, but it can not satisfy the physical meaning of the marker probability $0 \leq p_{eq} \leq 1$. However (11) is helpful for us to design a more reasonable AQM controller

$$p(t) = \left(\frac{\alpha c}{k} \left| \frac{r(t) - C}{r(t)} \right| + \beta \right) sign(S(t)) + \frac{\omega}{r(t)} \tag{12}$$

Theorem 1: If the control law (12) is used for system (5) and (6), the reaching condition is satisfied if $\alpha \geq 1, \beta > 0$.

Proof: When $S(t) > 0$,

$$\dot{S}(t) = c(r(t)-C) + k\left[\omega - r(t)\left(\frac{\omega}{r(t)} + \frac{\alpha c}{k}\left|\frac{r(t)-C}{r(t)}\right| + \beta\right)\right]$$
$$= c(r(t)-C) - \alpha c\left|r(t)-C\right| - kr(t)\beta$$
$$\le (1-\alpha)c\left|r(t)-C\right| - kr(t)\beta$$
$$< (1-\alpha)cr(t) - kr(t)\beta$$
$$= r(t)[(1-\alpha)c - k\beta]$$

So if $S(t) > 0$, choose $\alpha \ge 1, \beta > 0$ and the reaching condition $S(t)\dot{S}(t) < 0$ is satisfied.

When $S(t) < 0$,

$$\dot{S}(t) = c(r(t)-C) + k\left[\omega - r(t)\left(\frac{\omega}{r(t)} - \frac{\alpha c}{k}\left|\frac{r(t)-C}{r(t)}\right| + \beta\right)\right]$$
$$= c(r(t)-C) + \alpha c\left|r(t)-C\right| + kr(t)\beta$$
$$\ge (\alpha-1)c\left|r(t)-C\right| + kr(t)\beta$$
$$> (\alpha-1)cr(t) + kr(t)\beta$$
$$= r(t)[(\alpha-1)c + k\beta]$$

So if $S(t) < 0$, choose $\alpha \ge 1, \beta > 0$ and the reaching condition $S(t)\dot{S}(t) < 0$ is satisfied too.

Theorem 2: The sliding mode dynamics (9) can converge to origin point after the time

$$t_{\max} = \frac{\left|cx_1(0) + x_2(0)\right|}{k\beta r_{\min}} \tag{13}$$

where $r_{\min}$ is the lowest sending rate.

Proof: Choose a Lyapunov function candidate for (7) as follows:

$$V(x,t) = \frac{1}{2}S^2(t), \tag{14}$$

then the time derivative of $V(t)$ along system (5) and (6) is

$$\dot{V} = S\dot{S}$$
$$= S\left(cx_2 + k(\omega - (x_2 + C)p(t))\right)$$
$$= Scx_2 - \left|S\right|(\alpha c\left|x_2\right| + k\beta r(t)) \tag{15}$$
$$\le -\left|S\right|(\alpha c\left|x_2 + k\beta r(t)\right| + c\left|x_2\right|\left|S\right|)$$
$$\le -\left|S\right|((\alpha-1)c\left|x_2\right| + k\beta r(t))$$

From theorem 1 we can get $\alpha \ge 1, \beta > 0$, so

$$\dot{V} < -\left|S\right|k\beta r(t) < -\left|S\right|k\beta r_{\min} \tag{16}$$

By (14), we have

$$\left|S\right| = \sqrt{2V} \tag{17}$$

Substituting (17) into (16), we can obtain the following inequality

$$\dot{V} < -\sqrt{2V}(k\beta r_{\min}), \tag{18}$$

then

$$V(x,t) < \left(-\frac{k\beta r_{\min}}{\sqrt{2}} + \sqrt{V(0)}\right)^2. \tag{19}$$

So we can get

$$t_{\max} < \frac{\sqrt{V(0)}}{k\beta r_{\min}} = \frac{\left|cx_1(0) + x_2(0)\right|}{k\beta r_{\min}}. \tag{20}$$

C. Simulation Results

In this section we validate the effectiveness and performance of the controller proposed in this paper by simulations. We consider the dumbbell network topology with a single bottleneck link in figure 2.

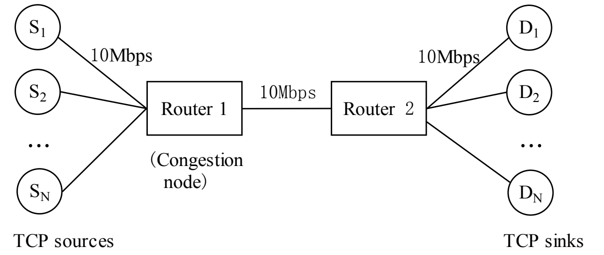

Fig. 2 Simulation network topology

Choose the parameters of network as follows: the maximum buffer of each router is 500 packets and $C = 1250$packets/s . The desired queue length q_d is 200 packets. The initial queue length is 400 packets. The PSMC-AQM controller parameters are $\alpha = 1$, $\omega = 10$, $\beta = 0.05$, $k = 15$, $c = 12$. To reduce the chattering problem, a saturation function is used. The RED algorithm is also simulated under the same network condition for the purpose of comparison. In addition, we use the parameters minimum 80packets and maximum 320packets.

The PSMC-AQM controller considers not only the queue length but also the matching condition of the assemble rate and the link capacity, so it reflects the network condition better than RED.

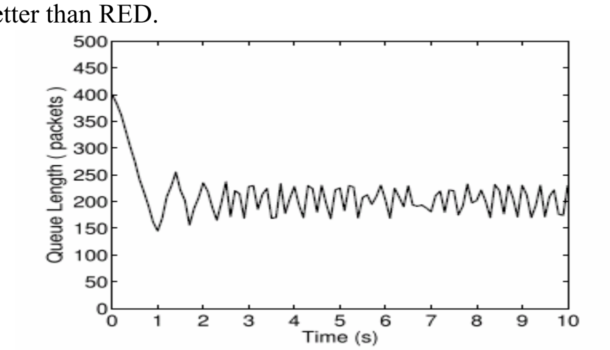

Fig. 3 Average queue length using RED

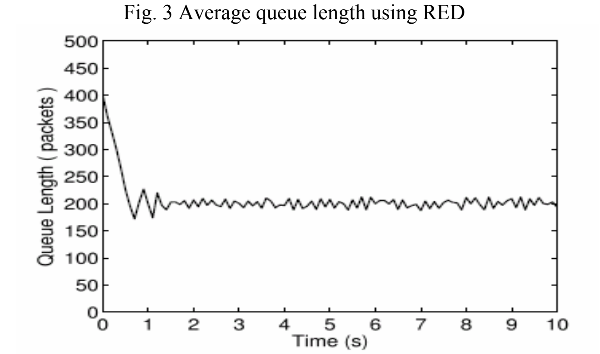

Fig. 4 Average queue length using PSMC

IV. Design of Terminal Sliding Mode Control Algorithm

Last section introduces sliding mode control into the optimization based internet congestion control model, and designs a linear sliding surface, and then validates the effectiveness of the algorithm in simulation. But the sliding mode along the sliding surface is asymptotically stable, that means the converging time could be quite long. For speediness is so important for a router algorithm, a special nonlinear sliding surface named terminal sliding surface is proposed in this section. The terminal sliding surface guarantees the finite reaching time to the sliding surface from initial states and the finite reaching time to the origin point. So the converging time is limited, and the speed of the sliding mode control system is enhanced, further the congestion control performance is improved.

A. Design of Terminal Sliding Surface

We design a nonlinear terminal sliding surface as follows:

$$S(t) = d_1 x_1(t) + d_2 x_2(t) + d_3 (x_1(t))^{q/p} \tag{21}$$

where $d_1 > 0$, $d_2 > 0$, $d_3 > 0$, p and q are odd positive integers and they satisfy $q < p < 2q$. The sliding surface $S(t)$ corresponds to a combination of the queue length error, the error between incoming traffic rate and link capacity.

When the system state trajectories are on the terminal sliding surface, $S(t)$ satisfies $S(t) = 0$. So we can obtain the following equality

$$x_2(t) = -d_2^{-1} [\, d_1 x_1(t) + d_3 (x_1(t))^{q/p} \,]. \tag{22}$$

Substituting (22) into (5), we can obtain the sliding mode dynamics as follows

$$\dot{x}_1(t) = -d_2^{-1} d_1 x_1(t) - d_2^{-1} d_3 (x_1(t))^{q/p} \tag{23}$$

In order to prove that (23) can converge to the equilibrium point in finite time, we introduce a lemma as follows

Lemma 1 [13]: Assume that a continuous, positive definite function $V(t)$ satisfies the following differential inequality

$$\dot{V}(t) \le -\alpha V^{\eta}(t), \quad \forall t \ge 0, \quad V(0) \ge 0 \tag{24}$$

where $\alpha > 0$, $0 < \eta < 1$ are constants. Then $V(t)$ satisfies the following inequality

$$V^{1-\eta}(t) \le V^{1-\eta}(0) - \alpha(1-\eta)t, \; 0 \le t \le t_r \tag{25}$$

and

$$V(t) = 0, \; \forall t \ge t_r \tag{26}$$

with t_r given by

$$t_r = \frac{V^{1-\eta}(0)}{\alpha(1-\eta)} \tag{27}$$

Theorem 3: The sliding mode dynamics (23) can converge to the equilibrium point after the time t_r and t_r satisfies

$$t_r = \frac{\|x_1(0)\|^{2(1-\eta)}}{2^{1-\eta}\alpha(1-\eta)} \tag{28}$$

where $x_1(0)$ is the initial value of $x_1(t)$ and $\alpha = 2^{\eta} d_2^{-1} d_3$, $\eta = \dfrac{q/p+1}{2}$.

Proof: Choose a Lyapunov function candidate for the system (23) as follows

$$V(t) = \frac{1}{2} x_1^{\mathrm{T}}(t) x_1(t) \tag{29}$$

then the time derivative of $V(t)$ along (23) is

$$\begin{aligned} \dot{V}(t) &= x_1^{\mathrm{T}}(t)\dot{x}_1(t) \\ &= x_1^{\mathrm{T}}(t)[-d_2^{-1} d_1 x_1(t) - d_2^{-1} d_3 (x_1(t))^{q/p}] \\ &= -d_2^{-1} d_1 \|x_1(t)\|^2 - d_2^{-1} d_3 \|x_1(t)\|^{q/p+1} \\ &\le -d_2^{-1} d_3 \|x_1(t)\|^{q/p+1} \end{aligned} \tag{30}$$

By (29), we have

$$\|x_1(t)\| = \sqrt{2V(t)} \tag{31}$$

Substituting (31) into (30), we can obtain the following inequality

$$\dot{V}(t) \le -\alpha V^{\eta}(t) \tag{32}$$

where $\alpha = 2^{\eta} d_2^{-1} d_3$, $\eta = \dfrac{q/p+1}{2}$.

According to lemma 1, we know that the sliding mode dynamics (23) can converge to the equilibrium point after the time t_r and t_r satisfies the following equality

$$t_r = \frac{V^{1-\eta}(0)}{\alpha(1-\eta)} = \frac{\|x_1(0)\|^{2(1-\eta)}}{2^{1-\eta}\alpha(1-\eta)} \tag{33}$$

So the system motion on the terminal sliding surface (21) can converge to the equilibrium point in finite time.

B. Design of Terminal Sliding Mode Controller

In the subsection, we design a robust terminal sliding mode controller to satisfy the reaching condition. We consider the following control structure of the form

$$p(t) = p_{eq}(t) + p_N(t) \tag{34}$$

Theorem 4: If the control law is used for system (5) and (6) as follows

$$\begin{aligned} p_{eq}(t) &= \frac{d_1 x_2 + (q/p) d_3 x_1^{q/p-1} x_2 + d_2 k\omega}{d_2 k(x_2 + C)} \\ &= \frac{d_1(r(t) - C)}{d_2 k r(t)} + \frac{q d_3 x_1^{q/p-1}}{p d_2 k r(t)} + \frac{\omega}{r(t)} \end{aligned} \tag{35}$$

$$p_N(t) = \frac{k_v S(t) + \varepsilon S^2 \operatorname{sgn}(S(t))}{d_2 k(x_2 + C)} \tag{36}$$

then the controller can satisfy the reaching condition

$$\dot{S}(t) = -\varepsilon S^2 \operatorname{sgn}(S(t)) - kS(t). \tag{37}$$

Proof: Recall (21), the time derivative of $S(t)$ along the trajectory of (5) and (6) under the control (34) is given as

$$\begin{aligned} \dot{S}(t) &= d_1 \dot{x}_1(t) + d_2 \dot{x}_2(t) + d_3 (q/p)(x_1(t))^{q/p-1} \dot{x}_1(t) \\ &= d_1 x_2(t) + d_2 (k\omega - (x_2(t) + C)) p(t)) \\ &\quad + d_3 (q/p)(x_1(t))^{q/p-1} x_2(t) \end{aligned} \tag{38}$$

Substitute (34) into (38), we can get

$$\dot{S}(t) = -k_v S(t) - \varepsilon S^2 \operatorname{sgn}(S(t)).$$

So the controller (34) can satisfy the reaching condition (37). That is to say, the controller can force system state trajectories toward the terminal sliding surface in finite time and maintain them on the sliding surface after then. Meanwhile, the reaching rate is fast and chattering is low.

C. Simulation Results

In this section the effectiveness and performance of the controller (TSMC) is validated by simulations. Common sliding mode controller (SMVS, Sliding Mode Variable Structure) is also simulated for the purpose of comparison with suggested parameter values $\alpha = 0.96$, $\beta = -0.96$, $w = 2$ given in [17]. We consider the same dumbbell network topology with a single bottleneck link as figure 2.

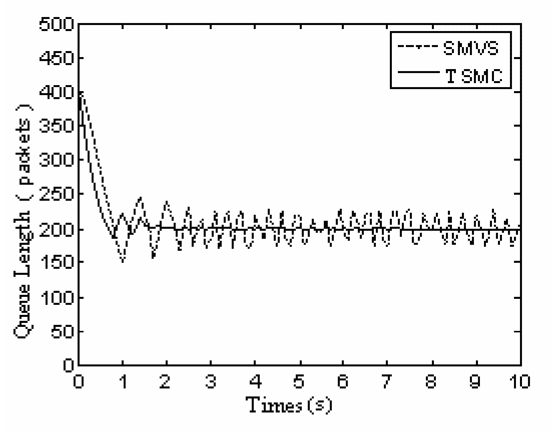

Fig. 5 The comparison with variable *N*

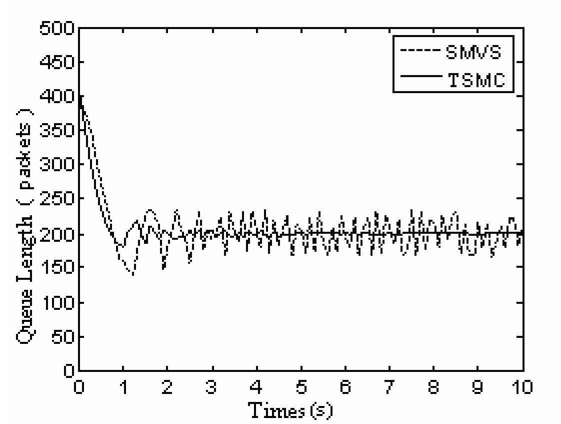

Fig. 6 The comparison with variable *C*

The network parameters are chosen the same as part C of section III. And the parameters of TSMC-AQM controller are chosen $d_1 = 1$, $d_2 = 1$, $d_3 = 1000$, $q = 3$, $p = 5$, $k = 5$, $\varepsilon = 0.1$. From theorem 3 we can calculate $t_r = 0.5506$ s.

As figure 5 and figure 6 show that the two controllers are insensitive to different TCP loads and link capacity, but TSMC has shorter regulating time and better steady performance than SMVS controller.

V. Conclusion

In this paper, effective AQM algorithms are proposed. We combine the Kelly's optimization scheme and the sliding mode control algorithm to analyze the convergence of the queue. We design two sliding mode algorithms: the linear and the terminal ones. The simulation results show that both of the algorithms can converge to the equilibrium point in finite time. Obviously, the terminal sliding mode control can obtain faster transients and less oscillatory responses under dynamic network conditions, which translates into higher link utilization, low packet loss rate and small queue fluctuations. And the proposed terminal controller has better stability and robustness than common sliding mode controller, which would be meaningful for the congestion control of Internet.

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