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Bayesian-Based Decision Making for Object Search and Characterization

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Abstract— **This paper focuses on the development of decision making criteria for autonomous vehicles where the tasks to be performed are competing under limited vehicle and sensory resources. More specifically, we are interested in the search and characterization of multiple objects given a limited number of autonomous sensor vehicles. In this case, search and characterization are two competing demands since an autonomous vehicle in the system can perform either the search task or the characterization task, but not both at the same time. This is a very critical decision as choosing one option over the other may mean missing other, more important objects not yet found, or missing the opportunity to satisfactorily characterize a found critical object. Building on previous deterministic-based work by the authors, in this paper we develop Bayesian-based search versus characterization decision making criteria that result in guaranteed detection and characterization of all objects in the domain.**

I. Introduction

This paper focuses on the management of autonomous sensor-equipped vehicles for the search and characterization of multiple objects, whose number is unknown beforehand, over a given domain. In the case where such objects are possibly in greater numbers than available sensor vehicles, search and characterization are two competing demands. This is because a sensor vehicle can perform either the characterization task or the search task, but not both at the same time (search requires mobility and characterization constrains the motion of the vehicle to that of the object). Hence, a sensor vehicle has to decide on whether to continue searching or stop and characterize once it finds an object. This decision may be very critical in some applications as in search and rescue, where, for example, finding and analyzing a nonhuman object may come at the cost of delaying or altogether missing a live human victim. Conversely, a vehicle may come across a human victim and, at the cost of missing it, and decides to continue the search task. Building on the deterministic framework developed by the authors in [1], in this work we develop Bayesian-based search and characterization metrics and decision making algorithms that guarantee that all objects in the domain will be found and satisfactorily characterized.

We first review some of the related literature. Inspired by work on particle filtering, in [2] the authors develop a strategy to dynamically control the relative configuration of sensor teams under a probabilistic framework. The goal is to get optimal estimates for target tracking through sensor fusion. In [3], the authors use the Beta distribution to model the level of confidence of target existence for an unmanned aerial vehicle (UAV) search task in an uncertain environment. The Beta distribution defined for each cell is a function of the prior probabilities which is updated through Bayes' theorem. In [4], the above uncertainty measurements are extended by using the Modified Bayes Factor, and prediction of future measurement is also taken into account to calculate the possible uncertainty reduction in UAV search operations. An alternate approach for searching in uncertainty environment is called SLAM. The paper [5] presents a paradox of combining mapping and localization at the same time, whose solution requires explicit representation of all the correlations between the estimated vehicle position and known geometric features.

Coordinated search and tracking under probabilistic framework has been studied mainly for optimal path planning in the literature. In [6], the authors investigate search-and-tracking using recursive Bayesian filtering with foreknown targets' positions with noise. A vehicle will keep searching until the target detection probability is above some preset threshold. However, the target might be lost and need to be found again due to measurement noises. The results are extended in [7] for dynamic search spaces based on forward reachable set analysis. In [8], the author proposes a Bayesian-based multisensor-multitarget sensor management scheme. The approximation strategy, based on probability hypothesis densities, maximizes the square of the expected number of targets. For the same objective, in [9] the authors seek to maximize the probability of finding a target with some foreknown location information in the presence of uncertainty. Vehicles are constrained to choose from a set of available controls and limited communication channels during each time step. However, there is no explicit decision making strategy for search and tracking proposed in the above literature.

In this paper, we propose effective decision making strategies for search and characterization under a probabilistic framework. During the search process, an uncertainty map is built based on the probability of object presence over the search domain. The probability of object presence over the domain is updated using Bayes' theorem given sensor measurements. A characterization uncertainty function is also defined for each found object. The metrics for both search and characterization are based on the corresponding uncertainty functions. A probabilistic framework is desirable to be able to take into account sensor errors, as well as allow for future incorporation of other tasks such as object tracking, data association, data fusion, sensor registration, and clutter resolution [10].

The paper is organized as follows. In Section II, Bernoulli distributions are first introduced to model search and

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characterization sensor models. We then derive expressions for the posterior probabilities of target present over the entire domain, and probabilities of classification for found object based on Bayes' theorem. Uncertainty search and characterization functions using information entropy, along with associated metrics are defined. In Section III, we develop a decision making strategy for search and characterization. In section IV, a set of simulation results are provided to show the effectiveness of the proposed decision-making strategy. The paper is concluded with a summary of current and future work in Section V.

II. Problem Formulation

A. Setup and Sensor Model

There are two basic objectives in a search and characterization operation. The first objective is to find each object and fix its position in space. The second objective is to observe each found object and collect the desired amount of information that is sufficient for its characterization. This paper focuses on the classification of static objects and future research will focus on mobile objects. Characteristics of interest for immobile objects may be geometric shape, and/or nature of electromagnetic emissions.

Let $\mathcal{D} \subset \mathbb{R}^2$ be a domain in which objects to be found and characterized are located. Let $\tilde{\mathbf{q}}$ be an arbitrary point in $\mathcal{D}$. Let $N_o > 1$ be the number of objects, however, both N_o and the positions of the objects in $\mathcal{D}$ are unknown beforehand. We will assume that there exists a single autonomous sensor-equipped vehicle (denoted by $\mathcal{V}$) that performs the search and characterization tasks. Future research will focus on a team of sensor-equipped vehicles. The current scenario is an extreme cases in which the resources available are at a minimum (a single sensor vehicle as opposed to multiple cooperating ones). At any time t, the vehicle can either perform the search task or the characterization task, but not capable of both at the same time. Initially, the vehicle starts in the search mode.

Assume that we are given some search versus characterization decision making strategy. Since the number of objects is potentially very large, with a poor choice of decision making strategy, the vehicle may end up excessively characterizing one single object while there may still exist unfound, and more important, objects in the domain. *In this paper, we investigate policies that guarantee that every static object within the domain will be found and each found object will be characterized by the autonomous vehicle until a minimum satisfactory classification performance is achieved.*

Let the position of the static object $\mathcal{O}_j$, $j \in \{1, 2, \ldots, N_o\}$, be $\mathbf{p}_j$, which is unknown beforehand. The vehicle $\mathcal{V}$ satisfies the following simple first order discrete-time equation of motion

$$\mathbf{q}(t+1) \quad = \quad \mathbf{q}(t) + \mathbf{u}(t),$$

where $\mathbf{q} \in \mathcal{D} \subset \mathbb{R}^2$ represents the position of $\mathcal{V}$, and $\mathbf{u} \in \mathcal{U} \subset \mathbb{R}^2$ is the control input, $\mathcal{U}$ is the set of allowable controls.

In this work, for both the search and characterization processes, we use a sensor model with Bernoulli distribution, which gives binary outputs, however, with different observation contents: object "present" or "not present" for search, and property "G" or "B" for characterization. This is a simplified but reasonable sensor model because it abstracts away the complexities in sensor noise, image processing algorithm errors, etc.

In the search process, let $V_s = \{v_1^s, v_2^s\}$ be the set of two possible sensor outputs, where v_1^s corresponds to object detected, and v_2^s corresponds to no object detected. Let $S_s = \{s_1^s, s_2^s\}$ be the set of two possible state types, where s_1^s corresponds to object present, and s_2^s corresponds to object not present. The actual observation V_s is taken according to the probability parameter β of a Bernoulli distribution. Since there are two states in all, two Bernoulli distributions are used and the following matrix (which is called the emission probability matrix in the Hidden Markov Model (HMM) literature [11]) for search task is given by

$$B = \begin{bmatrix} p_{s_1^s}^{v_1^s} = \beta_{1,s} & p_{s_1^s}^{v_2^s} = 1 - \beta_{1,s} \\ p_{s_2^s}^{v_1^s} = \beta_{2,s} & p_{s_2^s}^{v_2^s} = 1 - \beta_{2,s} \end{bmatrix} \tag{1}$$

where $p_{s_j^s}^{v_i^s}$, $i, j = 1, 2$, describes the probability of measuring v_i^s given state s_j^s. For the sake of simplicity, we can assume that the sensor probabilities of making a correct measurement are the same. That is, we have $p_{s_1^s}^{v_1^s} = p_{s_2^s}^{v_2^s} = \beta_s$. The value of β_s is assumed to depend on the range between the sensor and the observed point. Without loss of generality, here we assume a simple model for β_s that is a fourth order polynomial function of $s = \|\mathbf{q}(t) - \tilde{\mathbf{q}}\|$ within the sensor range r_s and $b_n = 0.5$ otherwise,

$$\beta_s(s) = \begin{cases} \frac{M}{r_s^4}\left(s^2 - r_s^2\right)^2 + b_n & \text{if } s \leq r_s \\ b_n & \text{if } s > r_s \end{cases}, \tag{2}$$

where $M + b_n$ gives the peak value of β_s if $\tilde{\mathbf{q}}$ being observed is located at the sensor vehicle's location, which indicates that the probability of sensing correctly is highest exactly where the sensor is. The parameter r_s is the range of the search sensor. The sensing capability decreases with range and becomes 0.5 outside of the limited sensory range $\mathcal{W}$, implying that the sensor returns an equally likely observation of "present" or "not present" regardless of the truth of whether there is an object at that location or not. Figure 1 shows the β_s function over a square domain of size 4×4. Note that β_s is a function of s, and s is a function of the vehicle's position $\mathbf{q}(t)$ and the location $\tilde{\mathbf{q}}$ of interest.

For the characterization process, we also define binary observation outputs for each found object: v_1^c and v_2^c that correspond to state types s_1^c (property "G"), and s_2^c (property "B"), respectively. The observation process is assumed to be Bernoulli and is assumed, for simplicity, to obey the same functional form as the detection model above:

$$\beta_c(s) = \begin{cases} \frac{M_c}{r_c^4}\left(s^2 - r_c^2\right)^2 + b_n & \text{if } s \leq r_c \\ b_n & \text{if } s > r_c \end{cases}, \tag{3}$$

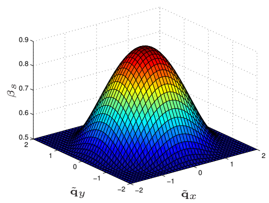


Fig. 1. Correct sensing probability function β_s with $\mathbf{q} = 0$, $M = 0.4$ and $r_s = 2$.

where $M_c + b_n$ and r_c are the peak sensory capacity and limited sensory range for the characterization process. When an object of interest is within the sensor's effective characterization radius $\tilde{r}_c < r_c$, this object is said to be found, and the vehicle has to decide whether to characterize it or continue searching.

Remarks:

1) If we have more than two possible classification properties, the binary-type Bernoulli sensor model is no longer appropriate. In this case, one can model the sensor observation using a multinomial random variable that can take one of K discrete values and expand the emission matrix to dimension $K \times K$.
2) A key feature of the proposed approach is that the sensor may have a limited range. Previous work on cooperative coverage control usually assumes that the sensors have an infinite range [12]. *This assumption is not made here*. Outside the sensor domain $\mathcal{W}$, the sensor is ineffective (with $\beta_s = 0.5$). This is very important in applications where $\mathcal{D}$ is large-scale (i.e., too large to be covered by a single set of static sensor agents). •

B. Bayesian Updates for Search and Characterization

For both search and characterization processes, we employ Bayes' theorem to update the probability of object presence at $\tilde{\mathbf{q}}$, or of a found object k having the property "G". Let us first consider the object detection Bayesian update equation.

Given an observation, Bayes' theorem gives, for each $\tilde{\mathbf{q}}$, the posterior probability $p_s(S_{s,t+1} = s_1^s | v_j^s; \tilde{\mathbf{q}}), 1 \leq j \leq 2$ after the observations have been taken at time step t:

$$p_s(S_{s,t+1} = s_1^s | v_j^s; \tilde{\mathbf{q}}) = \alpha_1 p_s(v_j^s | S_{s,t} = s_1^s; \tilde{\mathbf{q}}) \cdot p_s(S_{s,t} = s_1^s; \tilde{\mathbf{q}}) \qquad (4)$$

where $p_s(S_{s,t+1} = s_1^s | v_j^s; \tilde{\mathbf{q}})$ is the posterior probability of object being state type s_1^s given that observation v_j^s has just been taken at time step t, $p_s(v_j^s | S_{s,t} = s_1^s; \tilde{\mathbf{q}})$ is the probability of the particular observation v_j^s being taken given that the object state type at time step t is s_1^s, which is given by the emission matrix (1) and the β_s function (2), $p_s(S_{s,t} = s_1^s; \tilde{\mathbf{q}})$ is the prior probability of type s_1^s being correct at t, and α_1 serves as a normalizing function that ensures that the posterior probabilities $p_s(S_{s,t+1} = s_i^s | v_j^s; \tilde{\mathbf{q}})$ sum to one over the state type set $S_s = \{s_1^s, s_2^s\}$.

For brevity, we let $p_s(\tilde{\mathbf{q}}, t+1)$ denote $p_s(S_{s,t+1} = s_1^s | v_j^s; \tilde{\mathbf{q}})$ and $p_s(\tilde{\mathbf{q}}, t)$ denote $p_s(S_{s,t} = s_1^s; \tilde{\mathbf{q}})$. It can be shown that the object presence probability update equation at $\tilde{\mathbf{q}}$ is given by

$$p_s(\tilde{\mathbf{q}}, t+1) = y \frac{\beta_s p_s(\tilde{\mathbf{q}}, t)}{2\beta_s p_s(\tilde{\mathbf{q}}, t) - \beta_s - p_s(\tilde{\mathbf{q}}, t) + 1} + (1-y) \frac{(1-\beta_s) p_s(\tilde{\mathbf{q}}, t)}{-2\beta_s p_s(\tilde{\mathbf{q}}, t) + \beta_s + p_s(\tilde{\mathbf{q}}, t)}, \qquad (5)$$

where y is defined as follows

$$y = \begin{cases} 1 & \text{if } V = v_1^s \\ 0 & \text{if } V = v_2^s \end{cases}. \qquad (6)$$

The probability of object not present is $1 - p_s(\tilde{\mathbf{q}}, t+1)$.

For the characterization process, we use a similar update equation as (5) to express the posterior probability of a found object k at location $\tilde{\mathbf{q}}_k$ having property "G":

$$p_c(\tilde{\mathbf{q}}_k, t+1) = y \frac{\beta_c p_c(\tilde{\mathbf{q}}_k, t)}{2\beta_c p_c(\tilde{\mathbf{q}}_k, t) - \beta_c - p_c(\tilde{\mathbf{q}}_k, t) + 1} + (1-y) \frac{(1-\beta_c) p_c(\tilde{\mathbf{q}}_k, t)}{-2\beta_c p_c(\tilde{\mathbf{q}}_k, t) + \beta_c + p_c(\tilde{\mathbf{q}}_k, t)}, \qquad (7)$$

where $\tilde{\mathbf{q}}_k$ is the position of object k. The probability of having property "B" is $1 - p_c(\tilde{\mathbf{q}}_k, t+1)$. Recall that β_c is a function of $\mathbf{q}(t)$ and $\tilde{\mathbf{q}}$.

Remark about extension to multiple sensor vehicles. When we have multiple autonomous vehicles, each Berboulli type sensor will give its own observation for a certain location $\tilde{\mathbf{q}}$. Hence, there are 2^m combinations for the observation set V if we have m sensors in all. We will need to solve for the explicit expressions for $\alpha_l, l \in \{1, 2, \cdots, 2^m\}$ and obtain the corresponding update equations of the posterior probability $p_s(\tilde{\mathbf{q}}, t+1, l)$ similar to Equation (5). •

C. Uncertainty Map

For the search process, we use an information-based approach to construct the uncertainty map for every $\tilde{\mathbf{q}}$ within the search domain. The information entropy function of a probability distribution is used to evaluate uncertainty. The uncertainty map will be used to guide the vehicle in the search domain. Let the probability of the occurrence of an event be p, then the information is measured as

$$I(p) = \log_b(1/p) \geq 0, \qquad (8)$$

where b is the base (we use $b = e$ in in this paper). In our case, there are 2 distinct state values for the discrete probabilities. Therefore, the probability distribution P is given by $P = \{p_s, 1-p_s\}$. We define the weighted average of information $I(p)$ as the information entropy distribution for discrete probability distribution P at $\tilde{\mathbf{q}}$ at each time step t:

$$H_s(P, \tilde{\mathbf{q}}, t) = -p_s(\tilde{\mathbf{q}}, t) \ln p_s(\tilde{\mathbf{q}}, t) - (1 - p_s(\tilde{\mathbf{q}}, t)) \ln(1 - p_s(\tilde{\mathbf{q}}, t)). \qquad (9)$$

When $p_s(\tilde{\mathbf{q}}, t) = 1$ or 0, there is no uncertainty about object existence or lack thereof and, therefore, $H_s = 0$, which is the desired uncertainty level. Maximum uncertainty $H_{s,\max} = 0.6931$ is when $p_s(\tilde{\mathbf{q}}, t) = 0.5$. The initial "uncertainty" distribution is assumed to be $H_{s,\max}$ reflecting the fact that at the outset of the search mission there is poor certainty levels (in other words, a uniform distribution for object existence). The greater the value of H_s, the bigger the uncertainty is. Figure (2) shows the

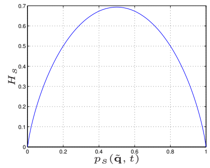


Fig. 2. Information entropy function H_s for the search process.

information entropy function (9) as a function of $p_s(\tilde{\mathbf{q}}, t)$. The information entropy distribution at time step t over the domain forms an uncertainty map at that time instant.

For the characterization process, we define a similar entropy function $H_c(P_c, \tilde{\mathbf{q}}_k, t)$, with $P_c = \{p_c, 1 - p_c\}$, for every found object k (located at $\tilde{\mathbf{q}}_k$) to evaluate classification uncertainty.

$$H_c(P_c, \tilde{\mathbf{q}}_k, t) = -p_c(\tilde{\mathbf{q}}_k, t) \ln p_c(\tilde{\mathbf{q}}_k, t) \\ -(1 - p_c(\tilde{\mathbf{q}}_k, t)) \ln(1 - p_c(\tilde{\mathbf{q}}_k, t)). \quad (10)$$

There are as many scalar H_c's as there are found objects k up to time t. The initial value for H_c for every *found* object k can also be set as $H_c = H_{c,\max} = 0.6931$. If the vehicle finds target k and decides to characterize it, H_c will decrease as the characterization probability increases according to the Bayesian update equations derived above. When a vehicle has collected enough information based on the available resources at that time step, it leaves the object and H_c remains constant until the vehicle comes back to characterize it when possible. This is repeated until the desired H_d (to be discussed below) characterization certainty is achieved.

D. Search and Characterization Metrics

In this section we develop metrics to be used for the search versus characterization decision making process. In the event of object detection and a decision not to proceed with the search process, but, instead, stopping to characterize the found object, the associated cost is defined as

$$\mathcal{J}(t) = \frac{\int_{\mathcal{D}} H_s(P, \tilde{\mathbf{q}}, t) d\tilde{\mathbf{q}}}{H_{s,\max} A_{\mathcal{D}}}. \quad (11)$$

The cost $\mathcal{J}$ is proportional to the total integral of the search uncertainty over $\mathcal{D}$. We divide the integral by the area of the domain $A_{\mathcal{D}}$ multiplied by $H_{s,\max}$ in order to normalize $\mathcal{J}(t)$. According to this definition, we have $0 \leq \mathcal{J}(t) \leq 1$. Initially, $\mathcal{J}(0) = 1$, since $H_s(P, \tilde{\mathbf{q}}, 0) = H_{s,\max}$ for all $\tilde{\mathbf{q}} \in \mathcal{D}$. If for some t_s we have $H_s(P, \tilde{\mathbf{q}}, t_s) = 0$ for all $\tilde{\mathbf{q}} \in \mathcal{D}$, then $\mathcal{J}(t_s) = 0$ and the entire domain has been satisfactorily covered and we know with 100% certainty that there are no objects yet to be found.

For the characterization process, let $\bar{N}_o(t)$ be the number of objects found by the autonomous sensor vehicle up to time t. For each found object $j \in \{1, 2, \cdots, \bar{N}_o(t)\}$, define the characterization metric $H_d(\tilde{\mathbf{q}}_j, t)$ to be

$$H_d(\tilde{\mathbf{q}}_j, t) = \epsilon_c \mathcal{J}(t), \quad (12)$$

where ϵ_c is a preset upper bound on the desired uncertainty level for characterization. H_d depends on how uncertain the vehicle is of the presence of more undetected objects in $\mathcal{D}$ through $\mathcal{J}(t)$. Initially, $\mathcal{J}(0) = 1$ and the vehicle will attempt to characterize it until the classification uncertainty is smaller than ϵ_c. On the other hand, if $\mathcal{J}(t_s) = 0$ for some time $t_s > 0$, the vehicle can spend as much time characterizing the object because the vehicle has achieved 100% certainty that it has found all critical and noncritical objects in the domain. If the vehicle finds an object $\mathcal{O}_j$ (i.e., within the effective characterization radius $\tilde{r}_c$) and decides to characterize it, the vehicle will continue characterizing until achieving the characterization condition

$$H_c(P_c, \tilde{\mathbf{q}}_j, t) < H_d(\tilde{\mathbf{q}}_j, t). \quad (13)$$

The vehicle $\mathcal{V}$ then stops characterizing the found object and switches to searching again. The vehicle can resume characterizing an object that has been detected and completely or partially characterized in the past if it finds it again during the search process. When this occurs, the value of H_d will be smaller than the last time the objected has been detected.

III. Search versus Characterization Decision-Making

We will consider a search/characterization decision making strategy that guarantees finding all objects in $\mathcal{D}$ (i.e., achieve $\mathcal{J} = 0$) and characterizing each object with an upper bound on the characterization uncertainty of ϵ_c. Let us first consider a search strategy. The goal in the search strategy is to attain an uncertainty level such that the search cost $\mathcal{J}(t) \leq \epsilon$ for all $\tilde{\mathbf{q}}$ within $\mathcal{D}$ and all $t \geq t_s$ for some $t_s > 0$.

Let the control $\mathbf{u}(t)$ be restricted to a set $\mathcal{U}$. For example, $\mathcal{U}$ could be the set of all controls $\mathbf{u}(t) \in \mathbb{R}^2$ such that $\|\mathbf{u}(t)\| < u_{\text{max}}$, where u_{max} is the maximum allowable control speed. Based on this constraint on the control, we define $\mathcal{Q}_{\mathcal{W}}(t)$ as the set of points in $\mathcal{W}$ reachable from the current location of the vehicle at time t:

$$\mathcal{Q}_{\mathcal{W}}(t) = \{\tilde{\mathbf{q}} \in \mathcal{W} : \tilde{\mathbf{q}} - \mathbf{q}(t) \in \mathcal{U}\}. \quad (14)$$

For the search process, we use a control law that drives the vehicle to some point $\tilde{\mathbf{q}} \in \mathcal{Q}_{\mathcal{W}}(t)$ that has the highest uncertainty, and switch to a perturbation control law when the vehicle is trapped in a region where no such point exists. Let us first consider the following condition, whose utility will become obvious shortly.

Condition C1. $H_s(P, \tilde{\mathbf{q}}, t) \leq \epsilon$, $\forall \tilde{\mathbf{q}} \in \mathcal{Q}_{\mathcal{W}}(t)$, where ϵ is a preset threshold of some small value.

Consider the following control law

$$\mathbf{u}^*(t) = \begin{cases} \bar{\mathbf{u}}(t) & \text{if } \mathbf{C1} \text{ does not hold} \\ \bar{\bar{\mathbf{u}}}(t) & \text{if } \mathbf{C1} \text{ holds} \end{cases} \quad (15)$$

where $\bar{\mathbf{u}}(t)$ is the *nominal control law*, and $\bar{\bar{\mathbf{u}}}(t)$ is the perturbation control law.

Let $\tilde{\mathbf{q}}_\star$ be the point that has the highest uncertainty within $\mathcal{Q}_{\mathcal{W}}(t)$, that is,

$$\tilde{\mathbf{q}}_\star(t+1) = \text{argmax}_{\tilde{\mathbf{q}} \in \mathcal{Q}_{\mathcal{W}}(t)} H_s(P, \tilde{\mathbf{q}}, t). \quad (16)$$

The nominal control law is then set to be

$$\bar{\mathbf{u}}(t) = \tilde{\mathbf{q}}_\star(t+1) - \mathbf{q}(t) \in \mathcal{U}. \quad (17)$$

This choice for the nominal control law is inspired by the nominal control law in [13].

Note that according to Equation (16), $\tilde{\mathbf{q}}_\star(t+1)$ might be a set of points holding the same maximum uncertainty value. Rules to pick the "best" point are immaterial as far as this work is concerned and in this paper we assume there is only one such point for the sake of simplicity.

When the uncertainty H_s of all the points $\tilde{\mathbf{q}} \in \mathcal{Q}_{\mathcal{W}}(t)$ is less than ϵ, Condition **C1** holds, and no such $\mathbf{q}_\star(t+1)$ exists. This means that the vehicle gets trapped in a region where $H_s \leq \epsilon$ if restricted to applying only the nominal control law $\bar{\mathbf{u}}$. Note that this does not imply that the entire domain $\mathcal{D}$ has been fully searched yet (hence, the need for the perturbation control law which will be discussed shortly). The reasons we restrict our choice of $\tilde{\mathbf{q}}_\star$ to $\mathcal{W}$ (as opposed to $\mathcal{D}$) in the definition of $\mathcal{Q}_{\mathcal{W}}(t)$ (causing $\bar{\mathbf{u}}$ to become a local controller) are as follows:

1) Using $\mathcal{W}$ instead of $\mathcal{D}$ limits the computations involved in finding $\tilde{\mathbf{q}}_\star$ to a smaller space and, hence, is more computationally efficient. This is especially true in the case of large scale domains, where much of the domain $\mathcal{D}$ is unreachable from where the vehicle is because of the restriction on the control $\mathbf{u} \in \mathcal{U}$.
2) Although in this paper we assume that the vehicle has full knowledge of the domain $\mathcal{D}$ and the search uncertainty function $H_s(P, \tilde{\mathbf{q}}, t)$ for all $\tilde{\mathbf{q}} \in \mathcal{D}$, $\mathcal{D}$ may not be known in real time. In this case, all the information the vehicle could obtain is within its limited sensory domain $\mathcal{W}$.

Note that the application of the local controller is consistent with our previous work [14]–[17].

If Condition **C1** holds, then the perturbation controller $\bar{\bar{\mathbf{u}}}(t)$ is used:

$$\bar{\bar{\mathbf{u}}}(t) = -\bar{\bar{k}}(\mathbf{q}(t) - \tilde{\mathbf{q}}^*)$$

where $0 < \bar{\bar{k}} \leq 1$ is the controller gain, and $\tilde{\mathbf{q}}^* \in \mathcal{Q}_{\mathcal{D}}(t) := \{\tilde{\mathbf{q}} \in \mathcal{D} : \tilde{\mathbf{q}} - \mathbf{q}(t) \in \mathcal{U}\}$ such that $H_s(P, \tilde{\mathbf{q}}^*, t) > \epsilon$. The controller is used to drive the vehicle out of the region with low uncertainty ϵ to some $\tilde{\mathbf{q}}^* \in \mathcal{Q}_{\mathcal{D}}(t)$ such that $H_s(P, \tilde{\mathbf{q}}^*, t) > \epsilon$, if such a point exists.

There are only two scenarios that can arise. The first is when the set $\mathcal{U}$ allows for motions from any point in $\mathcal{D}$ to any other point, and we have $\mathcal{Q}_{\mathcal{D}}(t) = \mathcal{D}$, $\forall t > 0$. If this condition is held for all t and if at some time t_f there is no point $\tilde{\mathbf{q}}^* \in \mathcal{Q}_{\mathcal{D}}(t_f) = \mathcal{D}$ such that $H_s(P, \tilde{\mathbf{q}}^*, t_f) > \epsilon$ then we say that the mission is complete as every point in the domain has been searched with a satisfactory certainty level (below ϵ). In the second scenario, the set $\mathcal{U}$ may be such that $\mathcal{Q}_{\mathcal{D}}(t) \subset \mathcal{D}$ (but $\mathcal{Q} \neq \mathcal{D}$) for some t. In other words, there are locations in $\mathcal{D}$ that the vehicle can not reach given the constraints on the control velocity $\bar{\bar{\mathbf{u}}}$. Under this scenario, the mission may never be completed. For the purpose of this paper, we will assume that $\mathcal{U}$ is such that $\mathcal{Q}_{\mathcal{D}}(t) = \mathcal{D}$ for all time t.

Since there may be many such points $\tilde{\mathbf{q}}^*$, a choice of only one such $\tilde{\mathbf{q}}^*$ needs to be made. There are several ways such a choice can be made. We provide one such choice that is efficient energy-wise than other possibilities. Let

$$\mathcal{D}_\epsilon(t) := \{\tilde{\mathbf{q}} \in \mathcal{Q}_{\mathcal{D}}(t) : H_s(P, \tilde{\mathbf{q}}, t) > \epsilon\},$$

which is an open set of all $\tilde{\mathbf{q}}$ for which $H_s(P, \tilde{\mathbf{q}}, t)$ is larger than a preset value ϵ. Let $\overline{\mathcal{D}}_\epsilon(t)$ be the closure of $\mathcal{D}_\epsilon(t)$. Let $\overline{\mathcal{D}}_{\epsilon,\mathcal{V}}(t)$ be the set of points in $\overline{\mathcal{D}}_\epsilon(t)$ that minimize the distance between the position vector of vehicle $\mathcal{V}$, $\mathbf{q}$, and the set $\overline{\mathcal{D}}_\epsilon(t)$:

$$\overline{\mathcal{D}}_{\epsilon,\mathcal{V}}(t) = \left\{\tilde{\mathbf{q}}^* \in \overline{\mathcal{D}}_\epsilon(t) : \tilde{\mathbf{q}}^* = \operatorname{argmin}_{\tilde{\mathbf{q}} \in \overline{\mathcal{D}}_\epsilon(t)} \|\tilde{\mathbf{q}} - \mathbf{q}(t)\|\right\}.$$

Other choices of the set $\overline{\mathcal{D}}_{\epsilon,\mathcal{V}}(t)$ may also be considered, but this choice is efficient since the perturbation maneuver seeks the minimum distance for redeployment. Similar as the choice of $\tilde{\mathbf{q}}_\star(t+1)$ in Equation (16), the set $\overline{\mathcal{D}}_{\epsilon,\mathcal{V}}(t)$ may contain more than a single point and we will simply assume that there will exist at most one such point. If $\overline{\mathcal{D}}_\epsilon(t)$ is empty, this means that the distribution $H_s(P, \tilde{\mathbf{q}}, t) < \epsilon$ everywhere over the domain and the search mission is complete.

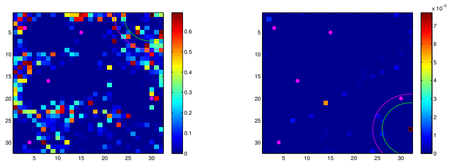
Once the vehicle finds an object and decides to characterize it, it switches to a characterization task and will not carry out any searching until $H_c(P_c, \tilde{\mathbf{q}}_j, t) < H_d(\tilde{\mathbf{q}}_j, t)$. After achieving at least the desired upper bound of characterization uncertainty ϵ_c, the vehicle will switch back to become a search vehicle and leave its characterization position to find new objects.

Under the assumption that $\mathcal{U}$ is such that $\mathcal{Q}_{\mathcal{D}}(t) = \mathcal{D}$ for all time t, the search and characterization control policy given by equations (15) and (13) will guarantee that $\mathcal{J}$ converges asymptotically to zero, which is equivalent to guaranteeing that all vehicles be found. The maximum value of characterization uncertainty acceptable is given by ϵ_c.

IV. Simulation Results

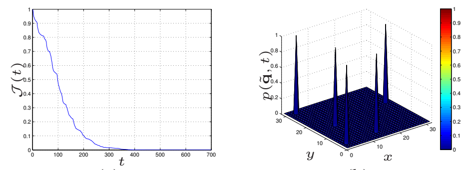
In this section we provide a numerical simulation, which illustrates the performance of the decision making strategy. We assume the domain $\mathcal{D}$ is square in shape with size 32×32 units length. There are 5 targets with objects 1, 3 and 5 have property "B", and objects 2 and 4 have Property "G", with a randomly selected initial deployment. We set $r_s = r_c = 8$ and $\tilde{r}_c = 6$ as shown by the magenta and green circle in Figure 3.The parameter $M = M_c$ of the sensor is set as 0.4, which gives the highest value for β_s as 0.9, i.e., there is 90% chance that the sensor is sensing correctly at the location of the vehicle and gradually to 0.5 according to the model discussed above (Equation (2), (3)). Let the desired upper bound for classification uncertainty ϵ_c be 0.01. Here we use the control law in equation (15) with control gain $\bar{\bar{k}} = 0.025$. The set $\mathcal{U}$ is chosen to be $\mathcal{D}$, so that $\mathcal{Q}_{\mathcal{W}}(t)$ is given by the intersection of $\mathcal{U}$ and $\mathcal{W}$, i.e., $\mathcal{W}$ and $\mathcal{Q}_{\mathcal{D}}(t) = \mathcal{D}$ which guarantees the full coverage of the entire domain.

Figure 3 shows the evolution of H_s. From Figure 3(b), we can conclude that at most $H_s = 1 \times 10^{-4}$ has been achieved everywhere within $\mathcal{D}$. Figure 4(a) shows the evolution of $\mathcal{J}(t)$ under the control strategy and can be



(a) (b)

Fig. 3. Uncertainty map (dark red for highest uncertainty and dark blue for lowest uncertainty) and the vehicle motion at $t = 200$ and 700 (with initial uncertainty $H_s = H_{\max}$ at $t = 0$): (a) Uncertainty at $t = 200$ and (b) Uncertainty at $t = 700$.



(a) (b)

Fig. 4. (a) Evolution of the search cost $\mathcal{J}(t)$, (b) Posterior probabilities for every $\tilde{\mathbf{q}}$ within $\mathcal{D}$ at 700.

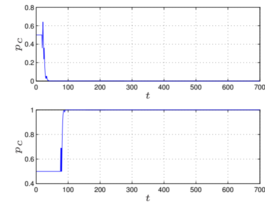


Fig. 5. Probability of object having Property "G" p_c for object 1 and object 2.

seen to converge to zero.

All the objects have been found with the probabilities of object present as 1 and zero search uncertainty. Those cells that do not contain an object end up with zero search probability and uncertainty. Figure 4(b) shows the detection of all objects at time step 700.

For all the 5 found objects, objects $2, 4$ have been characterized with probability of having Property "G" as 1 and zero classification uncertainty. Objects $1, 3, 5$ have been characterized with probability of having property "G" as 0 and zero classification uncertainty. Figure 5 shows the estimated characterization of the properties of object 1 (which has property "B") and of object 2 (which has property "G").

V. Conclusion

Based on a probabilistic framework, a decision-making and control strategy was developed to guarantee the detection of all objects in a domain and the characterization of each object until a preset small value of classification uncertainty is achieved. Numerical simulations demonstrated the operation of the strategies. Future research will focus on locating and characterizing dynamic objects. The question of unknown environment geometries (i.e., unknown $\mathcal{D}$) will also be addressed. Most importantly, optimizing the control and decision making laws with respect to some cost function will be investigated as the current result provides some solution to the decision making problem, albeit with guaranteed performance (i.e., guaranteed detection and characterization of all objects) under appropriate assumptions.

References

[1] Y. Wang, I. I. Hussein, and R. S. Erwin, "Awareness-Based Decision Making for Search and Tracking," *American Control Conference*, 2008, invited Paper.

[2] J. R. Spletzer and C. J. Taylor, "Dynamic Sensor Planning and Control for Optimally Tracking Targets," *The International Journal of Robotics Research*, no. 1, pp. 7–20, January 2003.

[3] L. F. Bertuccelli and J. P. How, "Robust UAV Search for Environments with Imprecise Probability Maps," *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*, December 2005.

[4] ——, "Bayesian Forecasting in Multi-vehicle Search Operations," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, August 2006.

[5] J. J. Leonard and H. F. Durrant-Whyte, "Simultaneous Map Building and Localization for an Autonomous Mobile Robot," in *IEEE/RSJ International Workshop on Intelligent Robots and Systems IROS '91*, Osaka, Japan, November 1991, pp. 1442 – 1447.

[6] T. Furukawa, F. Bourgault, B. Lavis, and H. F. Durrant-Whyte, "Recursive Bayesian Search-and-Tracking using Coordinated UAVs for Lost Targets," *Proceedings of the 2006 IEEE International Conference on Robotics and Automation*, May 2006.

[7] B. Lavis, T. Furukawa, and H. F. Durrant-Whyte, "Dynamic Space Reconfiguration for Bayesian Search-and-Tracking with Moving Targets," *Autonomous Robots*, vol. 24, pp. 387–399, May 2008.

[8] R. Mahler, "Objective Functions for Bayeisan Control-Theoretic Sensor Management, I: Multitarget First-Moment Approximation," *Proceedings of IEEE Aerospace Conference*, 2003.

[9] M. Flint, M. Polycarpou, and E. Fernández-Gaucherand, "Cooperative Control for Multiple Autonomous UAV's Searching for Targets," *Proceedings of the 41st IEEE Conference on Decision and Control*, December 2002.

[10] M. K. Kalandros, L. Trailovic, L. Y. Pao, and Y. Bar-Shalom, "Tutorial on Multisensor Management and Fusion Algorithms for Target Tracking," *American Control Conference*, 2004.

[11] L. R. Rabiner, "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition," *Proceedings of the IEEE*, vol. 77, no. 2, pp. 257 – 286, February 1989.

[12] B. Grocholsky, "Information-theoretic control of multiple sensor platforms," Ph.D. dissertation, The University of Sydney, 2002.

[13] I. I. Hussein, "A Kalman-filter based control strategy for dynamic coverage control," *Proceedings of the American Control Conference*, pp. 3271–3276, 2007.

[14] I. I. Hussein and D. Stipanović, "Effective Coverage Control for Mobile Sensor Networks with Guaranteed Collision Avoidance, special issue on multi-vehicle systems cooperative control with applications," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 4, pp. 642–657, 2007.

[15] Y. Wang and I. I. Hussein, "Awareness Coverage Control Over Large Scale Domains with Intermittent Communications," *2008 American Control Conference*, 2008.

[16] ——, "Cooperative Vision-based Multi-vehicle Dynamic Coverage Control for Underwater Applications," *Proceedings of the IEEE Multiconference on Systems and Control*, 2007, invited paper.

[17] ——, "Underwater Acoustic Imaging using Autonomous Vehicles," *2008 IFAC Workshop on Navigation, Guidance and Control of Underwater Vehilces*, April 2008.