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Control of Uncertain Nonlinear Systems against Actuator Faults Using Adaptive Fuzzy Approximation

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Abstract— **This paper studies the problem of fault-tolerant control (FTC) for a class of uncertain nonlinear systems which cannot be feedback linearized. These systems are unknown in their nonlinearities and subject to both lock-in-place and loss of effectiveness actuator faults. A novel fault-tolerant control approach is proposed by embedding adaptive fuzzy approximators into the backstepping design procedure. The designed controller can guarantee that all signals of the closed-loop system are uniformly ultimately bounded and the output tracking error converges to a small neighborhood of zero though there are uncertainties and actuator faults in the considered system. Simulation experiment is conducted and the simulation results demonstrate the effectiveness of the proposed control approach.**

I. Introduction

The design of fault tolerant control (FTC) is very important for safety and reliability of modern engineering systems. Since unexpected actuator faults may cause undesired system behavior and sometimes lead to system instability or even catastrophic accidents, it is necessary to develop control approaches that can deal with such faults during operation. So far, remarkable progresses have been made in controlling systems with actuator faults, and the developed approaches can be broadly classified as passive ones or active ones [1].

Active fault-tolerant control has been widely used since it can obtain better control performance compared with passive ones. As an effective active fault-tolerant control method, adaptive control has attracted much attention in recent years to accommodate unexpected actuator faults. For linear systems, [2] studies adaptive actuator failure compensation with redundant structure, [1] and [3]-[5] developed adaptive fault-tolerant control against loss of effectiveness of actuator in the framework of linear matrix inequality (LMI) approach; for nonlinear systems, [6]-[9] presented several adaptive methods to deal with unknown faults with filter or observer for fault detection and diagnosis (FDD), [10] presented a general framework for constructing automated fault diagnosis and accommodation architectures using on-line approximators and adaptive schemes, and [11]-[18] gave some adaptive neural network fault-tolerant control approaches. However, in the above mentioned works for nonlinear systems, FDD mechanism is needed for fault accommodation. The drawback of such designs is that if there are false alarms or omitted alarms of some faults from the FDD mechanism, the fault-tolerant control may fail. [2] also provides adaptive actuator failure compensation without the help of FDD for feedback linearizable and parametric-strict-feedback nonlinear systems, but further FTC researches are still demanded for more general nonlinear systems to accommodate more types of actuator faults without FDD.

Since adaptive fuzzy systems were proved to be universal approximators [19], and stable adaptive fuzzy control was developed for feedback linearizable systems in [20], many researchers have been interested in studying uncertain nonlinear systems with the help of fuzzy logic approximation. In recent years, adaptive fuzzy control for unknown nonlinear systems gets further development since it has been successfully used to control nonlinear systems which cannot be linearized by incorporating backstepping design procedure, see [21]-[26]. But there are still few works on adaptive fuzzy control for nonlinear systems to accommodate actuator faults as far as we know.

In this paper, an adaptive fuzzy fault-tolerant controller is designed for unknown nonlinear systems which cannot be feedback linearized. The designed controller can tolerate both loss of effectiveness and lock-in-place faults of the actuators under redundant actuation structure. In the backstepping design procedure, adaptive fuzzy systems are employed to approximate the unknown parts of the virtue or the real controllers in each step. The controller is obtained in the last step with fault-tolerant strategy based on some matching conditions. Though the parameters in the matching conditions are unknown and may change their values because the influence of faults, proper adaptive laws can be designed to estimate them on-line. The proposed controller can guarantee all signals of the closed-loop system uniformly ultimately bounded and the output tracking error converge to a small neighborhood of zero though there are uncertainties and actuator faults in the controlled system. So this paper develops an FTC scheme without FDD mechanism for structurally unknown nonlinear systems which cannot be feedback linearized to accommodate both loss of effectiveness and lock-in-place actuator faults.

This paper is organized as follows. The problem formulation and preliminaries are presented in Section II. Then, a systematic backstepping procedure for synthesis of the adaptive fuzzy fault-tolerant controller and the stability analysis are given in Section III. In Section IV, a simulation example demonstrates the effectiveness of the proposed scheme. Finally, Section V concludes the paper.

II. Problem formulation and preliminaries

Consider the following nonlinear system with m actuators:

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} && 1 \le i \le n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + \bar{g}_n^T(\bar{x}_n)u && n \ge 2 \\ y &= x_1 \end{aligned} \quad (1)$$

where x_i $i = 1, \cdots, n$ are the state variables, $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in R^i$, $u = [u_1, u_2, \cdots, u_m]^T \in R^m$ is the input vector whose components may fail during the operation, $y \in R$ is the system output, $\bar{g}_n(\bar{x}_n) = [g_{n1}(\bar{x}_n), g_{n2}, (\bar{x}_n), \cdots, g_{nm}(\bar{x}_n)]^T \in R^m$, f_i, g_i, f_n and g_{nj} for $i = 1, \cdots, n-1$, $j = 1, \cdots, m$ are unknown continuous nonlinear functions, g_i, g_{nj} are smooth. The state variable x_i is measurable,

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the reference output y_m and its up to nth order derivatives are bounded. The actuator faults considered in this paper are lock-in-place (stuck at some unknown place) and loss of effectiveness which are modeled as follows respectively.

Lock-in-place fault:

$$u_j(t) = \bar{u}_j \quad t \geq t_j, \quad j \in \{j_1, j_2 \cdots, j_p\} \subset \{1, 2, \cdots, m\} \tag{2}$$

where $\bar{u}_j$ is the constant value where the jth actuator is stuck at, t_j is the time instant at which the jth lock-in-place fault fault occurs.

Loss of effectiveness fault:

$$\begin{gathered} u_i(t) = \rho_i v_i(t) \quad t \geq t_i \quad i \in \overline{\{j_1, j_2, \cdots, j_p\}} \bigcap \{1, 2, \cdots, m\} \\ \rho_i \in [\underline{\rho}_i, 1], \quad 0 < \underline{\rho}_i \leq 1 \end{gathered} \tag{3}$$

where $v_i(t)$ is the applied control input, and t_i is the time when ith loss of effectiveness fault takes place. ρ_i is the effective proportion of the actuator after loss of effectiveness, $\underline{\rho}_i$ is the lower bound of ρ_i. When $\underline{\rho}_i$ is 1, the corresponding actuator is normal (that is completely effective). So, taking the actuator faults (2) and (3) into account, the input vector u(t) can be written as

$$u(t) = \rho v(t) + \sigma(\bar{u} - \rho v(t)) \tag{4}$$

where $v(t) = [v_1(t), v_2(t), \cdots, v_m(t)]^T$, $\bar{u} = [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_m]^T$, and

$$\begin{aligned} &\rho = diag\{\rho_1, \rho_2, \cdots, \rho_m\} \\ &\sigma = diag\{\sigma_1, \sigma_2, \cdots, \sigma_m\} \\ &\sigma_j = \begin{cases} 1 & if\ the\ jth\ actuator\ fails\ as\ (2)\ i.e.,\ u_j = \bar{u}_j \\ 0 & otherwise \end{cases} \end{aligned} \tag{5}$$

The control objective of this paper is to design a state feedback control law for system (1) to ensure that all signals in the closed-loop system are bounded and the output $y(t)$ tracks the given reference signal $y_m(t)$ as closely as possible though there are unknown actuator faults (2) and (3). For the control purpose, it is required that at least one actuator is not locked as (2), however, it can lose effectiveness as (3) only if $\rho_i \geq \underline{\rho}_i$. Of course there can be more actuators effective partly or completely. In order to accomplish the control task, the following assumptions are needed.

Assumption 1: The system (1) is so constructed that for any up to $m-1$ actuators fail as (2) and the effective proportions of the other actuators meet $\rho_i \in [\underline{\rho}_i, 1]$, the resulted system can still be forced to track the given reference signal closely.

The fault-tolerant control scheme for system (1) with faults (2) and (3) will be developed with the help of the solution to the nominal plant:

$$\begin{aligned} &\dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} && 1 \leq i \leq n-1 \\ &\dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u_0 && n \geq 2 \\ &y = x_1 \end{aligned} \tag{6}$$

where $g_n(\bar{x}_n)$ is a smooth unknown nonlinear function.

Assumption 2: There exit some constants $g_{i0} > 0$ and $g_{i1} > 0$ such that $g_{i0} \leq |g_i(\cdot)| \leq g_{i1}$, $\forall \bar{x}_i \in \Omega_i \subset R^i$, $i = 1, 2, \cdots, n$. The functions $g_1(\cdot)$, $g_2(\cdot)$, $\cdots$,$g_{n-1}(\cdot)$ are the same as the ones in system (1), and $g_n(\cdot)$ is defined in the nominal plant (6). Ω_i is a sufficient large compact set in R^i where $\bar{x}_i$ is included.

Assumption 3: There exit constants $g_{id} > 0$ such that $|\dot{g}_i(\cdot)| \leq g_{id}$, $i = 1, 2, \cdots, n$.

From Assumption 2 it can be seen that the smooth functions $g_i(\cdot)$ are strictly either positive or negative. Without loss of generality, we think that $g_i(\cdot) \geq g_{i0}$, $\forall \bar{x}_i \in \Omega_i \subset R^i$, $i = 1, 2, \cdots, n$. And based on Assumption 1, when all but one actuator have been stuck at zero, that is, $u_j = \bar{u}_j = 0$, $j = 1, 2, \cdots, i-1, i+1, \cdots, m$, the resulted system can still match the nominal plant (6), thus, it can be concluded that

$$g_{ni}(\bar{x}_n)u_i = g_n(\bar{x}_n)u_0 \tag{7}$$

This implies that there exit constants κ_{1j}^* such that $\kappa_{1j}^* g_{nj}(\bar{x}_n) = g_n(\bar{x}_n)$, $j = 1, 2, \cdots, m$. We do not know the value of κ_{1j}^*, but the sign of each κ_{1j}^* is needed for the design of stable parameter adaptive laws.

Assumption 4: The sign of the constant κ_{1j}^* which is represented by $sign[\kappa_{1j}^*]$ is known with $j \in \{1, 2, \cdots, m\}$.

The designed control scheme employs fuzzy logic systems to approximate the unknown nonlinear functions in the control design for it had been proved that fuzzy logic systems are universal approximators in [19]. So the approximation property of fuzzy logic systems should be presented before giving the detailed controller design procedure.

Lemma 1: For any given real continuous function $F(x)$, on a compact set $\Omega \subset R^n$, there exits a fuzzy logic system $Y(x) = \theta^T \xi(x)$ such that $\forall \varepsilon > 0$,

$$\left| F(x) - \theta^T \xi(x) \right| < \varepsilon \tag{8}$$

where $\theta = (\theta_1, \theta_2, \cdots, \theta_M)^T$ is the estimate parameter vector, and $\xi(x) = (\xi_1(x), \xi_2(x), \cdots, \xi_M(x))^T$ is the vector of fuzzy basis functions, M is the number of fuzzy rules.

Define the optimal parameter vector θ^* as:

$$\theta^* = \arg\min_{\theta \in R^n} \{\sup_{x \in \Omega} \left| F(x) - \theta^T \xi(x) \right| \} \tag{9}$$

which makes the fuzzy logic system $\theta^T \xi(x)$ approximate the unknown function $F(x)$ closest.

III. Controller Synthesis and Analysis

In this section, the fault-tolerant controller will be synthesized in detail. With the solution to the nominal plant, the controller structure can be constructed as

$$v(t) = \rho^{-1} d_1^* u_0 + \rho^{-1} d_2^* \tag{10}$$

where $d_1^* \in R^m$ and $d_2^* \in R^m$ are the controller parameter vectors which are meet some matching conditions for fault-tolerant control. Suppose that the lock-in-place faults (2) only occur at time instant t_k, $k = 1, 2, \cdots, q$, and $t_0 < t_1 < \cdots < t_q$. Up to t_k, there are p ($0 \leq p \leq m-1$) actuators being locked in some unknown places. In other words, during the time interval (t_k, t_{k+1}), $k = 0, 1, \cdots, q$, with $t_0 = 0$ and $t_{q+1} = \infty$, there are p control signals $u_{j_1}, \cdots, u_{j_p}$ cannot be available for the controlled system, furthermore, additional disturbances $\bar{u}_{j_1}, \cdots, \bar{u}_{j_p}$ are introduced into the system. Meanwhile the other actuators may lose their effectiveness, that is $u_j(t) = \rho_j v_j$, $j \neq j_1, j_2, \cdots, j_p$, as long as $\rho_j \in [\underline{\rho}_j, 1]$. From the expression (4) and (10), one can get

$$\begin{aligned} \bar{g}_n^T u &= \bar{g}_n^T(\rho v + \sigma(\bar{u} - \rho v)) \\ &= \bar{g}_n^T \rho(I-\sigma)\rho^{-1} d_1^* u_0 + \bar{g}_n^T \rho(I-\sigma)\rho^{-1} d_2^* + \bar{g}_n^T \sigma \bar{u} \\ &= \bar{g}_n^T(I-\sigma) d_1^* u_0 + \bar{g}_n^T(I-\sigma) d_2^* + \bar{g}_n^T \sigma \bar{u} \end{aligned} \tag{11}$$

If the considered system (1) can match the nominal plant (6) in any case of the faults which are allowed by Assumption 1, the following equations can be deduced.

$$\bar{g}_n^T(I-\sigma)d_1^* = \sum_{j \neq j_1 \cdots j_p} d_{1j}^* g_{nj} = \sum_{j \neq j_1 \cdots j_p} \frac{d_{1j}^*}{\kappa_{1j}^*} g_n = g_n \tag{12}$$

$$\begin{aligned} \bar{g}_n^T(I-\sigma)d_2^* + \bar{g}_n^T \sigma \bar{u} &= \sum_{j \neq j_1 \cdots j_p} d_{2j}^* g_{nj} + \sum_{j = j_1 \cdots j_p} \bar{u}_j g_{nj} \\ &= \sum_{j \neq j_1 \cdots j_p} \frac{d_{2j}^*}{\kappa_{1j}^*} g_n + \sum_{j = j_1 \cdots j_p} \frac{\bar{u}_j}{\kappa_{1j}^*} g_n = 0 \end{aligned} \tag{13}$$

Then the matching conditions for the controller parameters d_{1j}^* and d_{2j}^* for $j \in \{1,2,\cdots,m\}$ are needed.

$$\sum_{j \neq j_1 \cdots j_p} \frac{d_{1j}^*}{\kappa_{1j}^*} = 1 \tag{14}$$

$$\sum_{j \neq j_1 \cdots j_p} \frac{d_{2j}^*}{\kappa_{1j}^*} + \sum_{j = j_1 \cdots j_p} \frac{\bar{u}_j}{\kappa_{1j}^*} = 0 \tag{15}$$

The choice of d_{1j}^* and d_{2j}^* for $j = j_1, j_2, \cdots, j_p$ is irrelevant to the closed-loop system, and may be chosen as $d_{1j}^* = 0$, $d_{2j}^* = 0$ for $j \in \{j | j = j_1, j_2, \cdots, j_p\} \bigcap \{1,2,\cdots,m\}$.

Now through a backstepping procedure, the adaptive fuzzy control law for system (1) can be derived step by step to tolerate both faults (2) and (3).

Step 1: Let $x_{1d} = y_m$, $e_1 = x_1 - x_{1d}$, we have

$$\begin{aligned} \dot{e}_1 &= f_1(x_1) + g_1(x_1)x_2 - \dot{x}_{1d} \\ &= g_1(x_1)[g_1^{-1}(x_1)f_1(x_1) + x_2 - g_1^{-1}(x_1)\dot{x}_{1d}] \end{aligned} \tag{16}$$

The ideal virtual controller x_{2d}^* is

$$x_{2d}^* = -g_1^{-1}(x_1)f_1(x_1) + g_1^{-1}(x_1)\dot{x}_{1d} - k_1 e_1 \tag{17}$$

with $k_1 > 0$ being a constant. Take (17) into (16) we can get $\dot{e}_1 = -g_1(x_1)k_1e_1$, and there is a Lyapunov function $V_1 = \frac{1}{2}e_1^2$ that $\dot{V}_1 = -g_1(x_1)k_1e_1^2 \leq -g_{10}k_1e_1^2 \leq 0$. This shows that e_1 is asymptotically stable. Unfortunately, we can not get x_{2d}^* because the nonlinear functions $f_1(x_1)$ and $g_1(x_1)$ are unknown. A fuzzy logic system is used to approximate the unknown part of (17), then the estimate of the virtual control is obtained as

$$x_{2d} = \alpha_1(x_1 \mid \theta_1) = \theta_1^T \xi_1(x_1) - k_1 e_1 \tag{18}$$

Let $e_2 = x_2 - x_{2d}$, and rewritten (16) as

$$\begin{aligned} \dot{e}_1 &= f_1(x_1) + g_1(x_1)[e_2 + (x_{2d} - x_{2d}^*) + x_{2d}^*] - \dot{x}_{1d} \\ &= g_1(x_1)e_2 + g_1(x_1)\tilde{\theta}_1^T \xi_1(x_1) - g_1(x_1)k_1e_1 + g_1(x_1)w_1 \end{aligned} \tag{19}$$

where $\tilde{\theta}_1 = \theta_1 - \theta_1^*$ and w_1 is the minimal approximation error defined as

$$\begin{aligned} w_1 &= \alpha_1(x_1 \mid \theta_1^*) - x_{2d}^* \\ &= \theta_1^{*T} \xi_1(x_1) + g_1^{-1}(x_1)f_1(x_1) - g_1^{-1}(x_1)\dot{x}_{1d} \end{aligned} \tag{20}$$

where $\alpha_1(x_1 \mid \theta_1^*) = \theta_1^{*T}\xi_1(x_1) - k_1e_1$. From Lemma 1, there exits a constant ε_1 such that $|w_1| < \varepsilon_1$. Then consider Lyapunov function candidate

$$V_1 = \frac{1}{2g_1(x_1)}e_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1 \tag{21}$$

and choose parameter updating law as

$$\dot{\theta}_1 = -\gamma_1(\xi_1(x_1)e_1 + r_1\theta_1) \tag{22}$$

$\gamma_1 > 0$, $r_1 > 0$ are design constants. One can get that

$$\begin{aligned} \dot{V}_1 &= \tfrac{1}{g_1(x_1)}e_1\dot{e}_1 - \tfrac{\dot{g}_1(x_1)}{2g_1^2(x_1)}e_1^2 + \tfrac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 \\ &= e_1e_2 - k_1e_1^2 + e_1w_1 - \tfrac{\dot{g}_1(x_1)}{2g_1^2(x_1)}e_1^2 + \tilde{\theta}_1^T(\xi_1(x_1)e_1 + \tfrac{1}{\gamma_1}\dot{\theta}_1) \\ &= e_1e_2 - (k_{10} + \tfrac{\dot{g}_1(x_1)}{2g_1^2(x_1)})e_1^2 - k_{11}e_1^2 + e_1w_1 - r_1\tilde{\theta}_1^T\theta_1 \\ &\leq e_1e_2 - k_{10}^*e_1^2 + \tfrac{\varepsilon_1^2}{4k_{11}} - \tfrac{r_1}{2}\tilde{\theta}_1^T\tilde{\theta}_1 + \tfrac{r_1}{2}\theta_1^{*T}\theta_1^* \end{aligned} \tag{23}$$

where $\dot{\tilde{\theta}}_1 = \dot{\theta}_1$, $e_1w_1 - k_{11}e_1^2 \leq |e_1w_1| - k_{11}e_1^2 \leq \frac{\varepsilon_1^2}{4k_{11}}$ and $-r_1\tilde{\theta}_1^T\theta_1 = -r_1\tilde{\theta}_1^T(\tilde{\theta}_1 + \theta_1^*) \leq -\frac{r_1}{2}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{r_1}{2}\theta_1^{*T}\theta_1^*$ are considered, $k_1 = k_{10} + k_{11}$ with $k_{11} > 0$ and $k_{10}^* = k_{10} - \frac{g_{1d}}{2g_{10}^2} > 0$.

Step i ($2 \leq i \leq n-1$): The *ith* step is to make $e_i = x_i - x_{id}$ as small as possible with $x_{(i+1)d}$. Conduct the similar procedure, we can get the virtual control law and parameter updating law.

$$x_{(i+1)d} = \alpha_i(\bar{x}_i|\theta_i) = \theta_i^T\xi_i(\bar{x}_i) - e_{i-1} - k_ie_i \tag{24}$$

$$\dot{\theta}_i = -\gamma_i(\xi_i(\bar{x}_i)e_i + r_i\theta_i) \tag{25}$$

$\gamma_i > 0$, $r_i > 0$. Let $e_{i+1} = x_{i+1} - x_{(i+1)d}$ and choose Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{2g_i(\bar{x}_i)}e_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i \tag{26}$$

The following expression can be obtained.

$$\dot{V}_i \leq e_ie_{i+1} - \sum_{l=1}^{i}k_{l0}^*e_l^2 + \sum_{l=1}^{i}\frac{\varepsilon_l^2}{4k_{l1}} - \sum_{l=1}^{i}\frac{r_l}{2}\tilde{\theta}_l^T\tilde{\theta}_l + \sum_{l=1}^{i}\frac{r_l}{2}\theta_l^{*T}\theta_l^* \tag{27}$$

$k_i = k_{i0} + k_{i1}$ with $k_{i1} > 0$ and $k_{i0} - \frac{g_{id}}{2g_{i0}^2} = k_{i0}^* > 0$.

Step n: The controller structure (10) cannot be applied because d_1^*, d_2^* and ρ_j are all unknown. There might be a great challenge to estimate d_i^* ($i = 1,2$) and ρ_j ($j = 1,2,\cdots,m$) together since they are multiplied to each other. Let $\rho^{-1}d_1^* = \beta_1^*$ and $\rho^{-1}d_2^* = \beta_2^*$, the applied control input vector is taken as

$$v(t) = \beta_1u_0 + \beta_2 \tag{28}$$

with β_1 and β_2 being the estimate values of β_1^* and β_2^* respectively. Since there are p actuators stuck at some unknown places in (t_k, t_{k+1}), that is, $u_j(t) = \bar{u}_j$, $j = j_1, j_2, \cdots, j_p$, $1 \leq p \leq m-1$, and the others may lose effectiveness or be normal.

Take the derivative of $e_n = x_n - x_{nd}$, one can get

$$\begin{aligned} \dot{e}_n &= f_n(\bar{x}_n) + \bar{g}_n^T(\bar{x}_n)[\rho v + \sigma(\bar{u} - \rho v)] - \dot{x}_{nd} \\ &= f_n(\bar{x}_n) + \bar{g}_n^T(\bar{x}_n)\rho(I - \sigma)\beta_1u_0 + \bar{g}_n^T(\bar{x}_n)\rho(I - \sigma)\beta_2 \\ &\quad + \bar{g}_n^T(\bar{x}_n)\sigma\bar{u} - \dot{x}_{nd} \\ &= f_n(\bar{x}_n) + \sum_{j \neq j_1 \cdots j_p}\rho_jg_{nj}(\bar{x}_n)\beta_{1j}u_0 + \sum_{j \neq j_1 \cdots j_p}\rho_jg_{nj}(\bar{x}_n)\beta_{2j} \\ &\quad + \sum_{j = j_1 \cdots j_p}g_{nj}\bar{u}_j - \dot{x}_{nd} \\ &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u_0 - \dot{x}_{nd} + \sum_{j \neq j_1 \cdots j_p}\frac{\rho_j\tilde{\beta}_{1j}}{\kappa_{1j}^*}g_n(\bar{x}_n)u_0 \\ &\quad + \sum_{j \neq j_1 \cdots j_p}\frac{\rho_j\tilde{\beta}_{2j}}{\kappa_{1j}^*}g_n(\bar{x}_n) \end{aligned} \tag{29}$$

where $\tilde{\beta}_1 = \beta_1 - \beta_1^*$ and $\tilde{\beta}_2 = \beta_2 - \beta_2^*$ are the parameter estimate errors.

Then the ideal control law can be obtained as

$$u_0^* = -g_n^{-1}(\bar{x}_n)f_n(\bar{x}_n) + g_n^{-1}(\bar{x}_n)\dot{x}_{nd} - e_{n-1} - k_ne_n \tag{30}$$

However, u_0^* cannot be applied, so a adaptive fuzzy system is used to approximate the unknown part of u_0^*, and the real control law is designed as

$$u_0 = \alpha_n(\bar{x}_n|\theta_n) = \theta_n^T\xi_n(\bar{x}_n) - e_{n-1} - k_ne_n \tag{31}$$

The parameter updating law is chosen as

$$\dot{\theta}_n = -\gamma_n(\xi_n(\bar{x}_n)e_n + r_n\theta_n) \tag{32}$$

where $\gamma_n > 0$, $r_n > 0$. And the adaptive laws for the controller parameters β_1 and β_2 are established as

$$\begin{aligned} \dot{\beta}_{1j} &= -sign[\kappa_{1j}^*]\tfrac{1}{\tau_{1j}}u_0e_n - \tfrac{1}{\tau_{1j}}q_{1j}\beta_{1j}, \\ \dot{\beta}_{2j} &= -sign[\kappa_{1j}^*]\tfrac{1}{\tau_{2j}}e_n - \tfrac{1}{\tau_{2j}}q_{2j}\beta_{2j}, \quad j = 1,2,\cdots,m \end{aligned} \tag{33}$$

$\tau_{ij} > 0$ and $q_{ij} > 0$ for $i = 1,2$ and $j = 1,2,\cdots,m$. Then the main results can be summarized in the following theorem.

Theorem 1: The proposed control scheme which is constructed by the control structure (28), the control law (31), the adaptive laws (32) and (33), together with the virtual control variables (18) and (24) whose parameters are updated by (22) and (25) respectively, can guarantee the system (1) the following properties though there are unknown nonlinearities and actuator faults.

1) all signals in the closed-loop system remain bounded for bounded initial conditions;

2)The output tracking error $e = y(t) - y_m(t)$ converges to a small neighborhood of zero by choosing the design parameters appropriately.

Proof: Taking the control law (31) into account, we can rewrite $\dot{e}_n$ as

$$\begin{aligned}\dot{e}_n = g_n(\bar{x}_n)(&-e_{n-1} - k_n e_n + w_n + \tilde{\theta}_n^T \xi_n \bar{x}_n \\ &+ \sum_{j \neq j_1 \cdots j_p} \frac{\tilde{\beta}_{1j}}{\kappa_{1j}^*} u_0 + \sum_{j \neq j_1 \cdots j_p} \frac{\tilde{\beta}_{2j}}{\kappa_{1j}^*})\end{aligned} \tag{34}$$

with the minimal approximation error w_n being defined as

$$\begin{aligned}w_n &= \alpha_n(\bar{x}_n \mid \theta_n^*) - u_0^* \\ &= \theta_n^{*T} \xi_n(\bar{x}_n) + g_n^{-1}(\bar{x}_n) f_n(\bar{x}_n) - g_n^{-1}(\bar{x}_n) \dot{x}_{nd}\end{aligned} \tag{35}$$

Consider the Lyapunov function candidate of the *nth* step as

$$\begin{aligned}V = V_n = V_{n-1} &+ \frac{1}{2g_n(\bar{x}_n)} e_n^2 + \frac{1}{2\gamma_n} \tilde{\theta}_n^T \tilde{\theta}_n + \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{2|\kappa_{1j}^*|} \tau_{1j} \tilde{\beta}_{1j}^2 \\ &+ \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{2|\kappa_{1j}^*|} \tau_{2j} \tilde{\beta}_{2j}^2\end{aligned} \tag{36}$$

Take we can get

$$\begin{aligned}\dot{V} = \dot{V}_{n-1} &+ \frac{1}{g_n(\bar{x}_n)} e_n \dot{e}_n + \frac{\dot{g}_n(\bar{x}_n)}{2g_n^2(\bar{x}_n)} e_n^2 + \frac{1}{\gamma_n} \tilde{\theta}_n^T \dot{\theta}_n + \\ &\sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{|\kappa_{1j}^*|} \tau_{1j} \tilde{\beta}_{1j} \dot{\beta}_{1j} + \sum_{j \neq j_1 \cdots j_p} \frac{1}{|\kappa_{1j}^*|} \tau_{2j} \tilde{\beta}_{2j} \dot{\beta}_{2j} \\ \leq &- \sum_{i=1}^{n-1} k_{i0}^* e_i^2 + \sum_{i=1}^{n-1} \frac{\varepsilon_i^2}{4k_{i1}} + e_{n-1} e_n - \sum_{i=1}^{n-1} \frac{r_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i \\ &+ \sum_{i=1}^{n-1} \frac{r_i}{2} \theta_i^{*T} \theta_i^* - e_{n-1} e_n - k_n e_n^2 + e_n w_n + \tilde{\theta}_n^T \xi_n(\bar{x}_n) e_n \\ &+ \sum_{j \neq j_1 \cdots j_p} \frac{\tilde{\beta}_{1j}}{\kappa_{1j}^*} u_0 e_n + \sum_{j \neq j_1 \cdots j_p} \frac{\tilde{\beta}_{2j}}{\kappa_{1j}^*} e_n + \frac{g_{nd}}{2g_{n0}^2} e_n^2 + \frac{1}{\gamma_n} \tilde{\theta}_n^T \dot{\theta}_n \\ &+ \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{|\kappa_{1j}^*|} \tau_{1j} \tilde{\beta}_{1j} \dot{\beta}_{1j} + \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{|\kappa_{1j}^*|} \tau_{2j} \tilde{\beta}_{2j} \dot{\beta}_{2j} \\ \leq &- \sum_{i=1}^{n} k_{i0}^* e_i^2 + \sum_{i=1}^{n} \frac{\varepsilon_i^2}{4k_{i1}} - \sum_{i=1}^{n} \frac{r_i}{2} \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^{n} \frac{r_i}{2} \theta_i^{*T} \theta_i^* \\ &- \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j q_{1j}}{2|\kappa_{1j}^*|} \tilde{\beta}_{1j}^2 + \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j q_{1j}}{2|\kappa_{1j}^*|} \beta_{1j}^{*2} \\ &- \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j q_{2j}}{2|\kappa_{1j}^*|} \tilde{\beta}_{2j}^2 + \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j q_{2j}}{2|\kappa_{1j}^*|} \beta_{2j}^{*2}\end{aligned} \tag{37}$$

where $k_n = k_{n0} + k_{n1}$ with $k_{n1} > 0$ and $k_{n0}^* = k_{n0} - \frac{g_{nd}}{2g_{n0}^2} > 0$, (32), (33) and (34) have been taken into account.

For any given $\mu > 0$, choose parameters such that $k_{i0}^* > \frac{\mu}{2g_{i0}}$, $r_i > \frac{\mu}{\gamma_i}$, $q_{1j} > \mu \tau_{1j}$ and $q_{2j} > \mu \tau_{2j}$, $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$, we have

$$\begin{aligned}\dot{V} \leq &- \sum_{i=1}^{n} \mu (\frac{1}{2g_{n0}} e_i^2 + \frac{1}{2\gamma_i} \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{2|\kappa_{1j}^*|} \tau_{1j} \tilde{\beta}_{1j}^2 \\ &+ \sum_{j \neq j_1 \cdots j_p} \frac{\rho_j}{2|\kappa_{1j}^*|} \tau_{2j} \tilde{\beta}_{2j}^2) + \delta \\ \leq &-\mu V + \delta\end{aligned} \tag{38}$$

with $\delta = \sum_{i=1}^{n} \frac{\varepsilon_i^2}{4k_{i1}} + \sum_{i=1}^{n} \frac{r_i}{2} \theta_i^{*T} \theta_i^* + \sum_{j=1}^{m} \frac{\mu q_{1j}}{2|\kappa_{1j}^*|} \beta_{1j}^{*2} + \sum_{=1}^{m} \frac{\mu q_{2j}}{2|\kappa_{1j}^*|} \beta_{2j}^{*2}$. It follows that

$$V \leq (V(0) - \frac{\delta}{\mu}) e^{-\mu t} + \frac{\delta}{\mu} \tag{39}$$

So, it can be obtained $V \leq max(V(0), \frac{\delta}{\mu})$, δ is a finite constant from its definition. Therefore, if $V(0)$ is bounded, the signals e_i, $\tilde{\theta}_i$, $\tilde{\beta}_{1j}$ and $\tilde{\beta}_{2j}$ are all bounded and belong to the compact set $\Omega = \{(e_i, \tilde{\theta}_i, \tilde{\beta}_{1j}, \tilde{\beta}_{2j}) | V \leq max(V(0), \frac{\delta}{\mu})\}$. Furthermore, it is obvious that θ_i, β_{1j} and β_{2j} are bounded, which implies x_{id} and u_0 are bounded, thus x_i and $v(t)$ can be concluded bounded.

Besides, it can be obtained from (39) that $\lim_{t \to \infty} V = \frac{\delta}{\mu}$, and for any given small positive constant ε, one can appropriately choose μ, k_i, r_i, q_{1j} and q_{2j} such that $\lim_{t \to \infty} V = \frac{\delta}{\mu} \leq \varepsilon$ It is obvious that

$$\lim_{t \to \infty} e_1^2 \leq 2g_1(x_1) \lim_{t \to \infty} V_1 \leq 2g_1(x_1) \lim_{t \to \infty} V \leq 2g_1(x_1) \varepsilon$$

From Assumption 2, there exists a constant g_{11} such that $g_1(x_1) \leq g_{11}$. So the output tracking error e_1 satisfies

$$\lim_{t \to \infty} |e_1| \leq \sqrt{2g_{11}\varepsilon} \tag{40}$$

Therefore, if ε is chosen small enough, e_1 will converge to a neighborhood which is very close to zero.

So far, we have showed that during the time interval (t_k, t_{k+1}), the results in Theorem 1 are ensured. At $t = t_{k+1}$, some actuators are stuck. This will cause a new pair of parameters (d_{1j}^*, d_{2j}^*) for $j \in \{j | j \neq j_1, j_2, \cdots, j_p\} \bigcap \{1, 2, \cdots, m\}$ to match the conditions (14) and (15), consequently, β_{1j} and β_{2j} will be adapted to estimate new β_{1j}^* and β_{2j}^*, then the estimate errors $\tilde{\beta}_{1j}$ and $\tilde{\beta}_{2j}$ will have a sudden jump, which will cause a finite change of V. However, from the above analysis, $V(t_{k+1}^-)$ is bounded if $V(t_k^+)$ is bounded. After a finite change, the value of V at (t_{k+1}^+) is still bounded, which implies that the control task can be achieved in (t_{k+1}, t_{k+2}). Thus in turn, the closed-loop stability and the convergence of the tracking error to a small neighborhood of zero can be realized in $(0, +\infty)$ as long as $V(0)$ is chosen bounded though there are lock-in-place actuator faults. Similar analysis can be made for loss of effectiveness faults and the same results can be obtained. Then results in Theorem 1 are ensured. This completes the proof. ∎

Remark 1: Using backstepping design procedure, adaptive fuzzy approach is introduced to deal with actuator faults in nonlinear systems with unknown structures. So the fault tolerant control technique in [2] for nonlinear systems can be applicable to more general systems. This development is obviously important because most real physical systems are nonlinear and uncertain in their structures. Furthermore, the fault set which can be tolerated has been enlarged to one that contains both lock-in-place and loss of effectiveness of actuators.

Remark 2: Because we do not use the inverse of $\hat{g}_i(\bar{x}_i) = \theta_{gi}^T \xi_{gi}(\bar{x}_i)$ to construct the control law as some existing adaptive fuzzy control approaches did, the singularity problem of the designed controller is avoided effectively.

IV. Simulation Example

An example is given to show the effectiveness of the proposed fault-tolerant control scheme in this paper. The considered system is

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + g_1(x_1) x_2 \\ \dot{x}_2 &= f_2(\bar{x}_2) + g_{21}(\bar{x}_2) u_1 + g_{22}(\bar{x}_2) u_2 \\ y &= x_1\end{aligned} \tag{41}$$

where x_1 and x_2 are states, y is the output of the system. The nonlinear functions of (41) for simulation are: $f_1(x_1) = 0.5x_1$, $f_2(\bar{x}_2) = x_1 x_2$, $g_1(x_1) = (1 + 0.1x_1^2)$, $g_{21}(\bar{x}_2) = (2 + cos(x_1))$, $g_{22}(\bar{x}_2) = 6 + 3sin(x_1))$. The initial conditions are chosen as $x(0) = [1, 0]^T$ and the reference signal is $y_m(t) = sin(t)$.

Selecting fuzzy membership functions as $\mu_{F_i^1}(x_i) = 1/(1 + exp(5(x_i + 2)))$, $\mu_{F_i^2}(x_i) = exp(-(x_i + 1.5)^2)$, $\mu_{F_i^3}(x_i) = exp(-(x_i + 0.5)^2)$, $\mu_{F_i^4}(x_i) = exp(-0.5x_i^2)$, $\mu_{F_i^5}(x_i) = exp(-(x_i - 0.5)^2)$, $\mu_{F_i^6}(x_i) = exp(-(x_i - 1.5)^2)$, $\mu_{F_i^7}(x_i) = 1/(1 + exp(-5(x_i - 2)))$.

Design the adaptive fuzzy controller by the procedure presented above with the parameters $k_1 = 8$, $k_2 = 10$, $\gamma_i = 0.2$, $r_i = 0.05$, $\beta_{1j}(0) = 0.7$, $\beta_{2j}(0) = 0$, $\tau_{ij} = 0.01$, $q_{ij} = 0.001$ for $i = 1,2$ and $j = 1,2$. The initial parameters of the fuzzy approximate systems are $\theta_1 = 0_{7\times1}$ and $\theta_2 = 0_{1\times49}$. The actuator faults introduced for simulation are $u_2(t) = 2$ when $t \geq 4$, and $u_1(t) = 0.4v_1(t)$ for $t \geq 10$. Figures 1-3 show the simulation results of applying the proposed control scheme to system (41) for tracking the reference signal $y_m(t)$. We can see that all the closed-loop signals are bounded and good tracking performance is obtained though the nonlinear system functions and the actuator fault information are all unknown. In order to emphasize the fault-tolerant capability of the control scheme, we also plot the output tracking curve of x_1 to the reference sinal y_m without fault-tolerant control strategy in Figure 4. It can be seen from Figure 4 that the controlled system becomes unstable after the first considered actuator fault, but in Figure 1, the output tracks the reference signal smoothly and closely after both the considered faults.

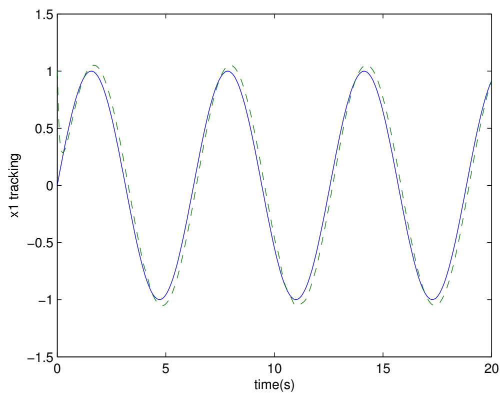

Fig. 1. The curves of x_1 (dash) and y_m (solid)

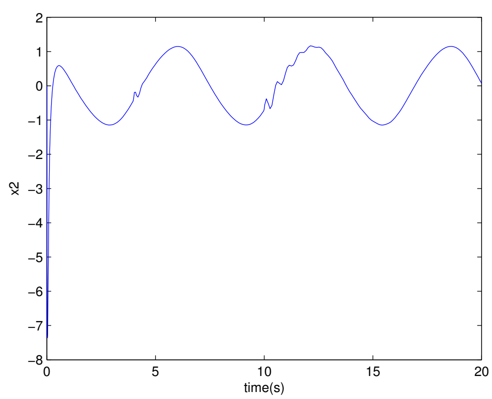

Fig. 2. The curve of x_2

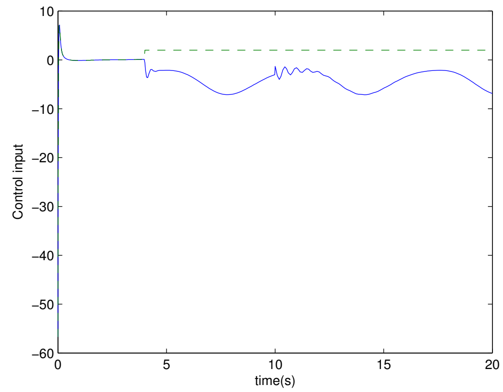

Fig. 3. The curves of u: u_1 (solid); u_2 (dash)

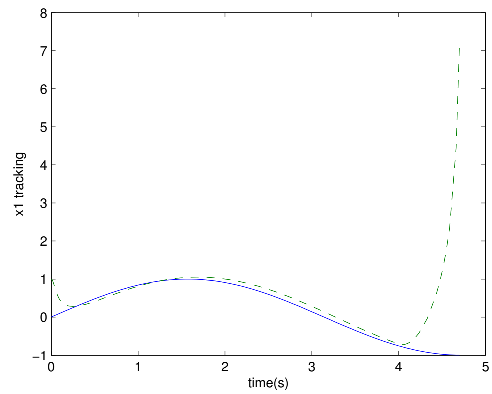

Fig. 4. The curves of x_1 (dash) and y_m (solid) without fault-tolerant control strategy

V. Conclusion

In this paper, a novel fault-tolerant control approach based on the combination of adaptive fuzzy approximation and backstepping design procedure is proposed for structural unknown nonlinear system with redundant actuators. Each actuator may be stuck at some place or lose its effectiveness as long as the resulted system can still be driven to get the desired control performance. The designed control scheme can guarantee that all signals of the closed-loop system uniformly ultimately bounded and the tracking error between the system output and the reference signal converge to an arbitrarily small neighborhood of zero, though the nonlinearities of the controlled system and the information of the occurred faults are all unknown. Besides, the controller singularity problem is avoided perfectly. The results of simulation example show the effectiveness of the control scheme.

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