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Synthesis of switching H_2 and H_∞ output-feedback controllers: A fuzzy supervisor approach

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***Abstract*— In this paper the synthesis of switching multiple static output-feedback controllers for discrete-time LTI systems with state-multiplicative noise is considered which achieves a minimum bound on either the stochastic H_2 or the H_∞ performance levels. The proposed hybrid control scheme is based on a fuzzy supervisor which manages the combination of controllers. A convex formulation of the two controllers leads to a structure which benefits from the advantages of both controllers to ensure a good tracking performance in both the transient state (H_2) and the steady state (H_∞). The stability analysis uses the Lyapunov technique, inspired from switching system theory, to prove that the system with the proposed controller remains globally stable despite the configuration changing.**

I. INTRODUCTION

SYSTEMS with stochastic nature have received much attention in the last decade, mainly in the H_∞ control theory framework. Solutions to various control and estimation problems that ensure a worst case performance bound in the H_∞ sense have been derived, in both, the continuous-time framework and the discrete-time counterpart (see [10] and the references therein). The modeling of parameter system uncertainties as white noise processes in a linear setting is encountered in many areas of applications such as: nuclear fission and heat transfer, population models and immunology. In control theory, such models are encountered in gain scheduling when the scheduling parameters are corrupted with measurement noise.

Following the research of the 1960s and 1970s where the main issues were stability and control of continuous-time state multiplicative systems in the stochastic H_2 framework (see [17] and the references therein), research in the last decade has focused on the H_∞ control setting. Thus, the continuous time stochastic state-multiplicative bounded real lemma (BRL) was obtained in [19] and the discrete-time counterpart was derived in [8]. [1]

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The problem of H_∞ state-multiplicative state- and measurement feedback control was solved in [13, 22, 17] and [2, 8], respectively. In [9], a discrete-time stochastic estimation for a guidance motivated tracking problem was solved and its results were shown to achieve better results than those achieved by the Kalman-filter. In [6], a practical continuous-time estimation problem was solved where a white-noise modeled parameter uncertainty exists in the measurement of a radar altimeter.

Recently, the continuous and discrete-time preview tracking control problems were solved by [5, 7].

The deterministic static output-feedback problem has attracted the attention of many in the past. The main advantage of the static output-feedback is the simplicity of its implementation and the ability it provides for designing controllers of prescribed structure such as PI and PID. An algorithm has been presented recently by [20], which under some assumptions, is found to converge in stationary infinite horizon examples without uncertainty. A sufficient condition for the existence of a solution to a special case of the static output-feedback problem has been obtained in [3]. This condition is in some cases quite conservative.

A necessary and sufficient condition for the existence of a solution to the deterministic problem without uncertainty in terms of matrix inequalities readily follows from the standard BRL. It is, however, bilinear in the decision variable matrices and this is the reason why standard convex programming procedures could not be used in the past to solve the problem, even in the case where the system parameters were all known, and various methods have been proposed to deal with this difficulty [12].

Hybrid dynamical systems include continuous and discrete dynamics and a mechanics (supervisor) managing the interaction between these dynamics. This paper is concerned with a particular class of hybrid systems where the hybrid nature of the control scheme developed consists of a fuzzy supervisor managing the combination between two controllers (H_2 and H_∞). The switching action is gradual and is related to the system evolution between the consecutive transient and steady state modes.

An abrupt switch is not used in the proposed control scheme in order to attenuate the controllability and the instability problems related to the induced jump phenomenon. So, in this work, the supervisor determines the adequate mixing between the two controllers in each mode. Furthermore, the proof of the global stability of a

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closed loop system using the proposed method is related to the ones developed for switching systems theory based on multiple Lyapunov functions [18].

Combination of different techniques to obtain the best performances is widely used today. Wong et al. [14] proposed a combination of three methods: SMC, fuzzy logic control (FLC), and PI control. The resulting controller eliminates the chattering and the steady error introduced by the FLC. Lin and Chen [19] used Genetic algorithms to optimize the mixing of SMC and FLC, and hence to reduce chattering in the system. Barrero et al. [11] developed a FLC-based hybrid controller to manage the switching between a SMC and a fuzzy PI controller. Nevertheless, the above-mentioned works use a fixed combination or restrictive assumptions for the stability analysis. Many combinations of fuzzy logic intelligence and H_∞ technique efficiency have been proposed in the literature [23, 15, 1, 16].

In the present paper we solve the switching H_2 and the H_∞ static control problems for discrete-time linear systems that contain stochastic white-noise parameter uncertainties in the matrices of the state-space model that describes the system.

We apply the simple design method of [21, 10] for deriving the static output-feedback gain that satisfies prescribed H_2 and H_∞ performance criteria. A parameter dependent Lyapunov function (LPD) is applied and a sufficient condition is obtained by adopting a stochastic counterpart of a recent LPD stabilization method. We propose a fuzzy supervisor for hybrid combination of H_2 and H_∞ controllers to use their advantages, and to ensure the robustness and the stability of the closed loop system.

The contribution of the work presented in this paper is combining H_2 and H_∞ controllers using a supervisor, which manages the gradual transition from one controller to another. This method is applied to use the advantages of each controller. The control signal is obtained via a weighting sum of the two signals given by the H_2 and the H_∞ controllers. This weighting sum is managed thanks to a fuzzy supervisor, which is adapted to obtain the desired closed loop system performances by benefiting from the robustness of the H_2 in the approaching phase, minimizing the energy of impulse response and the ability of the H_∞ control to eliminate the chattering and to guarantee the system robustness. So, the $\mathcal{H}_2$ mainly acts in the transient state providing a fast dynamic response and enlarging the stability limits of the system, while the H_∞ control acts mainly in the steady state to reduce chattering and maintain the tracking performances. This method is particularly attractive since it can result in many cases in invariant control systems, i.e. systems completely insensitive to parametric uncertainties and external disturbances. Furthermore, the global stability of the system even if the system switches from one configuration to another (transient to steady state and vice versa) is guaranteed. Section 2 presents the system definition, and the controllers used. In Section 3, the fuzzy supervisor, and the proposed control law and its stability analysis are described. An example of a two-output one-input system is given in Section 4 to illustrate the efficiency of the proposed method.

Notation: Throughout the paper the superscript 'T' stands for matrix transposition, R^n denotes the n dimensional Euclidean space, $R^{n \times m}$ is the set of all $n \times m$ real matrices, N is the set of natural numbers and the notation $P > 0$, (respectively, $P \geq 0$) for $P \in R^{n \times n}$ means that P is symmetric and positive definite (respectively, semi-positive definite). The variables $\{\zeta_k\}$ and $\{\nu_k\}$ are zero-mean real scalar white-noise sequences that satisfy

$$E\{\nu_k \nu_j\} = \delta_{kj},\ E\{\zeta_k \zeta_j\} = \delta_{kj},\ E\{\zeta_k \nu_j\} = \delta_{kj},\ \forall k, j \geq 0$$

We denote by $L^2(\Omega, R^n)$ the space of square-summable R^n valued functions on the probability space $(\Omega, \mathrm{F}, \mathrm{P})$, where Ω is the sample space, F is a σ algebra of a subset of Ω called events and P is the probability measure on F. By $(\mathrm{F}_k)_{k \in N}$ we denote an increasing family of σ-algebras $\mathrm{F}_k \subset \mathrm{F}$ which is generated by $\nu_j, \zeta_j, j \leq k-1$. We also denote by $\tilde{l}^2(N; R^n)$ the space of non-anticipative stochastic processes $\{f_k\} = \{f_k\}_{k \in [0,\infty]}$ in R^n with respect to $(\mathrm{F}_k)_{k \in [0,\infty]}$ satisfying

$$\|f_k\|_{\tilde{l}^2}^2 = E\left\{\sum_0^\infty \|f_k\|^2\right\} = \sum_0^\infty E\left\{\|f_k\|^2\right\} < \infty, \{f_k\} \in \tilde{l}^2(N; R^n) \quad (1)$$

Where $\|\cdot\|$ is the standard Euclidean norm. We denote by δ_{ij} the Kronecker delta function.

II. PROBLEM FORMULATION

We consider the following linear system:

$$x_{k+1} = (A + D\nu_k)x_k + B_1\omega_k + (B_2 + G\zeta_k)u_k,\ x_0 = 0,$$
$$y_k = C_2 x_k + D_{21} n_k, \quad (2)$$

With the objective vector

$$z_k = C_1 x_k + D_{12} u_k, \quad (3)$$

where $\{x_k\} \in R^n$ is the system state vector, $\{\omega_k\} \in R^q$ is the exogenous disturbance signal, $\{n_k\} \in R^p$ is the measurement noise sequence, $\{u_k\} \in R^l$ is the control input, $\{y_k\} \in R^m$ is the measured output and $\{z_k\} \in R^r \subset R^n$ is the state combination (objective function signal) to be regulated. The state-multiplicative white-noise sequences are defined in the Notation section. The matrices in (2), (3) are assumed to be constant matrices of appropriate dimensions.

In each state we seek a constant output-feedback controller

$$u_k = K y_k \quad (4)$$

That achieves a certain performance requirement. We treat the following two different performance criteria:

• The stochastic H_2 control problem: Assuming that $\{\omega_k\},\{n_k\}$ are realizations of a unit variance, stationary, white noise sequences that are uncorrelated with $\{\nu_k\},\{\zeta_k\}$, the following performance index should be minimized:

$$J_2 = \underset{\omega,n}{E}\left\{\|z_k\|_{\tilde{l}_2}^2\right\} \tag{5}$$

• The stochastic H_∞ control problem: Assuming that the exogenous disturbance signal is energy bounded, a static control gain is sought which, for a prescribed scalar $\gamma > 0$ and for all non-zero $\{\omega_k\} \in R^q$, $\{n_k\} \in R^p$, guarantees that $J_\infty < 0$ where

$$J_\infty = \|z_k\|_{\tilde{l}_2}^2 - \gamma^2\left[\|\omega_k\|_{\tilde{l}_2}^2 + \|n_{k+1}\|_{\tilde{l}_2}^2\right] \tag{6}$$

Augmenting systems (2) and (3) to include the measured output y_k we define the augmented state vector $\xi_k = col\{x_k, y_k\}$ and obtain the following representation to the closed-loop system:

$$\xi_{k+1} = \tilde{A}\xi_k + \tilde{B}\tilde{\omega}_k + \tilde{D}\xi_k\nu_k + \tilde{G}\xi_k\zeta_k,\ \xi_0 = 0$$
$$z_k = \tilde{C}\xi_k \tag{7a,b}$$

Where

$$\tilde{\omega}_k = \begin{bmatrix}\omega_k\\ n_{k+1}\end{bmatrix}, \tilde{A} = \begin{bmatrix}A & B_2K\\ C_2A & C_2B_2K\end{bmatrix}, \tilde{D} = \begin{bmatrix}D & 0\\ C_2D & 0\end{bmatrix},$$
$$\tilde{G} = \begin{bmatrix}0 & GK\\ 0 & C_2GK\end{bmatrix},\ \tilde{B} = \begin{bmatrix}B_1 & 0\\ C_2B_1 & D_{21}\end{bmatrix}, \tilde{C} = \begin{bmatrix}C_1 & D_{12}K\end{bmatrix} \tag{8a-g}$$

We consider the following Lyapunov function:

$$V_L = \xi^T\tilde{P}\xi \text{ With } \tilde{P}_2 = \begin{bmatrix}P & -\alpha^{-1}PC_2^T\\ \alpha^{-1}C_2P & \hat{P}\end{bmatrix}, \tilde{P} > 0 \tag{9a-c}$$

Where $P \in R^{n\ n}$ and $\hat{P} \in R^{m\ m}$. The parameter α is a positive scalar tuning parameter.

A. The stochastic H_2 control problem

Applying (9) to the derivation of the stochastic H_2 control results [10] it is obtained that $J_2 < \delta^2$ for a prescribed δ if there exist a positive definite solution $\tilde{Q} = \tilde{P}^{-1}$, where $\tilde{P}$ is of the structure (9b), and $H \in R^{(q+p)\ (q+p)}$ that solve the following linear matrix inequalities (LMIs):

$$\begin{bmatrix}-\tilde{Q} & \tilde{A}\tilde{Q} & 0 & 0 & 0\\ * & -\tilde{Q} & \tilde{Q}\tilde{C}^T & \tilde{Q}\tilde{D}^T & \tilde{Q}\tilde{G}^T\\ * & * & -I_r & 0 & 0\\ * & * & * & -\tilde{Q} & 0\\ * & * & * & * & -\tilde{Q}\end{bmatrix} < 0, \tag{10a-c}$$
$$\begin{bmatrix}H & \tilde{B}^T\\ \tilde{B} & \tilde{Q}\end{bmatrix} > 0,\ trace(H) < \delta^2$$

We note that the LMIs of (10a–c) provide, in the limit where the variances of the multiplicative noise tend to zero (i.e. $\tilde{D} = 0, \tilde{G} = 0$) the standard H_2 criterion for deterministic systems.

Applying [21] it is found that $\tilde{Q}$ possesses the following structure:

$$\tilde{Q} = \begin{bmatrix}Q & C_2^T\hat{Q}\\ \hat{Q}C_2 & \alpha\hat{Q}\end{bmatrix}, \tag{11}$$

Where $Q \in R^{n\ n}, \hat{Q} \in R^{m\ m}$.

Substituting for $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}, \tilde{G}$ into the latter LMIs we obtain the following:

Lemma 1. Consider systems (2), (3). The output-feedback control law (4) achieves a prescribed H_2- norm bound $0 < \delta$, if there exist $Q \in R^{n\ n}, \hat{Q} \in R^{m\ m}, Y \in R^{l\ m}$ and $H \in R^{(q+p)\ (q+p)}$ that, for some tuning scalar $0 < \alpha$, satisfy the following LMIs:

$$\tilde{\Gamma} = \begin{bmatrix}-Q & -C_2^T\hat{Q} & \tilde{\Gamma}_{13} & \tilde{\Gamma}_{14} & 0 & 0 & 0 & 0 & 0\\ * & -\alpha\hat{Q} & \tilde{\Gamma}_{23} & \tilde{\Gamma}_{24} & 0 & 0 & 0 & 0 & 0\\ * & * & -Q & -C_2^T\hat{Q} & \tilde{\Gamma}_{35} & QD^T & \tilde{\Gamma}_{37} & C_2^TY^TG^T & \tilde{\Gamma}_{39}\\ * & * & * & -\alpha\hat{Q} & \tilde{\Gamma}_{45} & \hat{Q}C_2D^T & \tilde{\Gamma}_{47} & \alpha Y^TG^T & \tilde{\Gamma}_{49}\\ * & * & * & * & -I_r & 0 & 0 & 0 & 0\\ * & * & * & * & * & -Q & -C_2^T\hat{Q} & 0 & 0\\ * & * & * & * & * & * & -\alpha\hat{Q} & 0 & 0\\ * & * & * & * & * & * & * & -Q & -C_2^T\hat{Q}\\ * & * & * & * & * & * & * & * & -\alpha\hat{Q}\end{bmatrix}$$

$$\tilde{\Gamma} < 0, \begin{bmatrix}H_{11} & H_{12} & B_1^T & B_1^TC_2^T\\ * & H_{22} & 0 & D_{21}^T\\ * & * & Q & C_2^T\hat{Q}\\ * & * & * & \alpha\hat{Q}\end{bmatrix} > 0, trace(H) < \delta^2 \tag{12a-c}$$

Where $H = \begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}$,

$$\tilde{\Gamma}_{13} = AQ + B_2YC_2, \tilde{\Gamma}_{14} = \alpha B_2Y + AC_2^T\hat{Q},$$
$$\tilde{\Gamma}_{23} = \left[C_2AQ + C_2B_2YC_2 - \hat{Q}C_2\right] + \hat{Q}C_2,$$
$$\tilde{\Gamma}_{37} = QD^TC_2^T, \tilde{\Gamma}_{39} = C_2^TY^TG^TC_2^T\tilde{\Gamma}_{47} = \hat{Q}C_2D^TC_2^T, \tag{13a-k}$$
$$\tilde{\Gamma}_{24} = \left[C_2AC_2^T + \alpha C_2B_2Y - \alpha\hat{Q}\right] + \alpha\hat{Q}, \tilde{\Gamma}_{49} = \alpha Y^TG^TC_2^T$$
$$\tilde{\Gamma}_{45} = \alpha Y^TD_{12}^T + \hat{Q}C_2C_1^T, \tilde{\Gamma}_{35} = QC_1^T + C_2^TY^TD_{12}^T$$

If a solution to the latter LMIs exists, the gain matrix K that stabilizes the system and achieves the required performance is given by $K = Y\hat{Q}^{-1}$ (14)

B. The stochastic H_∞ problem

The LMIs of lemma 1 provide a sufficient condition for the existence of a static output-feedback gain that achieves a prescribed H_2- norm for system (7). A similar result can be obtained if the H_∞-norm of the latter system is considered. Given a prescribed desired bound $0 < \gamma$ on the H_∞- norm of the system, the inequalities in (10) are replaced by the following BRL condition [8].

$$\begin{bmatrix} -\tilde{Q} & \tilde{A}\tilde{Q} & \tilde{B} & 0 & 0 & 0 \\ * & -\tilde{Q} & 0 & \tilde{Q}\tilde{C}^T & \tilde{Q}\tilde{D}^T & \tilde{Q}\tilde{G}^T \\ * & * & -\gamma^2 I_{q+p} & 0 & 0 & 0 \\ * & * & * & -I_r & 0 & 0 \\ * & * & * & * & -\tilde{Q} & 0 \\ * & * & * & * & * & -\tilde{Q} \end{bmatrix} < 0, \quad (15)$$

Using the definition of (11), multiplying (15), from both sides, by $\mathrm{diag}\{\tilde{Q}, \tilde{Q}, I_{q+p}, I_r, \tilde{Q}, \tilde{Q}\}$, where $\tilde{Q}$ is defined in (11), and substituting for $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}, \tilde{G}$ in the latter LMI we obtain the following:

Lemma 2 [10]. Consider the system of (2), (3). The control law (4) achieves a prescribed H_∞-norm bound $0 < \gamma$, if there exist, $Q \in R^{n\ n}, \hat{Q} \in R^{m\ m}, Y \in R^{l\ m}$ that, for some scalar $0 < \alpha$, satisfy the following LMI:

$$\begin{bmatrix} \tilde{\Gamma} & \begin{bmatrix} B_1 & 0 \\ C_2 B_1 & D_{21} \\ 0 & 0 \end{bmatrix} \\ * & -\gamma^2 I_{q+p} \end{bmatrix} < 0 \quad (16)$$

Where $\tilde{\Gamma}$ is defined in (12a).

If a solution to the latter LMI exists, the gain matrix K that stabilizes the system and achieves the required performance is given by (14).

III. FUZZY SUPERVISOR

H_2 control provides a fast dynamic response, a stable control system, and a simple implementation. Conversely, this control strategy leads to some drawbacks that appear in the steady state. The H_∞ techniques can be an alternative for guaranteeing the robustness and the global stability. In order to take advantage of both controllers, H_2 during the transient time, and H_∞ control during the steady state, their control actions are combined by means of a weighting factor, $\alpha \in [0 \quad 1]$, representing the output of a fuzzy logic supervisor that takes the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ as inputs. The global control scheme of the proposed approach is illustrated in Fig. 1.

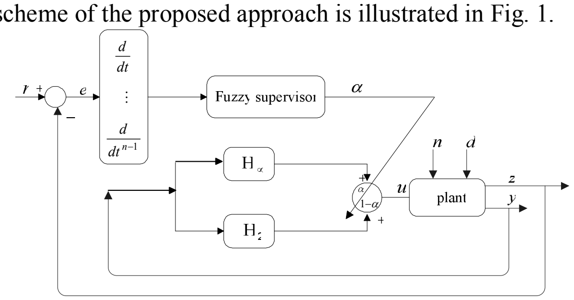

Fig. 1 The control scheme of the proposed method

The fuzzy system is constructed from a collection of fuzzy rules whose j th component can be given in the form

If e *is* H_1^j *And* … *And* e^{n-1} *is* H_n^j *Then* $\alpha = \alpha_j$

where H_i^j is a fuzzy set, and α_j is a singleton.

It is easy to see that it can be considered as a fuzzy rule of a Takagi–Sugeno fuzzy system.The fuzzy implication uses the product operation rule. The connective *AND* is implemented by means of the minimum operation, whereas fuzzy rules are combined by algebraic addition. Defuzzification is performed using the centroid method, which generates the gravity centre of the membership function of the output set. Since the membership functions that define the linguistic terms of the output variable are singletons, the output of the fuzzy system is given by

$$\alpha = \frac{\sum_{i=1}^{m} \alpha_i \prod_{j=1}^{n} {}_i^j}{\sum_{i=1}^{m} \prod_{j=1}^{n} {}_i^j} \quad (17)$$

Where ${}_i^j$ is the degree of membership of H_i^j, and m is the number of fuzzy rules used.

The objective of this fuzzy supervisor is to determine the weighting factor, α, which gives the participation rate of each control signal. Indeed, when the norm of the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ are small (near to zero), the plant is governed by the H_∞ controller, $\alpha = 1$. Conversely, if the error and its derivatives are large, the plant is governed by the H_2, $\alpha = 0$. The variation of α for $n = 2$ is depicted in Fig. 2.

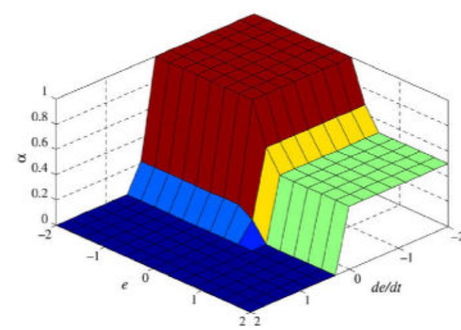

Fig. 2 The variation of α in the state space

The control action, u is determined by

$$u = (1-\alpha) u_{H_2} + \alpha u_{H_\infty} \quad (18)$$

Remark. In the case of a large rule base, some techniques can be employed to significantly reduce the number of rules activated at each sampled time by using the system position in the state space. Indeed, it is demonstrated that using a strict triangular partitioning allows guaranteeing that, at each sampling time, each input variable is described with two linguistic terms at the most [16]. Thus, the output generated by the fuzzy system with n inputs is then reduced to that produced by the subsystem composed of the 2^n fired rules.

A. Stability analysis

The theorem of Essounbouli et al. [16] will be used to prove the global stability of the system governed by the control law (18). Similar to [16], using H_2 and H_∞ control, this theorem can be rewritten as follows:

Theorem 1. Consider a combined fuzzy logic control system as described in this work. If

(1) There exists a positive definite, continuously differentiable, and radially unbounded scalar function V

(2) Every fuzzy subsystem gives a negative definite $\dot{V}$ in the active region of the corresponding fuzzy rule,

(3) The weighted sum defuzzification method is used, such that for any output u we have

$\min(u_{H_2}, u_{H_\infty}) \le u \le \max(u_{H_2}, u_{H_\infty})$

Then the resulting control u, given by (18), guarantees the global stability of the closed loop system.

Satisfying the two first conditions guarantees the existence of a Lyapunov function in the active region which is a sufficient condition for ensuring the asymptotic stability of the system during the transition from the H_2 control to the H_∞ one.

So, let us consider the Lyapunov function $V_\infty = \xi^T P_\infty \xi$ where $P_\infty = P$ is a positive definite matrix and the solution of (16) and we have $\xi^T P_\infty \xi \le \lambda_{\max}(P_\infty)\xi^T \xi$, where $\lambda_{\max}(P_\infty)$ is the maximal eigen value of P_∞. Section 2.2 has shown that the synthesized H_∞ control ensures the decrease of the Lyapunov function V_∞.

We consider the Lyapunov function $V_2 = \xi^T P_2 \xi$ where $P_2 = P$ is a positive definite matrix and the solution of (12)) and we have $\lambda_{\min}(P_2)\xi^T \xi \le \xi^T P_2 \xi$, where $\lambda_{\min}(P_2)$ is the minimal eigen value of P_2. Section 2.1 has shown that the synthesized H_2 control ensures the decrease of the Lyapunov function V_2.

To satisfy the second condition of the theorem, it is enough to choose P_2, P_∞ such that

$$\lambda_{\max}(P_\infty) \le \lambda_{\min}(P_2) \tag{19}$$

This condition guarantees that in the neighborhood of the steady state (H_∞ control), the value of the Lyapunov function V_2 is greater than that of V_∞.

To guarantee third condition, the balancing term α takes its values in the interval [0 1].

Consequently, the three conditions of the above theorem are satisfied, and both the global stability of the system and the error convergence towards zero are guaranteed.

B. Design procedure

In order to minimize the on-line computing time of the proposed method and to simplify its real time implementation, the design procedure implies an off-line processing step and an on-line step during control execution. In the off-line step, the gains are defined in order to satisfy the stability criterion (19). The supervisor design is essentially based on the available information of the process under study. Indeed, when a sufficient amount of information is available, it becomes possible to reduce the number of inputs and the fuzzy rules.

In order to construct the fuzzy supervisor, we define firstly the fuzzy sets for each input (the error and its derivatives) and output; then the rule base is elaborated.

For the on-line step, the error vector is computed and then injected in the supervisor to determine the value of α to apply the global control signal.

IV. EXAMPLE

To demonstrate the solvability of the various LMIs in this paper we bring a third-order, two-output, one-input example where we seek switching output feedback controllers. We consider the system of (2), (3), where

$$A = \begin{bmatrix} 0.9813 & 0.342 & 1.3986 \\ 0.0052 & 0.984 & -0.1656 \\ 0 & 0 & 0.5488 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0198 & 0.0034 & 0.0156 \\ 0.0001 & 0.0198 & -0.0018 \\ 0 & 0 & 0.015 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -1.47 \\ -0.0604 \\ 0.4512 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D_{21} = 0, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, G = 0 \tag{26a-i}$$

Obtain the following results:

• The stochastic H_2 controller: Applying the result of lemma 1 and solving (12) a minimum H_2-norm bound of $\delta = 0.0449$ is obtained for $\alpha = 2.4$. The corresponding static output-feedback controller of (14) is $K = [0.3469 \quad 0.6216]$.

• The stochastic H_∞ controller: Using lemma 2 and solving (16) a minimum value of $\gamma = 0.8916$ is obtained for $\alpha = 2.4$. The corresponding static output-feedback controller of (14) is $K = [0.3567 \ 1.2622]$.

The fuzzy supervisor is constructed by using three fuzzy sets zero, medium, and large for the tracking error and its time derivative. The corresponding membership functions are triangular, as shown in Fig. 3. For the output, five singletons are selected; very large (VL), large (L), medium (M), small (S), and zero (Z), corresponding to 1, 0.75, 0.5, 0.25, and 0, respectively. The fuzzy rule base is depicted in Fig. 4. Rules are defined by a table; for example, a rule in the table can be stated as follows: "If the norm of the error is medium AND the norm of the error derivative is large, and then α is zero".

Results show that H_2 and the combined controller provide a fast dynamic response compared to H_∞, and that H_∞ and the combined controller provide a smooth variation of the control signal. Hence, the proposed control set-up benefits from their advantages of both H_∞ and H_2, and in terms of tracking performance and the robustness to external perturbations, which is ensured by H_∞ control in the steady state (The fuzzy supervisor favors H_∞ to reach the steady state with a fast dynamic). The applied control signal forces the system to remain stable and to attain the desired trajectory. Thus, we obtain an intermediate dynamics whose advantage is to have a compromise between the settling time and the actuator solicitations. Comparing the results shows that the proposed controller ensures a good convergence towards the desired trajectory. The conditions of Theorem 1 are satisfied and the system global stability is guaranteed despite the configuration changing.

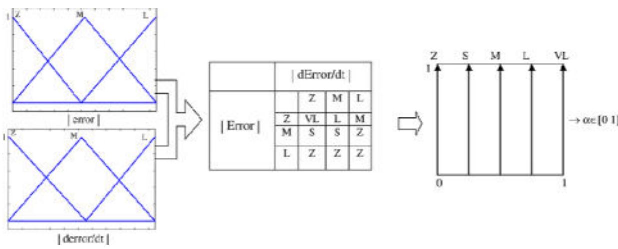

Fig. 3 The structure of the proposed fuzzy supervisor

V. Conclusions

A convex programming method is presented which provides an efficient design of switching robust static output-feedback controllers for linear systems with state multiplicative noise. Sufficient conditions are derived for the existence of switching controller that stabilizes the system and achieves a prescribed bound on its performance. Both stochastic H_2 and H_∞ performance criteria have been considered.

In this work, we have developed a hybrid robust controller. The main idea is the use of a fuzzy supervisor to manage efficiently the action of two controllers based on H_2 and H_∞, such that the system remains stable and robust despite the plant switching from one mode to a new one. Furthermore, this structure allows us to take advantage of both controllers and to efficiently eliminate their drawbacks. Simulation results showed the efficiency and the design simplicity of the proposed approach. Indeed, the H_2 provides good performances in the transient state (a fast dynamic response, enlarged stability limits of the system), while the H_∞ control acts mainly in the steady state to reduce chattering and the effect of the external disturbances. This work can be generalized to multiple controllers, more than two, managed by the same fuzzy supervisor. Indeed, the structure of the fuzzy supervisor allows partitioning the state into different sub states. An adequate controller can be defined for each sub state to ensure the desired performances. The rule base of the fuzzy supervisor will be reconstructed so that the premise part defines the subspace and the conclusion part the corresponding control law. Thus the applied control signal will be a weighted sum of all the controllers used.

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