2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

A multirate autotuning PI with improved static performance

Alberto Leva and Luigi Piroddi, *Member, IEEE*

Abstract—**Among the performance degradation effects that occur with finite precision implementations of controllers, a seldom addressed one is static imprecision. This paper proposes a multirate realization of a PI controller, relying on the frequency separation between the integral action and the remaining controller dynamics, that improves static precision, while minimally affecting dynamic performance. Suitable feasibility conditions for this approach are discussed, and its integration in existing PI tuning methods is also addressed. Conveniently enough, the proposed multirate realization of the controller can be implemented with standard single rate hardware machinery. Simulation results are reported to illustrate the effectiveness of the proposed methodology applied to a well known and widely used tuning procedure.**

I. Introduction

The digital implementation of controllers designed in the continuous time domain notoriously gives rise to a degradation of the achieved control results. The main reasons for this are the process of sampling and holding, the need for anti–aliasing filters, the quantizations introduced by analog to digital and digital to analog conversions, and the presence of finite precision computations [1]. The above problems are of particular relevance for embedded control implementations on low–end architectures, that are nowadays a significant part of the overall control market. Indeed, in most applications of that type (*e.g.*, in household appliances) the difficulties posed by the digital implementation are severe enough to hinder any use of autotuning techniques, which on the other hand would be desirable as a means to achieve (and maintain over time) good performance.

The typical architecture used in the mentioned applications is a microcontroller with 16 (or sometimes 32) bit fixed–point internal arithmetics. Performance degradation and possibly instability may result when a controller designed with infinite precision is actually implemented with a fixed–point digital processor (see, *e.g.*, [2, 3, 4, 5]). This paper, in particular, sets the focus on a seldom studied fact, namely the joint role of sampling/holding and finite precision computations in degrading the static precision of a feedback control system, with reference to regulators endowed with integral action (the PI structure will be considered for simplicity, without loss of conceptual generality).

Manuscript received September 22, 2008.

A. Leva and L. Piroddi are with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Via Ponzio 34/5, 20133, Milano, Italy (phone: +39–02–23993556; fax: +39–02–23993412; e–mail: {leva piroddi,}@elet.polimi.it).

The key fact here is that the finite precision arithmetics of the controller determines the minimum error amplitude that triggers a variation of the integral action. That minimum error therefore equals the actual achievable steady state error, and interestingly enough, it can be significantly larger than the quantum of information corresponding to one bit of the digital implementation. As discussed in this work, a possible way to reduce such error consists in implementing the integral action separately and at a lower sampling rate than the proportional one (adjusting accordingly the discretized integral gain to match the designed continuous one), on the grounds that a slower updating of the integral action makes it reactive to comparatively smaller errors while hardly modifying the overall control dynamics. The suggested scheme results in a multirate digital PI implementation where the proportional and integral actions are managed at different sampling times and summed in a parallel scheme.

Multirate control systems have been the subject of extensive research. Various formalization techniques have been developed for their analysis, both in the z–transform domain and the time domain (using state–space systems and the so called “lifting” technique), and various theoretical results and design methods have been presented in the literature (see, *e.g.*, [6, 7, 8, 9, 10]). Motivations for using multirate control systems range from the need to integrate input–output paths operating with heterogeneous sampling machinery in multivariable systems [6], to the implementation of integrated communication and control systems [11]. A typical circumstance in which multirate control systems are employed arises when there are limitations in the measurement sampling, but the control can be updated faster, [9, 10, 12, 13]. Specific attention has been dedicated to multirate PID controllers in several works (see, *e.g.*, [12, 13, 14]). References [13] and [14], in particular, discuss a dual rate PID implementation that resembles the one proposed here, although a series composition of the PI and PD blocks is there selected instead of a parallel scheme, and the finite precision problem is not addressed.

Moreover, to the best of the authors’ knowledge, while the idea of operating the controller at higher sampling speeds than allowed by the measurement devices has been widely exploited, the reverse idea that slowing down some computations can sometimes improve the (static) performance of the control system is hardly mentioned in the literature of multirate control systems.

Notice also that while multirate control systems often deal with multi–loop implementations, actually involving hardware elements operating at different sampling times, a single control loop is here considered, the regulator only being implemented in a multirate fashion, using single rate hardware machinery.

The outcome of the analysis is a modified PI tuning method that takes into account an additional downsampling parameter for the integral action, and can be applied under suitable conditions (detailed in the following) related both to performance and sampling. It is important to point out that the idea proposed here can be applied to *existing* tuning rules, thus facilitating its use as a complement for already well assessed autotuners.

The paper is structured as follows. Section II illustrates the static performance degradation problem due to finite precision computations, which is subsequently addressed in Section III by means of a multirate PID implementation. Section IV illustrates the proposed tuning procedure for the multirate PI, evidencing that it can be seamlessly coupled with existing tuning methods, therefore facilitating its use. Section V provides some simulation results to illustrate the operation of the proposed multirate PI and tuning method. Finally, in Section VI, some conclusions are drawn, and future developments are envisaged.

II. Problem Statement

In the following, we will assume that the controller operates with fixed point internal arithmetics using B bits, and that its input and output variables vary in a range $[-R/2, R/2]$, so that the least significant bit corresponds to a quantum of information equal to $Q = R/2^B$ (note, by the way, that the above assumptions are consistent with the normalized nature of the signals typically managed in industrial controllers). On a 16 bit machine this would correspond to a 3–digit resolution with $R = 100$ (8 digits with 32 bits).

Converter quantization is not taken into account here, since we are interested in the computations taking place *inside* the controller. It is also worth noticing that – in some sense contrary to intuition – the problem addressed here is not dominated by converter quantization, as will be shown in the following examples.

Assume that a controller with integral action has been designed, *e.g.*, as a result of a PI/PID tuning process. The digital version of the controller (discretized with the backward Euler method) yields at each step an increment of the integral action computed as follows:

$$\Delta u_{int}(k) = K_i T_s e(k),$$

where K_i is the integral gain ($K_i = K/T_i$ for a PI/PID, K being the proportional gain and T_i the integral time), and T_s is the sampling time. Now, the minimum value $\overline{e}$ of the error signal that will trigger a nonzero variation of the integral action solves the following expression:

$$Q = K_i T_s \overline{e},$$

which yields:

$$\overline{e} = \frac{Q}{K_i T_s}.$$

The minimum achievable error value at steady state is therefore constrained to be smaller than $\overline{e}$, in order for the integral action to stop: $|e(\infty)| < \overline{e}$. This threshold is not necessarily lower than the minimum resolution attainable with the given machine arithmetics. On the contrary, if $K_i T_s << 1$ the minimum error may actually correspond to several bits. For example, a unitary integral gain on a 16 bit machine operating at $T_s = 10^{-3}$ s, would result in a 1.5% steady state error, due only to finite precision computations. Notice that $\overline{e}$ is inversely proportional to the sampling time T_s, which is typically chosen very small for various requirements, related to the sampling theorem and other known issues.

It is therefore of interest to investigate if there are methods for reducing $\overline{e}$, while substantially preserving all the designed closed loop features of the nominal continuous time controller. Notice that the latter requirement rules out the increase of T_s, since this would have a direct impact on the overall dynamic performance. However, a trade–off can be attempted by implementing separately the integral action at a slower sampling rate nT_s, with $n > 1$, integer, on the grounds that it accounts for low frequency dynamics, while the rest of the controller is implemented at the standard sampling rate T_s. This amounts to designing a multirate controller, as discussed in the next section with reference to the PI case, for simplicity.

III. Multirate PI Implementation

Let $R_c(s) = K(1+1/sT_i)$ be the transfer function of a standard PI regulator and assume that parameters K and T_i have already being designed. The regulator discretization involves the introduction of sample and hold devices, as in figure 1, and requires sampling at an appropriately high frequency $\omega_s = 2\pi/T_s = \alpha\omega_{cn}$, $\alpha >> 1$, ω_{cn} being the nominal cut–off frequency [1]. Zero order holder (ZOH) devices will be used in the following, and synchronous operation of samplers and holders is assumed.

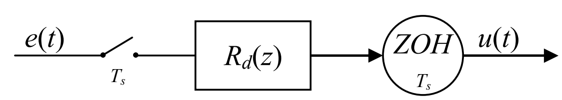

Fig. 1. Single rate digital PI regulator.

The discretized version of the regulator can then be obtained with the backward difference method (the most widely used in applications) as:

$$R_d(z) = \frac{K}{T_i}\frac{(T_i+T_s)z - T_i}{z-1}.$$

The proposed multirate implementation of the PI regulator is depicted in figure 2, and includes two separate blocks:

$$R_{mp}(z) = K,$$

operating with sampling time T_s, and

$$R_{mi}(z) = \frac{K}{T_i}\frac{nT_s z}{z-1},$$

downsampled at nT_s, with $n > 1$, integer.

Notice that this does not require the actual implementation of two discretization paths, since the downsampling can be easily managed via software, by picking one input datum every n sampled, and modifying the relative part of the output only in the corresponding time period.

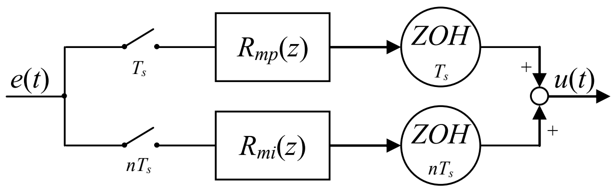

Fig. 2. Multirate digital PI regulator.

In order to investigate the difference between the two schemes depicted in figures 1–2, let us first compare the respective frequency responses (referring to the continuous time domain, *i.e.*, including sampling and holding). With reference to the single rate scheme and restricting the analysis to the frequency band [0 ω_N), where $\omega_N = \pi/T_s$ is the Nyquist frequency, one obtains:

$$U_d(j\omega) = C_d(j\omega)E(j\omega) = \frac{1}{T_s}H_{0,T_s}(j\omega)R_d(e^{j\omega T_s})E(j\omega),\ \ \omega < \omega_N,$$

where $C_d(j\omega)$ denotes the (continuous time) frequency response of the single rate controller, and $H_{0,T}(s) = \dfrac{1-e^{-sT}}{s}$ can be interpreted as the "transfer function" of a zero–order–holder operating at sampling time T (more precisely, it is the rate between the Laplace transform of the output of the ZOH and the z–transform of its input, evaluated at $z = e^{sT}$).

Analogously, the Fourier transform of the control variable in the multirate scheme of figure 2 can be written as:

$$U_m(j\omega) = C_m(j\omega)E(j\omega) = \left\{\frac{1}{T_s}H_{0,T_s}(j\omega)R_{mp}(e^{j\omega T_s}) + \right.$$
$$\left. + \frac{1}{nT_s}H_{0,nT_s}(j\omega)R_{mi}(e^{j\omega nT_s})\right\}E(j\omega),\ \ \omega < \frac{\omega_N}{n},$$

where $C_m(j\omega)$ denotes the (continuous time) frequency response of the multirate controller.

It is easy to verify that the two frequency responses are equal in the frequency band where both are valid, *i.e.* [0 ω_N/n):

$$C_d(j\omega) - C_m(j\omega) = \frac{1-e^{-j\omega T_s}}{j\omega T_s}\frac{K}{T_i}\frac{T_s e^{j\omega T_s}}{e^{j\omega T_s}-1} - \frac{1-e^{-j\omega nT_s}}{j\omega nT_s}\frac{K}{T_i}\frac{nT_s e^{j\omega nT_s}}{e^{j\omega nT_s}-1}$$
$$= \frac{1}{j\omega T_s}\frac{K}{T_i}T_s - \frac{1}{j\omega nT_s}\frac{K}{T_i}nT_s = 0$$

In that frequency band, the only difference between the two schemes is generated if an anti–aliasing filter is introduced in the downsampled path of the multirate scheme that introduces significant phase distortion at the closed loop cut–off frequency. To avoid this, the designer should ensure that

$$\omega_c << \frac{\omega_N}{n}. \tag{1}$$

In view of this, the multirate scheme preserves the dominant dynamic behavior of the control system, but may display some differences at frequencies higher than ω_N/n, especially concerning the control variable dynamics. Such differences are more evident in the time domain, since the integral action is modified only every n samples with the multirate scheme, resulting in a stair–like behavior of the control signal superimposed to the fundamental dynamics of the signal. Notice that this does not imply that significant differences are experienced also with the controlled variable, provided correct sampling conditions are met.

Consider the updating of the integral action during a time period nT_s. Assuming a quasi–constant nonzero error $e_{\Delta I}$ in a time period nT_s – consistently with the correct sampling assumption (1) – the integral action is increased by $\dfrac{KT_s e_{\Delta I}}{T_i}$ at every sample with the single rate version of the controller, while the downsampled version only updates it after n samples, adding the quantity $\dfrac{KnT_s e_{\Delta I}}{T_i}$. Therefore, the two integral actions are equal at $k = n$, while their maximum difference equals

$$\Delta I = \frac{KT_s e_{\Delta I}}{T_i}(n-1), \tag{2}$$

at time step $k = n-1$. This quantity must be kept small, in order to reduce the differences in the control signal between the two implementations of the controller.

Concerning the value of $e_{\Delta I}$ in expression (2), simple considerations allow to state that it equals the maximum amplitude of the expectable step variations of the set point and of output or feedback disturbances. A worst case design, yet extremely conservative choice, can be $e_{\Delta I} = R$, but this is hardly ever advisable. In most practical cases, setting $e_{\Delta I}$ to a few percent of R is more than reasonable.

In conclusion, in order to select appropriately the parameters n and T_s the following pair of conditions should be met

$$n > \frac{R}{2^B}\frac{1}{K_i e_{max}(\infty)}\frac{1}{T_s}, \tag{3}$$

$$n < 1 + \frac{1}{K_i e_{\Delta I}}\frac{1}{T_s}\Delta I_{max}, \tag{4}$$

where $e_{max}(\infty)$ denotes the maximum tolerable error amplitude at steady state, and ΔI_{max} is the maximum allowed difference in the integral action due to downsampling, in addition to the two conditions for correct sampling:

$$\frac{\pi}{T_s} \geq \alpha\omega_c, \tag{5}$$

$$\frac{\pi}{nT_s} \geq \beta\omega_c, \tag{6}$$

where $\alpha > \beta >> 1$. The last two constraints actually define an upper bound for n and T_s. Precisely,

$$T_s \le \frac{\pi}{\alpha\omega_c}, \tag{5'}$$

and condition (6) is necessarily satisfied by ensuring that

$$n \le \frac{\alpha}{\beta}. \tag{6'}$$

The previously stated conditions define a region of admissible choices in the space (T_s,n):

$$\Omega = \{(T_s,n)\in\mathbf{R}^+\times\mathbf{Z}^+ \mid (3) \wedge (4) \wedge (5') \wedge (6')\}. \tag{7}$$

Given the numerical characteristics of the target machine and the parameters of the continuous time version of the controller, parameters $e_{max}(\infty)$, $e_{\Delta I}$, ΔI_{max}, α, β should be properly defined by the designer so as to ensure that $\Omega \neq \emptyset$.

IV. A TUNING METHOD FOR THE MULTIRATE PI

In view of an autotuning application of the PI regulator, the tuning method of choice can be extended to allow for a multirate implementation, the only prerequisite being that the tuning method provides a nominal closed loop cutoff frequency ω_{cn}:

Extended PI tuning method:

Step 1) Apply the tuning method of choice with a suitable selection of the design variables, if any.

Step 2) Select $e_{max}(\infty)$, $e_{\Delta I}$, ΔI_{max}, α, β so that $\Omega \neq \emptyset$.

Step 3) Given the regulator parameters K, T_i and ω_{cn} obtained at Step (1), select a pair $(T_s,n) \in \Omega$.

With respect to the design variables of the original PI tuning method, and to the sampling thresholds α and β, the proposed implementation adds the variables $e_{max}(\infty)$, $e_{\Delta I}$, ΔI_{max}. While the rationale behind the choice of $e_{max}(\infty)$ is obvious, a reasonable choice of ΔI_{max} can be made based on the characteristics of the problem at hand, particularly the admissible actuator upset. As a rule of thumb, a few percent of the control full scale can be selected.

Naturally, if region Ω is large, different admissible choices of n and T_s are possible. Stated otherwise, the expectations of the control design can be increased, by updating $e_{max}(\infty)$, $e_{\Delta I}$, ΔI_{max} to make constraints (3) and (4) more stringent.

V. SIMULATION RESULTS

Consider the simple FOPDT process model

$$G(s) = e^{-sL}\frac{\mu}{1+sT}$$

where $\mu = 10$, $T = 1$, and $L = 0.2$, and apply the classical IMC–PID tuning method [15] with design parameter $\lambda = 0.5$ to parameterize a PI regulator. Recall that λ is interpreted as the desired (dominant) closed loop constant.

The obtained parameters are given by:

$$T_i = T,$$

$$K = \frac{T}{\mu\,(L+\lambda)}.$$

It is trivial to verify that the nominal closed loop system has a cutoff frequency equal to:

$$\omega_{cn} = \frac{1}{L+\lambda}.$$

Accordingly, the sampling time for the single rate PI implementation is selected as

$$T_s = \frac{\pi}{40\omega_{cn}}.$$

Now, assuming conventionally that R = 100 and B = 16, the least significant bit equals the quantum of information $Q = R/2^B = 0.0015$. However, it is apparent from figure 3, which depicts the closed loop performance of the discretized single rate PI against the nominal one designed in the continuous time domain, that the steady state error is at least 2 orders of magnitude larger than Q and is unsatisfactory by typical control performance standards. As explained in Section II, this is related to the fact that $K_iT_s = 0.0079 << 1$.

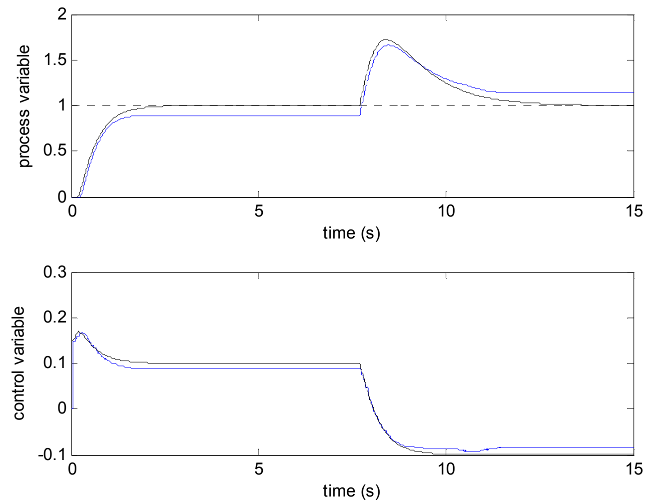

Fig. 3. Closed loop performance of the discretized single rate PI (blue) vs. the nominal regulator (black): response to a reference signal unit step variation at $t = 0$ s and a load disturbance 0.5 step variation at $t = 7.5$ s, process variable (top) and control variable (bottom).

Fixed point operations are here emulated using integer computations and suitable rounding and truncation operations. Real numbers are pre–divided by Q and rounded to the nearest integer. Then, *e.g.*, the product (see also figure 4) is obtained by multiplying two integer operands, multiplying the result by Q and truncating the final number to the integer nearest to 0. The truncated integer is then reconverted back to real by multiplying it again by Q. Analogous reasoning applies to the sum operator.

A multirate implementation of the designed PI regulator can be attempted to overcome the static precision degradation problem. Precisely, setting $e_{max}(\infty)$ = 0.05 and $\frac{\Delta I_{max}}{e_{\Delta I}} = 0.05$, $\alpha = 40$, and $\beta = 4$, a nonempty admissible region for the multirate implementation is obtained, as shown in figure 5. In particular, if – for simplicity sake –one keeps the already chosen sampling time T_s (corresponding to the right vertical edge of Ω), n can be chosen in the range [4, 7].

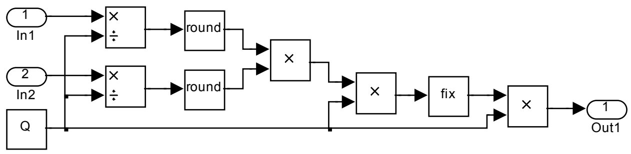

Fig. 4. Emulation of fixed point product.

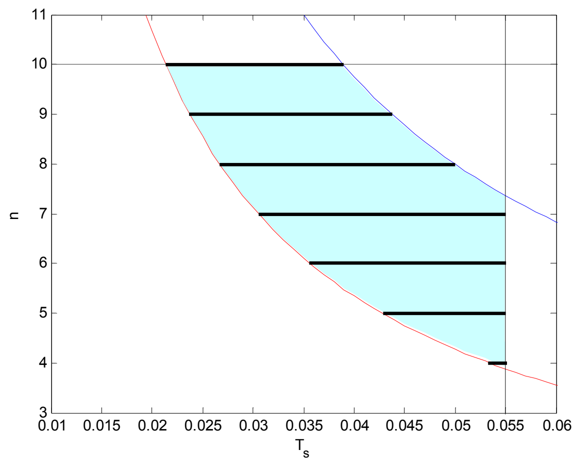

Fig. 5. The admissible region Ω defined by constraints (3) (red line), (4) (blue line), (5') (vertical black line), and (6') (horizontal black line): the horizontal black segments indicate the admissible values of T_s for each n.

Using $n = 7$ with $T_s = 0.055$ s should be particularly effective in reducing the steady state error, since it corresponds to a point of Ω at nearly the maximum admissible distance from the red line, associated to constraint (3). Indeed, the control performance (compare figures 3 and 6) displays a significant improvement in the steady state behavior of the process variable, both in response to reference signal and load disturbance step variations, with almost negligible effects on the transients. It is worth stressing that the improvement is obtained by leaving the (continuous-time equivalent) integral gain unchanged: of course the unwanted effect of finite precision could also be cured by altering that gain, but in that case there would also be a dynamic effect.

This also confirmed by the analysis in the frequency domain, since the single rate and multirate discretizations of the PI regulator have equal frequency response in the frequency band where both are valid (see figure 7). Therefore, it is expected that both realizations have identical low frequency behavior, at least up to the Nyquist frequency related to the slower sampling.

Concerning the higher frequency behavior, figure 6 shows that the low–pass dynamics of the process are enough to filter out any high frequency chattering taking place at frequencies higher than π/nT_s. However, the downsampling of the integral action is not totally without consequences on the control signal dynamics, where the integral action reacts only every n samples, generating a stair–like behavior (see figure 6, bottom).

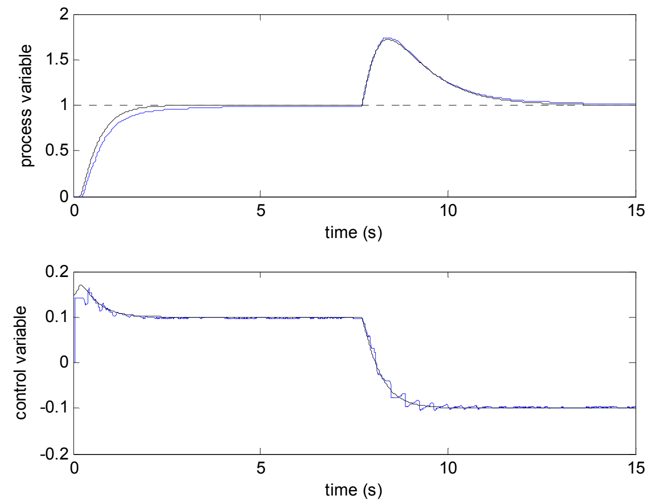

Fig. 6. Closed loop performance of the discretized dual rate PI (blue) vs. the nominal regulator (black): response to a reference signal unit step variation at $t = 0$ s and a load disturbance 0.5 step variation at $t = 7.5$ s, process variable (top) and control variable (bottom).

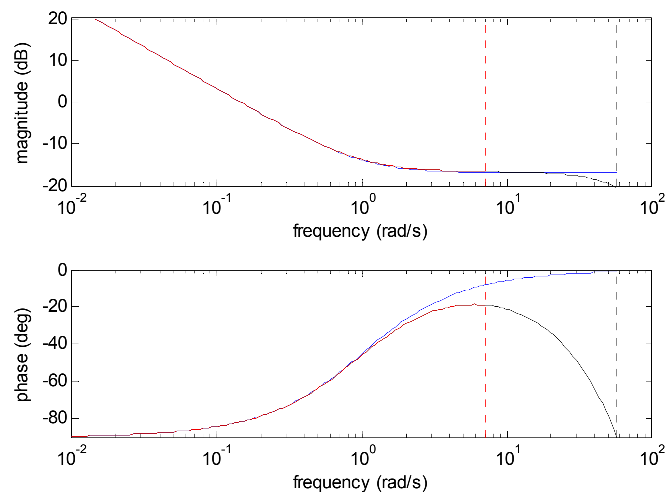

Fig. 7. Frequency response functions of the PI regulator: nominal continuous time controller (blue line), single rate discretized regulator (black line), multirate discretized regulator (red line).

If necessary, such undesired rapid variations of the control signal can be explicitly filtered out using a reconstruction filter with cutoff at frequency π/nT_s (or higher). To see how this works, compare figure 6 with figure 8, where the first order low–pass filters

$$F_{T_s}(s) = \frac{1}{1 + \frac{T_s}{\pi}s},$$

and

$$F_{nT_s}(s) = \frac{1}{1 + \frac{nT_s}{\pi}s},$$

have been used as anti–aliasing and reconstruction filters in the high and low sampling frequency discretization path, respectively.

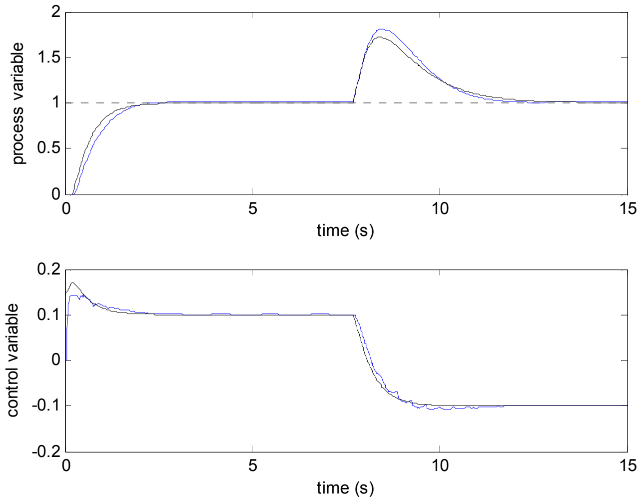

Fig. 8. Closed loop performance of the discretized dual rate PI with anti–aliasing and reconstruction filters (blue) vs. the nominal regulator (black): response to a reference signal unit step variation at $t = 0$ s and a load disturbance 0.5 step variation at $t = 7.5$ s, process variable (top) and control variable (bottom).

Notice how the low and medium frequency behavior is hardly affected, while the control chattering is much reduced. Obviously, the loss in performance due to the addition of a low pass filter with low cutoff frequency in the closed loop depends on how close is the filter cutoff frequency to ω_c, since the nearest they are, the more phase loss is introduced by the filter.

VI. Conclusions

The implementation of controllers on finite precision machines can reveal several unwanted precision degradation effects, such as a static error much larger than the machine and the I/O converters' resolution. A possible solution to this problem has been put forward in this paper, in the form of a multirate implementation of PI/PID controllers, where the integral action is isolated and updated with a slower sampling period.

It is shown that the multirate implementation preserves exactly the low and medium frequency behavior, and presents minimal high frequency variations of the controlled variable, although some high frequency chattering occurs in the control signal. Such chattering can be reduced by proper filtering in the control loop. The required downsampling can easily be performed by software operations, so that it is not actually necessary to set up multiple hardware sample and hold paths.

A methodology was also proposed to make existing PI tuning rules capable of accounting for the multirate implementation and tuning the corresponding additional parameters. This demonstrates that the multirate implementation can be easily included as a feature of autotuning control systems.

The proposed scheme can be generalized to regulators not of the PID family and of higher order, to multirate schemes with more than two sampling periods, and to non multiple (*e.g.*, rationally related) sampling times. Such extensions are currently matter of research, together with a more formal analysis of the obtained schemes in both the continuous and discrete time domains.

Acknowledgements

The authors gratefully acknowledge Prof. M. Lovera for the fruitful discussions on multirate and periodic systems.

References

[1] K.J. Åström and B. Wittenmark, *Computer Controlled Systems: Theory and Design*. Prentice Hall, 1984.

[2] S. Chen, J. Wu, R. H. Istepanian, and J. Chu, "Optimizing Stability Bounds of Finite–Precision PID Controller Structures," *IEEE Transactions on Automatic Control*, vol. 44, no. 11, 2149–2153, 1999.

[3] J. F. Whidborne and R.S.H. Istepanian, "Genetic algorithm approach to designing finite–precision controller structures," *IEE Proceedings – Control Theory and Applications*, vol. 148, no. 5, pp. 377–382, 2001.

[4] W.–S. Yu and H.–J. Ko, "Improved eigenvalue sensitivity for finite–precision digital controller realisations via orthogonal Hermitian transform," *IEE Proceedings – Control Theory and Applications*, vol. 150, no. 4, pp. 365–375, 2003.

[5] J. Wu, S. Chen, G. Li, and J. Chu, "Optimal realizations of fixed–point implemented digital controllers with the smallest dynamic range," *International Journal of Control*, vol. 79, no. 10, pp. 1297–1312, 2006.

[6] M. Araki and K. Yamamoto, "Multivariable Multirate Sampled–Data Systems: State–Space Description, Transfer Characteristics and Nyquist Criterion," *IEEE Transactions on Automatic Control*, vol. 31, no. 2, pp. 145–154, 1986.

[7] S. Samadi, M. Omair Ahmad, A. Nishihara, and M.N.S. Swamy, "Eigensignals of Downsamplers in Time and Transform Domains," *IEICE Transactions Fundamentals*, vol. e90–a, no. 9, pp. 1904–1912, 2007.

[8] S. Samadi, M. Omair Ahmad, and M.N.S. Swamy, "Z–Transform of Quantized Ramp Signal," *IEEE Transactions on Signal Processing*, vol. 53, no. 1, pp. 380–383, 2005.

[9] J. Salt and P. Albertos, "Model–based multirate controllers design," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 6, pp. 988–997, 2005.

[10] Y. Gu, M. Tomizuka, and J. Tornero, "Digital Redesign of Continuous Time Controller by Multirate Sampling and High Order Holds," *Proc. of the 38th Conference on Decision and Control*, Phoenix (AZ), USA, pp. 3422–3427, 1999.

[11] V. Casanova and J. Salt, "Multirate control implementation for an integrated communication and control system," *Control Engineering Practice*, vol. 11, pp. 1335–1348, 2003.

[12] M.J. Er, S.C. Lee, and L.L. Tan, "Digital–Signal–Processor–Based Multirate PID Control of a Two–Link Flexible– Joint Robot," *IEEE TENCON – Digital Signal Processing Applications*, pp. 900–905, 1996.

[13] J. Salt, A. Cuenca, V. Casanova, and V. Mascarós, "A PID Dual Rate Controller Implementation over a Networked Control System," *Proceedings of the 2006 IEEE International Conference on Control Applications*, Munich, Germany, pp. 1343–1349, 2006.

[14] J. Tornero, M. Tomizuka, C. Camiña, E. Ballester, and R Pizá, "Design of Dual–rate PID Controllers," *Proc. of the IEEE Conference on Control Applications*, Mexico City, Mexico, pp. 859–865, 2001.

[15] M. Morari and E. Zafiriou, *Robust Process Control*. Englewood Cliffs: Prentice–Hall, 1989.