2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Analytical $\mathcal{H}_\infty$ design for a Smith-type inverse-response compensator

S. Alcántara, C. Pedret, R. Vilanova* and W. D. Zhang**

Abstract— **In this paper a control configuration for inverse-response processes is presented. It results in a Smith-type predictor scheme that aims to put the non-minimum phase dynamics out of the feedback loop. The design is carried out analytically by solving an $\mathcal{H}_\infty$ weighted optimization problem assuming a second order stable process with one positive zero. The performance of the proposed control configuration is compared by simulation with two different approaches to show its applicability.**

I. INTRODUCTION

A process exhibits inverse-response behavior when the initial response of the output variable is in the opposite direction with respect to the steady-state value. In chemical process industry this phenomenon occurs in several systems, such as drum boilers in distillation columns [12]. The reason for the inverse response is that the process transfer function has zeros in the open right half plane (RHP) [17]. This non-minimum phase (NMP) characteristic of the process affects the achievable closed-loop performance because the controller operates on wrong sign information at the beginning of the transient. This fact introduces essential limitations in terms of achievable performance. For a clear discussion of such limitations see [13].

Two categories of control structures can be found in the literature. The first uses Proportional-Integral-Derivative (PID) controllers with many kinds of tuning methods [11],[10], among others. The good results obtained with such PID approaches is due to a positive feature of the derivative action in trying to correct the wrong direction of the system's response [14]. However, performance of PID control usually degrades to keep the stability margin. The second category uses the so-called inverse-response compensators.

Common inverse-response compensators are [7], where an empirical controller is proposed and the Internal Model Controller (IMC) [8] where the controller minimizes an $\mathcal{H}_2$-norm based performance criterium. Both of the referred approaches are not directly applicable in case of unstable plants. Despite the IMC configuration exists for unstable plants, it is much more involved than for stable plants [8],[6].

The control configuration presented in this paper is, with minor changes, that introduced in [1], which results in a Smith-type predictor scheme that aims to put the non-minimum phase dynamics out of the feedback loop and face the, possibly unstable, minimum phase dynamics in the closed loop. As opposed to [1], where a simple Ziegler-Nichols based tuning was suggested, this time the design is carried out analytically by solving a $\mathcal{H}_\infty$ weighted optimization problem in the line of [17]. Although the presented design is not valid in the unstable plant case neither, it could be adapted for handling unstable systems by adopting the Observer-Controller configuration [2], which could be regarded as a Smith-type inverse response compensator valid in the unstable plant case. This fact would permit to look at the problem at hand from an unified point of view. In this work, however, only the stable plant case is considered.

The paper is organized as follows. Section II introduces the condition of an inverse-response behavior and briefly summarizes the limitations imposed by RHP-zeros. Section III reviews common control configurations for these kind of processes. The proposed architecture and the design procedure is presented in Section IV. An illustrative example to show the applicability of the proposed approach is given in Section V. Concluding remarks are made in Section VI.

II. PROBLEM STATEMENT

As it is common in the literature, we will assume a second-order process with inverse response (SOPIR) resulting from two parallel first-order stable processes having opposite gain

$$P(s) = P_1(s) - P_2(s) = \frac{K_1}{\tau_1 s + 1} - \frac{K_2}{\tau_2 s + 1} \tag{1}$$

where K_1, K_2, τ_1 and τ_2 are positive constants. The overall transfer function $P(s)$ in equation (1) can be posed as

$$P(s) = K_p \frac{-\alpha s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{2}$$

where

$$K_p = K_1 - K_2 \tag{3}$$

and

$$\alpha = \frac{(K_2\tau_1 - K_1\tau_2)}{(K_1 - K_2)} \tag{4}$$

In this context, inverse response appears due to competing effects of slow and fast dynamics [14]. In concrete terms, it appears when the slower process has higher gain. Therefore, the condition for inverse response reads as:

$$\frac{\tau_1}{\tau_2} > \frac{K_1}{K_2} > 1, \quad \alpha > 0 \tag{5}$$

The open-loop response to a step input presents an undershoot, the more pronounced the larger the value of α. Then, the difficulties associated with the control of this kind of processes become more stringent as α approaches the origin.

This work was supported by the Spanish CICYT programme under grants DPI2007-64570 and DPI2007-63356.

*S.Alcántara, C.Pedret and R.Vilanova are with the Systems Engineering and Automatic Control Group. ETSE. Universitat Autònoma de Barcelona. 08193 Bellaterra, Spain. E-mail: salvador.alcantara@uab.es

**Weidong D. Zhang is with the Department of Automation, Shanghai Jiaotong University, Shanghai 200240, P. R. China. Tel: +86.21.34204019; Fax: +86.21.54260762; E-mail: wdzhang@sjtu.edu.cn

978-1-4244-4524-0/09/$25.00 ©2009 AACC

For stable SISO systems with n real RHP-zeros, the output to a step change in the input will cross the original value at least n times. For instance, the output of a system with two real RHP-zeros will initially increase, then decrease below the original value, and then increase to its positive steady-state value. Another important limitation due to the presence of RHP-zeros is the high-gain instability. As it is well-known from classical root-locus analysis, as the feedback gain increases towards infinity the closed-loop poles move to the positions of the open-loop zeros. Additional limitation imputable to the presence of RHP-zeros entails bandwidth limitations. Then, the frequency response of the closed-loop system has an upper limitation (this fact limits the tight control at low frequencies) and also has a lower limitation (this fact limits the tight control at hight frequencies). At last, the presence of RHP-zeros makes impossible the condition of perfect control by any stable and causal controller. Here, we have briefly pointed out the gist of the limitations imposed by RHP-zeros. Nevertheless, for a comprehensive, more in-depth discussion see [13].

III. COMMON CONTROL APPROACHES

Let us consider the process $P(s)$ in (2) and the feedback control scheme shown in Fig. 1.

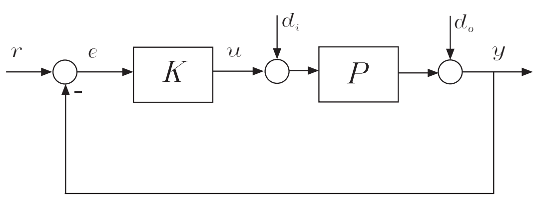

Fig. 1. Feedback control sheme

The open-loop response of the system is

$$y(s) = K(s)K_p \frac{-\alpha s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)} e(s) \tag{6}$$

where K_p and α were defined in equation (3) and equation (4) respectively. The condition for inverse response was indicated by (5). Therefore, the process has a RHP-zero at the point

$$z = \frac{(K_1 - K_2)}{(K_2\tau_1 - K_1\tau_2)} > 0 \tag{7}$$

In order to minimize the input to output inverse response it is necessary to eliminate the unstable zero (7) from the open-loop relation in (6).

A large number of methods can be used to control inverse-response processes. Nevertheless, there are only two popular approaches in the literature and can be grouped into two categories [14].

One of the categories uses PID controllers with many kinds of tuning methods. In [16], it is demonstrated that the Ziegler-Nichols classical tuning for a PID controller can yield good performance for systems with inverse response. However, their robustness margins are not always satisfactory.

The other category uses what is referred to as inverse-response compensators. They have their origin in the Smith predictor [9], i.e. a well-known control configuration for dead time processes which aim is to cancel the dead time within the loop. This concept was used in [7] to cope with the SOPIR plant in (2) and the scheme in Fig. 2 was proposed.

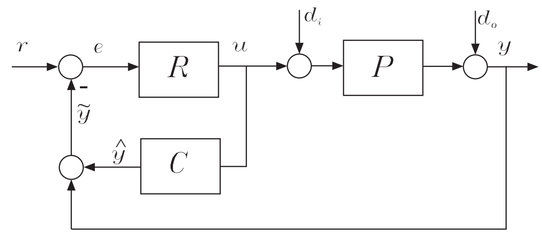

Fig. 2. Inverse-response compensation scheme

The inverse-response compensator $C(s)$ in Fig. 2 was selected as

$$C(s) = k\left(\frac{1}{\tau_2 s + 1} - \frac{1}{\tau_1 s + 1}\right) \tag{8}$$

The compensator $C(s)$ predicts the inverse behavior of the process and provides a corrective signal to eliminate it:

$$\hat{y}(s) = R(s)C(s)e(s) \tag{9}$$

Then, from equation (6), with $K(s) = R(s)$, equation (9) and the controller $C(s)$ in (8) it was easily found that

$$\begin{aligned} \tilde{y}(s) &= y(s) + \hat{y}(s) \\ &= R(s) \times \\ &\frac{[(K_1\tau_2 - K_2\tau_1) + k(\tau_1 - \tau_2)]s + (K_1 - K_2)}{(\tau_1 s + 1)(\tau_2 s + 1)} e(s) \end{aligned} \tag{10}$$

and for

$$k \geq \frac{K_2\tau_1 - K_1\tau_2}{\tau_1 - \tau_2} \tag{11}$$

it is found that the zero of the open-loop transfer function is in the open left half plane (LHP):

$$z = \frac{K_1 - K_2}{(K_1\tau_2 - K_2\tau_1) + k(\tau_1 - \tau_2)} \leq 0 \tag{12}$$

To end the design of the control configuration in Fig. 2 it rests to choose the controller $R(s)$. In [14] it is recommended to select it as a PI controller.

Another control configuration suitable for the control of inverse response processes is the well-known Internal Model Controller (IMC) [8]. The IMC structure is illustrated in Fig. 3, where $P(s)$ denotes the actual plant, $P_o(s)$ is the nominal model and $Q(s)$ is the IMC controller.

It is an open-loop structure in the nominal case ($P(s) = P_o(s)$ and $d_i = d_o = 0$) and thus allowing a direct design of the $Q(s)$ controller by factoring the process model $P_o(s)$ in a minimum phase factor, $M^{-1}(s)$, and in a non-minimum phase, all-pass part, $N(s)$, as

$$P_o(s) = N(s)M^{-1}(s) \tag{13}$$

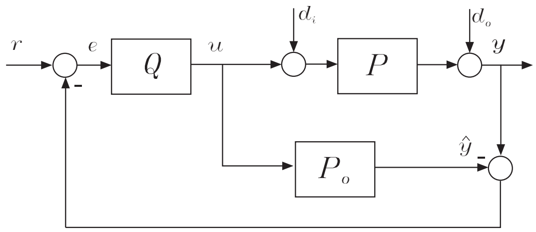

Fig. 3. Internal model control structure

The IMC controller is found by minimizing an $\mathcal{H}_2$-norm criterium. In general, for step inputs we have that:

$$Q(s) = M(s) \tag{14}$$

The robust controller is obtained by augmenting the nominal controller (14) with a filter:

$$Q(s) = M(s)F(s) \tag{15}$$

where $F(s)$ has a low-pass shape. For step inputs it has the following structure:

$$F(s) = \frac{1}{(\lambda s + 1)^n}, \quad \lambda > 0 \tag{16}$$

The exponent n is chosen to make the controller (14) proper, while the parameter λ is chosen to enhance the robustness properties. By increasing λ, the robustness margins increase at the expense of a slower response.

A simple factorization for the plant in (2) is

$$N(s) = \frac{-\alpha s + 1}{\alpha s + 1} \tag{17}$$

and

$$M^{-1}(s) = \frac{K_p(\alpha s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{18}$$

For the plant given in (2) and the above factorization the IMC controller reads as

$$Q(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K_p(\alpha s + 1)(\lambda s + 1)} \tag{19}$$

The IMC structure can be implemented as the feedback scheme in Fig. 1 if we choose

$$K(s) = \frac{Q(s)}{1 - P_o(s)Q(s)} \tag{20}$$

and, from equation (19) and the model $P_o(s)$ as in (2) we can write

$$K(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K_p(\alpha\lambda s + 2\alpha + \lambda)s} \tag{21}$$

It is easy to show that the resulting feedback controller (21) is a PID augmented with a first order filter, so it is equivalent to the *commercial* PID regulator [11]. We consider the IMC in this section because it can also be seen as an inverse-response compensation scheme. Indeed, if we compare Fig. 2 and Fig. 3 and we choose $R(s) = Q(s)$ and $C(s) = -P_o(s)$, both schemes become equivalent.

To finish this section, we will mention the inverse-response compensator proposed by [17]. The configuration scheme follows that of Fig. 2, with $R(s)$ designed to minimize an $\mathcal{H}_\infty$-norm criterium and the compensator $C(s)$ customized for the specific plant (1) as

$$C(s) = \frac{K_2}{\tau_2 s + 1} - \frac{K_1}{\tau_1 s + 1} \tag{22}$$

Since $C(s) = -P_o(s)$, this proposal results in a $\mathcal{H}_\infty$-norm based IMC . Then, the limitations of the structure for unstable plants is still a challenge, as well as for the IMC.

IV. PROPOSED CONTROL CONFIGURATION

In this section we present a control configuration that can be regarded as a Smith-type predictor for the control of inverse-response processes. The proposed scheme is represented in Fig. 4. Model following and model reference configurations have been widely used in literature [4], [3]. The proposed control scheme can be thus regarded since what the controller $R(s)$ sees is the actual plant $P(s)$ in parallel with a model $P_o(s)$ factored in a minimum phase part, $M^{-1}(s)$, and a non-minimum phase, all-pass part $N(s)$, as in equation (13). The output of the minimum phase block, $\xi(s)$, is fed back to form the error signal $e(s)$. To this main feedback, a residues signal $y(s) - \hat{y}(s)$ is added to take into account the effects of the disturbances and modeling errors.

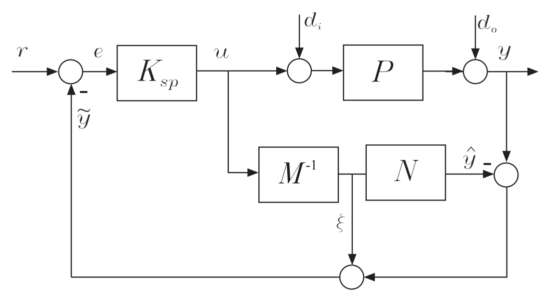

Fig. 4. Proposed control configuration

From the proposed control scheme, we can compute the transfer functions that relate the input signals and the output signals. Let us define first

$$R \doteq \frac{K_{sp}}{M + K_{sp}} \tag{23}$$

Then we have that

$$\begin{pmatrix} u \\ y \end{pmatrix} = \begin{pmatrix} -NR & -RM & MR \\ P(1-NR) & 1-NR & NR \end{pmatrix} \begin{pmatrix} d_i \\ d_o \\ r \end{pmatrix} \tag{24}$$

where the Laplace variable has been dropped for clarity. Note that the R defined in (23) has nothing to do with that appearing in Section III. From now on it should be clear from the context to which one we are referring. Fig. 5 shows the net result of ideal inverse-response compensation that would be accomplished in a nominal situation, i.e. $P(s) = P_o(s)$ and $d_i(s) = d_o(s) = 0$. It can be seen immediately that the transfer function from $r(s)$ to $\xi(s)$ is given by R in (23).

It is important to note, in view of the ideal net result in Fig. 5, that the objective of the proposed control system for inverse-response processes is to get non-minimum phase

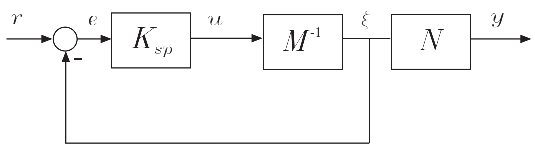

Fig. 5. Ideal net result of inverse-response compensation

dynamics out of the feedback loop and consequently, in the ideal scenario, the controller $K_{sp}(s)$ just faces the minimum-phase factor $M^{-1}(s)$.

Looking at the overall control configuration in Fig. 4, the closed loop system exhibits internal stability if all transfer functions in (24) are stable ,i.e. the injection of bounded external signals at any point in the system results in bounded output signals measured anywhere in the system [13]. It is easy to see from (24) that internal stability is guaranteed if and only if

$$R \in \mathcal{RH}_\infty \tag{25}$$

On the other hand, let us consider the performance specification in terms of the following weighted sensitivy $\mathcal{H}_\infty$-norm:

$$\min_{R(s)} \|W(s)T_{yd_o}(s)\|_\infty \tag{26}$$

This $\mathcal{H}_\infty$ optimization problem means that the effect of the disturbance signal $d_o(s)$ on the output $y(s)$ is to be minimized. In order to focus on setpoint disturbances [17] we choose

$$W(s) = 1/s \tag{27}$$

and the sensitivity function $T_{yd_o}(s)$ is taken from equation (24). From equations (17) and (18) it is possible to obtain the sensitivity function for the plant (2) as

$$T_{yd_o}(s) = 1 - \frac{K_{sp}(s)K_p(-\alpha s+1)}{(\tau_1 s+1)(\tau_2 s+1) + K_{sp}(s)K_p(\alpha s+1)} \tag{28}$$

The control configuration predicts the inverse behaviour of the process and provides a corrective action to eliminate it. In the nominal case, the open-loop response reads as:

$$\begin{aligned}
\tilde{y}(s) &= y(s) - \hat{y}(s) + \xi(s) \\
&= R(s)[P_o(s) - M^{-1}(s)N(s) + M^{-1}(s)]e(s) \\
&= K_{sp}(s)M^{-1}(s)e(s) \\
&= K_{sp}(s)\frac{K_p(\alpha s+1)}{(\tau_1 s+1)(\tau_2 s+1)}e(s)
\end{aligned} \tag{29}$$

Therefore, it is found that the zero of the open-loop transfer function is in the open LHP, so the inverse response has been eliminated from the loop gain. To design $K_{sp}(s)$ analytically let us define

$$z \doteq 1/\alpha \tag{30}$$

We will make use of the maximum modulus principle [5], [13]. Let us consider that $f(s)$ is stable, i.e. it is analytic in the complex RHP. Then, the maximum value of $|f(s)|$ for s in the RHP is attained on the analiticity region boundary, i.e. somewhere along the $j\omega$-axis, consequently

$$\|f(s)\|_\infty = \max_\omega |f(j\omega)| \geq |f(w)| \quad \forall w \in \text{RHP} \tag{31}$$

Assuming $W(s)T_{yd_o}(s)$ is a stable transfer function (this will be the case if internal stability holds and the T_{yd_o} transfer function incorporates a zero in the origin) and attending to the well-known zero constraint $|T_{yd_o}(z)| = 1$, [13] we can write

$$\|W(s)T_{yd_o}(s)\|_\infty \geq |W(z)| \tag{32}$$

Therefore,

$$\min_{R(s)} \|W(s)T_{yd_o}(s)\|_\infty \geq \alpha \tag{33}$$

The minimum value solution implies

$$\frac{1}{s}(1 - NR) = \alpha \tag{34}$$

which leads to

$$R(s) = \alpha s + 1 \tag{35}$$

For internal stability (25) and realization issues it is necessary to extend the optimum R in (35) with a filter. The following suboptimum R is proposed:

$$R(s) = (\alpha s+1)F(s) \tag{36}$$

where

$$F(s) = \frac{1}{(\lambda s+1)^n} \tag{37}$$

From (23), (36) and (37) it follows that

$$K_{sp} = RM(1-R)^{-1} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p\left((\lambda s+1)^n - (\alpha s+1)\right)} \tag{38}$$

It can be seen that by choosing $n = 2$ in (38) the resulting K_{sp} controller is proper and thus physically realizable.
As in the IMC design, the parameter λ is chosen to enhance the robustness properties, i.e. by increasing λ the robustness margins increase at the expense of a deterioration of the performance. The following proper, suboptimal, controller is finally obtained by selecting $n = 2$.

$$K_{sp}(s) = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p s(\lambda^2 s + 2\lambda - \alpha)} \tag{39}$$

The corresponding R block is obtained from (36) and (37) by choosing $n = 2$:

$$R(s) = \frac{(\alpha s+1)}{\lambda^2 s^2 + 2\lambda s + 1} \tag{40}$$

The proposed control configuration can be implemented as the inverse-response scheme in Fig. 2 if we choose

$$C(s) = [1 - N(s)]M^{-1}(s) \quad (41)$$

$$= \frac{K_p 2\alpha s}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (42)$$

and

$$R(s) = K_{sp}(s) \quad (43)$$

Certainly, it is more interesting to implement it as the feedback scheme in Fig. 1. Simple straightforward blocks algebra gives the unity feedback controller depicted in Fig. 6. From Fig. 6 and (39) the following equivalent unity feedback controller is obtained:

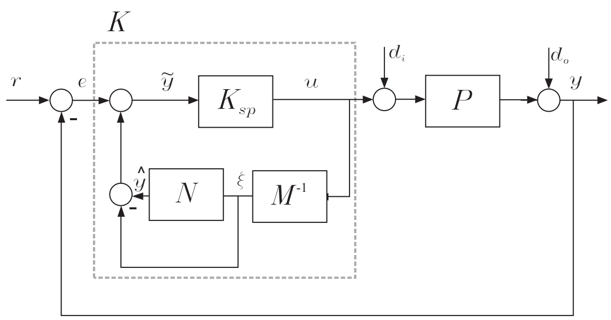

Fig. 6. Proposed inverse-response compensator as a classical unity feedback controller

$$K(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K_p s(\lambda^2 s + 2\lambda + \alpha)} \quad (44)$$

The resulting $K(s)$ is a PID controller augmented with a first order filter, so it is equivalent to the *commercial* PID regulator [11]. More precisely, $K(s)$ in (44) can be cast into the conventional PID form:

$$K_{PID}(s) = K_C(1 + \frac{1}{T_I s} + T_D s)\frac{1}{T_F s + 1} \quad (45)$$

according to the following tuning rules

$$K_C = \frac{\tau_1 + \tau_2}{K_p(2\lambda + \alpha))} \qquad T_I = \tau_1 + \tau_2 \quad (46)$$

$$T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \qquad T_F = \frac{\lambda^2}{2\lambda + \alpha} \quad (47)$$

It should be noted that the IMC controller tends to cancel the poles and the zero of the minimum-phase factor in which the plant is factored, yielding an open loop with the non-minimum phase, all-pass, factor. It is not the case of the proposed $\mathcal{H}_\infty$ controller since the non-minimum phase all-pass factor is isolated by the control structure, as it can be seen in the ideal net result shown in Fig. 5.

Despite the IMC configuration exists for unstable plants, it becomes more involved, to say the least. The proposed control configuration can not tackle unstable processes neither, since it is based on the Smith Predictor. The extension of the presented ideas to the unstable plant case is being currently investigated by means of using the so-called *Observer-Controller* configuration [15],[2], which provides an ideal net result as that in Fig. 5 and can be used for controlling unstable systems.

V. ILLUSTRATIVE EXAMPLE

To show the applicability of the proposed control configuration let us consider the inverse-response process in (2). This system has uncertainty on four parameters: $K_p = K_{p_o} + \delta_k$, $\alpha = \alpha_o + \delta_\alpha$, $\tau_1 = \tau_{1_o} + \delta_1$ and $\tau_2 = \tau_{2_o} + \delta_2$. The nominal values are: $K_{p_o} = 3$, $\alpha_o = 2$, $\tau_{1_o} = 2$ and $\tau_{2_o} = 1$. Then the nominal process model is,

$$P_o(s) = 3\frac{-2s + 1}{(2s + 1)(s + 1)} \quad (48)$$

First, the nominal model (48) is factored as in (17) and (18) yielding:

$$N(s) = \frac{-2s + 1}{2s + 1} \quad (49)$$

and

$$M^{-1}(s) = \frac{3}{s + 1} \quad (50)$$

Once the factorization of the nominal process model is performed, the suboptimal controller $R(s)$ is chosen from (40) with $\lambda = 2.2$ to assure a good compromise between performance and robustness.

In order to complete the example two other control approaches have been used. First, the feedback control configuration in Fig. 1 with a PI controller tuned with the Ziegler-Nichols method (PI-ZN), with $K_c = 0.22$ and $T_i = 4.67$. Second, the control configuration with inverse-response compensation shown in Fig. 2 stated by Iinoya *et al.* (IA), [7], with a compensator $C(s)$ as in (8) with $k = 6$ and $R(s)$ as a PI controller tuned with the Ziegler-Nichols method also with $K_c = 0.22$ and $T_i = 4.67$.

The time responses of the nominal system are shown in Fig. 7. We have supposed that the set-point is a unit step signal at $t = 0$ and the output disturbance is a step signal with amplitude 0.5 in the controlled variable at $t = 50$ sec.

Time responses of the perturbed process with 20% uncertainty on the parameters of the process are shown in Fig. 8

With the control configuration proposed in this paper we obtain faster nominal responses, as it can be seen in Fig. 7. The simulations with the uncertain case show that time responses achieved with the proposed scheme are fast and less oscillatory, as it can be seen in Fig. 8.

The proposed controller can be equivalent to the IMC [8] and the inverse-response compensator in [17] by selecting different filters. This is the reason why these approaches has not been considered in this example.

VI. CONCLUSIONS

A review of the most significant approaches to control inverse-response processes has been done. Also, a new control configuration for these kind of processes has been presented. It results in a Smith-type predictor scheme that

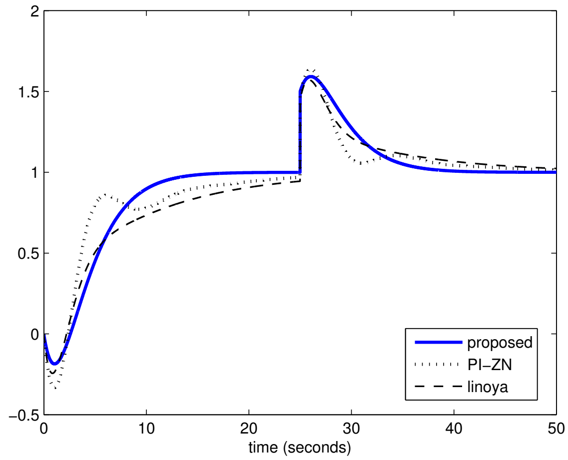

Fig. 7. Time responses of the nominal process model: PI-ZN (dot), IA (dash) and proposed (solid).

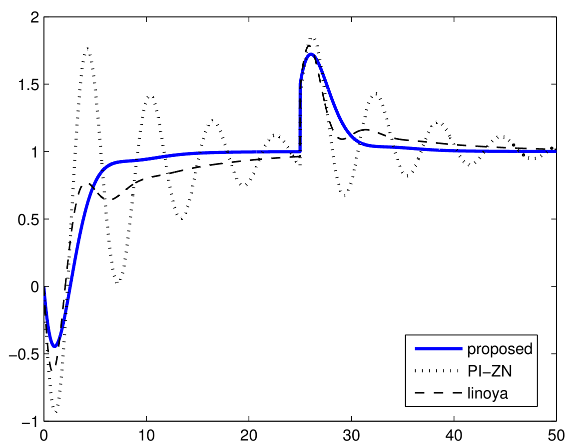

Fig. 8. Time responses of the perturbed process ($\delta = 20\%$): PI-ZN (dot), IA (dash) and proposed (solid).

aims to put the nom-minimum phase dynamics out of the feedback loop. The design has been done analytically by solving an $\mathcal{H}_\infty$ weighted optimization problem. It has been shown that the proposed control configuration can be simplified and implemented in a standard feedback configuration. By means of simulation, the performance of the proposed control configuration has been compared with two different approaches. Since the proposed control configuration can not be used with unstable processes, further work will attempt to adapt the design procedure for the unstable plant case.

REFERENCES

[1] S. Alcántara, C. Pedret, R. Vilanova, P. Balaguer, and A. Ibeas. Control configuration for inverse response processes. *Proc. of the 16th Mediterranean Conference on Control and Automation (MED08)*, 2008.

[2] S. Alcántara, C. Pedret, R. Vilanova, A. Ibeas, and P. Balaguer. A 2dof $\mathcal{H}_\infty$ robust tracking design for a special type of observed state feedback controllers. *Proceedings of the 47th IEEE Conference on Decision and Control (CDC 2008)*, 2008.

[3] B.D.O. Anderson and J.B. Moore. *Optimal Control: Linear Quadratic Methods*. Prentice-Hall, 1989.

[4] K.J. Astrom and B. Wittenmark. *Computer Controlled Systems: Theory and Design*. Prentice-Hall, 1984.

[5] R. Churchill and J. Brown. *Complex Variable and Applications*. McGraw Hill, 1986.

[6] Arvin Dehghani, Alexander Lanzon, and Brian D.O. Anderson. $\mathcal{H}_\infty$ design to generalize internal model control. *Automatica*, 42(11):1959 – 1968, 2006.

[7] K. Iinoia and R. J. Alperter. Inverse response in process control. *Ind. Eng. Chem.*, 54:39–43, 1962.

[8] M. Morari and E. Zafirou. *Robust Process Control*. Prentice-Hall International, 1989.

[9] J. E. Normey-Rico and E. F. Camacho. *Control of Dead-time Processes*. Springer-Verlag, 2007.

[10] C. Pedret, R. Vilanova, R. Moreno, and I. Serra. A refinement procedure for PID controller tuning. *Computers and Chemical Engineering*, 26:903–908, 2002.

[11] C. Scali and A.Rachid. Analytical design of proportional-integral-derivative controllers for inverse response processes. *Ind. Eng. Chem. Res.*, 37:1372–1379, 1998.

[12] F. G. Shinskey. *Process control systems*. McGraw-Hill, 2nd. edition, 1979.

[13] S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control*. Wiley, 1997.

[14] G. Stephanopoulos. *Chemical Process Control: An Introduction to Theory and Practice*. Prentice-Hall, 1984.

[15] M. Vidyasagar. *Control System Synthesis. A factorization approach*. MIT Press. Cambridge, Massachusetts, 1985.

[16] K. T. V. Waller and C. G. Nygardas. On inverse response in process control. *Ind. Eng. Chem. Fundam.*, 14(3):221–223, 1975.

[17] W. Zhang, X. Xu, and Y. Sun. Quantitative performance design for inverse-response processes. *Ind. Eng. Chem. Res.*, 39:2056 – 2061, 2000.