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Sensitivity of Operator-based Nonlinear Feedback Control Systems

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Abstract— **This paper studies sensitivity of operator-based nonlinear feedback control systems. Nonlinear systems under bounded perturbations can be stabilized by using a condition of robust right coprime factorization. In this paper, based on the stabilized systems, a new condition is proposed to guarantee that the stabilized systems are insensitive to the effect of the perturbations.**

I. INTRODUCTION

Nonlinear control system design problems have attracted more attentions in different fields, especially stability of the nonlinear control system. There are two different ways of formulating the notion of stability of control systems, that are bounded input bounded output (BIBO) stability approach and input-to-state stability (ISS) approach. Recently, relying on operator-theoretic method, [1] has proposed a method for nonlinear control, and it has led to BIBO stability of the nonlinear feedback control systems depending on robust right coprime factorization of the nonlinear systems [4]. More recently, an applicable condition for the robust right coprime factorization has been proposed [2], and robust stabilization of the nonlinear control system can be obtained by using this condition. However, sensitivity of the operator-based nonlinear control feedback control systems has not been discussed in the above literatures. In this paper, with guaranteeing BIBO stability of the considered systems, the effect of bounded perturbations of the considered systems on the sensitivity is considered. According to the theorem of [3], a new condition is proposed to guarantee that the stabilized systems are insensitive to the effect of the perturbations.

II. PROBLEM STATEMENT

Consider a nonlinear plant $P:U\rightarrow Y$, where U and Y are the input and output spaces, respectively. It's described as

$$P = ND^{-1} \tag{1}$$

where $D: W \rightarrow U$ and $N: W \rightarrow Y$ are stable operators from the quasi-state space W to the input and output spaces, and D is invertible. A feedback control system is said to be well-posed if every signal in the control system is uniquely determined for any input signal in U. For the above nonlinear plant (1), under the condition of well-posedness, N and D are said to be right coprime factorization if there exist two stable operators $S: Y \rightarrow U$ and $R: U \rightarrow U$ satisfying the following Bezout identity

$$SN + RD = M, \text{ for some } M \in \mathcal{S}(W,U) \tag{2}$$

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where $\mathcal{S}(W,U)$ is the set of unimodular operator [1].

Considering that real nonlinear plants must deal with uncertainties and disturbances, the overall plant $\tilde{P}$ with the above bounded perturbation ΔP is defined as

$$\tilde{P} = P + \Delta P \tag{3}$$

where $\tilde{P}$ and P are nonlinear operators. As above mentioned, we assume that the right coprime factorization of $\tilde{P}$ is

$$\tilde{P} = P + \Delta P = (N + \Delta N)D^{-1} \tag{4}$$

and ΔN is unknown but the upper and lower bounds of it are known. According to (2), we can obtain

$$S(N + \Delta N) + RD = M, \text{ for some } M \in \mathcal{S}(W,U) \tag{5}$$

if the range of ΔN is included in the null set of S, where ΔP is perturbation of the plant which can represent only ΔN. The reason is that ΔP is an additive uncertainty. However, in some cases, (5) is not satisfied since ΔN is unknown. The stability of the nonlinear feedback system with bounded perturbation can be guaranteed by Theorem 1 of [2], and the operator S should satisfy

$$\|[S(N + \Delta N) - SN]M^{-1}\| < 1. \tag{6}$$

According to the above mentioned condition, designed operators S and R can make the output of the nonlinear plant with bounded perturbation be bounded when the input of the nonlinear plant is bounded. The detail of proof can be found in [2]. However, the sensitivity of the above stabilized nonlinear control system to the perturbation is not discussed. In the next section, it will be considered, and a new condition on insensitivity will be proposed.

III. MAIN RESULT

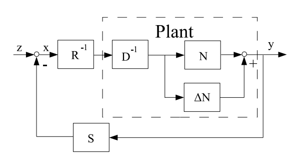

Fig. 1. Operator-based nonlinear feedback control system

In the above section, the mentioned operator-based nonlinear feedback control system can be described as Fig. 1. In this figure, z, x and y represent the input, the state and the output, respectively. When the considered plant P varies to be $\tilde{P}$ under the effect of bounded perturbation ΔP, if the output y corresponding to a given input z will not blow up, we say that this operator-based nonlinear feedback control system is insensitive to the perturbation of the plant P. To make the above system insensitive to the perturbation ΔP, the designed operators S and R need to be modified.

Remark 1: In this paper, z, x and y are elements of Hilbert spaces. Since a Hilbert space is always a Banach space, Theorem 1 of [2] also can be used.

Before modifying operators S and R, some notations and a definition on insensitivity should be introduced. If H, K are real Hilbert spaces and the set $E \subset H$ is nonempty, let $\mathcal{N}(E,K)$ be the linear space of all (not necessarily linear) operators $J: E \to K$. Also let $Lip(E,K)$ be the set of all operators J in $\mathcal{N}(E,K)$ such that $\|J\|^* = sup\{\|Jx_1 - Jx_2\| \cdot \|x_1 - x_2\|^{-1} : x_1, x_2 \in E, x_1 \neq x_2\} < \infty$, and $\mathcal{B}(E,K)$ be the set of all operators J in $\mathcal{N}(E,K)$ such that $\|J\|_0 = sup\{\|Jx\| \cdot \|x\|^{-1} : x \in E, x \neq 0\} < \infty$ and $J0 = 0$ if $0 \in E$. It is easy to see that $Lip(E,K)$ is a linear subspace of $\mathcal{N}(E,K)$, and that $\|\cdot\|^*$ is a seminorm on $Lip(E,K)$. To define input-output system, let H_i, H_s, H_o, H be fixed real Hilbert spaces, and the sets $E_i \subset H_i$ and $E_s \subset H_s$ be nonempty. The sets E_i, E_s, H_o are interpreted as the sets of the inputs, states, and outputs, respectively, while H is an auxiliary space required in the system description. Furthermore, let $\mathcal{L}$ be a real linear space, $\mathcal{L}^+$ be a linear subspace of equipped with a seminorm $\|\cdot\|^+$, and $\mathcal{P} \subset \mathcal{L}^+$ be nonempty. $\mathcal{P}$ is interpreted as the set of admissible perturbations of the system.

Definition 1: Let $[T,V]$ be to describe an input-output system, where $T: \mathcal{L} \times E_i \to \mathcal{N}(E_s,H)$, $V: \mathcal{L} \times E_i \to \mathcal{N}(E_s,H_o)$, and $G_0 \in \mathcal{L}$. The system $[T,V]$ is called uniformly insensitive with respect to $G_0 + \mathcal{P}$ if there exist fixed constants $r > 0$ and $\alpha, \beta \geq 0$ such that $\|y - y_0\| \leq (\alpha + \beta\|z\|)\|G - G_0\|^+$, whenever $z \in E_i$, $x, x_0 \in E_s$, $y, y_0 \in H_o$, $G \in G_0 + \mathcal{P}$, $\|G - G_0\|^+ \leq r$, and

$$T(G_0,z)x_0 = 0, \quad y_0 = V(G_0,z)x_0 \tag{7}$$

$$T(G,z)x = 0, \quad y = V(G,z)x \tag{8}$$

hold. Then we summarized the theorem about sensitivity of [3] in Lemma 1.

Lemma 1: Let $\tilde{T}: \mathcal{L} \to \mathcal{N}(E_s,H)$, $\mathcal{P} \subset \mathcal{L}^+$, and $G_0 \in \mathcal{L}$ be fixed. Assume that (i) there exist $v > 0$ and $Q \in Lip(E_s,H)$ such that $|\langle \tilde{T}(G_0)x_1 - \tilde{T}(G_0)x_2, Qx_1 - Qx_2\rangle| \geq v\|x_1 - x_2\|^2$ for all $x_1, x_2 \in E_s$, (ii) there exists $\lambda > 0$ so that $\tilde{T}(G) - \tilde{T}(G_0) \in \mathcal{B}(E_s,H)$ and $\|\tilde{T}(G) - \tilde{T}(G_0)\|_0 \leq \lambda\|G - G_0\|^+$, whenever $G \in G_0 + \mathcal{P}$.

If $T(G,z)x = \tilde{T}(G)x - z$, then the system $[T,I]$ is uniformly insensitive with respect to $G_0 + \mathcal{P}$. Moreover T satisfies $\|x_0\| \leq \sigma + \omega\|z\|$ whenever $x_0 \in E_s$, $z \in E_i$, and $T(G_0,z)x_0 = 0$, where constants $\sigma, \omega \geq 0$.

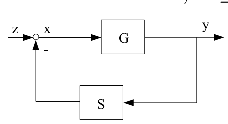

Fig. 2. Operator based feedback system

To simplify the notation, we put $\mathcal{H}_0 = \mathcal{N}(E,H)$, $\mathcal{H} = \mathcal{N}(H,H)$, and $Lip(E) = Lip(E,H)$, $\mathcal{B}(E) = \mathcal{B}(E,H)$ for any $E \subset H$. G is an operator in $\mathcal{H}_0$, so we let $\mathcal{L} = \mathcal{H}_0$, $\mathcal{L}^+ = \mathcal{B}(E)$ with the seminorm $\|\cdot\|_0 = \|\cdot\|^+$ and $\mathcal{P} = \mathcal{B}(E)$. Let $\mathcal{M}(E)$ be the set of all operators $J \in \mathcal{H}_0$ such that $\mu_J = \inf\{\langle Jx_1 - Jx_2, x_1 - x_2\rangle \|x_1 - x_2\|^{-2} : x_1, x_2 \in E, x_1 \neq x_2\} > -\infty$. Fig. 2 is the simplified block diagram of Fig. 1, where $G = (N + \Delta N)D^{-1}R^{-1}$. When $\Delta N = 0$, we have that $G = G_0$. The system can be represented as

$$(I + SG)x = z, \quad y = Gx \tag{9}$$

Then we have the following theorem to guarantee that the above system is insensitive to the perturbation.

Theorem 1: Let $x \in E$, $G_0 \in Lip(E)$, $G \in G_0 + \mathcal{B}(E)$ and operator $S \in Lip(H)$. If $\mu_{G_0} > 0$ and $1 + \mu_{G_0}\mu_S > 0$, then the above system (see Fig. 2) is uniformly insensitive with respect to $G_0 + \mathcal{B}(E)$.

Proof: The above system (see Fig. 2) is represented as

$$T(G,z)x = (I + SG)x - z, \quad V(G,z) = G \tag{10}$$

and let $\tilde{T}(G) = I + SG$. Referring to (i) of Lemma 1, put $Q = G_0$ and let $x_1, x_2 \in E$. Then we have

$$\begin{aligned} & \langle \tilde{T}(G_0)x_1 - \tilde{T}(G_0)x_2, Qx_1 - Qx_2\rangle \\ = \; & \langle (I + SG_0)x_1 - (I + SG_0)x_2, G_0x_1 - G_0x_2\rangle \\ \geq \; & \mu_{G_0}(1 + \mu_{G_0}\mu_S)\|x_1 - x_2\|^2 \end{aligned} \tag{11}$$

Hence, (i) in Lemma 1 holds with $v = \mu_{G_0}(1 + \mu_{G_0}\mu_S) > 0$. Then we get

$$\begin{aligned} & \|[\tilde{T}(G) - \tilde{T}(G_0)]x\| \\ = \; & \|(I + SG)x - (I + SG_0)x\| \\ \leq \; & \|S\|^*\|(G - G_0)x\| \leq \|S\|^*\|G - G_0\|^+\|x\| \end{aligned} \tag{12}$$

This inequality shows that (ii) in Lemma 1 also holds with $\lambda = \|S\|^*$. Then we can obtain that the system $[T,I]$ is insensitive with respect to $G_0 + \mathcal{B}(E)$, and T satisfies $\|x_0\| \leq \sigma + \omega\|z\|$. In the above system $[T,I]$, $V(G,z) = I$, so $\|y - y_0\| = \|x - x_0\| \leq (\alpha + \beta\|z\|)\|G - G_0\|^+$. Since in our considered system (10), $V(G,z) = G$, we have

$$\begin{aligned} \|y - y_0\| &= \|Gx - G_0x_0\| \\ &\leq \{r + \|G_0\|^*\}(\alpha + \beta\|z\|)\|G - G_0\|^+ \\ &\quad + \|G - G_0\|^+(\sigma + \omega\|z\|) \\ &= (\tilde{\alpha} + \tilde{\beta}\|z\|)\|G - G_0\|^+ \end{aligned} \tag{13}$$

According to Definition 1, we obtain that the system (10) is uniformly insensitive with respect to $G_0 + \mathcal{B}(E)$. ■

IV. CONCLUSION

Sensitivity of operator-based nonlinear feedback control systems is studied, and a new condition is proposed to guarantee the insensitivity of the considered systems to the bounded perturbations.

REFERENCES

[1] G. Chen and Z. Z. Han, Robust right factorization and robust stabilization of nonlinear feedback control systems, *IEEE Trans. Autom. Control*, vol. 43, no. 10, pp. 1505–1510, 1998.

[2] M. Deng, A. Inoue and K. Ishikawa, Operator based nonlinear feedback control design using robust right coprime factorization, *IEEE Trans. Autom. Control*, vol. 51, no. 4, pp. 645–648, 2006.

[3] V. Dolezal, Some results on sensitivity of general input-output systems, *Circuits, Systems, and Signal Processing*, vol. 10, no. 1, pp. 71-89, 1991.

[4] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, MIT Press, Cambridge, MA, 1985.