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Delay-dependent robust stability criteria for neutral singular systems with time-varying delays and nonlinear perturbations

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Abstract— **This paper concerns the problem of the delay-dependent robust stability for neutral singular systems with time-varying delays and nonlinear perturbations. Based on integral inequalities, some both discrete-delay-dependent and neutral-delay-dependent stability criteria are obtained and formulated in the form of linear matrix inequalities(LMIs). Neither model transformation nor bounding technique for cross terms is involved. Numerical examples are given to show the less conservatism of the proposed results.**

I. INTRODUCTION

Recently, the stability analysis of neutral differential systems, which have delays in both its state and the derivatives of its states, has been widely investigated by many researchers [1]–[8]. Current stability criteria for the neutral systems can be roughly divided into two categories, namely delay-independent criteria and delay-dependent criteria. While the delay-independent stability criteria guarantee the asymptotic stability irrespective to the size of time-delay, delay-dependent stability criteria give the admissible maximum delay bounds for guaranteeing the asymptotic stability of system. In general, delay-dependent stability criteria are less conservative than delay-independent ones when the size of time-delay is small. So, more attention has been paid to delay-dependent criteria.

In recent years, the problem of the robust stability of nonlinear perturbed systems has also received considerable attention [9]–[14]. These results were obtained by using model transformation [9]–[13] or bounding technique for cross terms [9], [10] or free matrices technique [13] or the properly chosen Lyapunov-Krasovskii functionals [14]. Besides, for neutral systems with mixed discrete and neutral delays, most of the aforementioned methods can only provide **discrete-delay-dependent** and **neutral-delay-independent** results. In [6], some both discrete-delay-dependent and neutral-delay-dependent stability criteria were obtained by introducing some free matrices variables. However, some of the free matrices did not serve to reduce the conservatism of the results that was obtained.

On the other hand, singular systems have been extensively studied in the past years due to the fact that singular systems better described physical systems than state-space ones. Depending on the area application, these models are also called descriptor systems, semi-state systems, differential-algebraic systems or generalized state-space systems [15], [16]. Therefore, the study of robust stability problem for neutral singular system with nonlinear perturbations is of theoretical and practical importance.

It should be pointed out that when the robust stability problem for singular systems is investigated, the regularity and absence of impulses are required to be considered simultaneously [17], [18]. Hence, the robust stability problem for neutral singular systems is much more complicated than that for state-space ones. In [19], the authors studied the robust stability problem for the neutral singular system and presented delay-independent stability criteria without considering the regularity and absence of impulses.

To the best of the authors' knowledge, the delay-dependent robust stability problem for neutral singular systems with time-varying delays and nonlinear perturbations remains open, which motivates this paper. Since model transformation may introduce additional dynamics [20], [21], and using bounding techniques for cross terms appearing in the derivative of corresponding Lyapunov functional may introduce additional conservativeness, neither model transformation nor bounding technique for cross terms is applied in analyzing the considered systems, which may yield a less conservative robust stability condition. By using integral inequalities in [2] and [22], some both **discrete-delay-dependent** and **neutral-delay-dependent** stability conditions in terms of linear matrix inequalities are obtained. Numerical examples illustrate the effectiveness of the obtained results.

II. PROBLEM FORMULATION

Consider the following neutral singular system with time-varying discrete delay:

$$\begin{cases} E\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_d x(t-d(t)) \\ \qquad\qquad + f(x(t),t) + g(x(t-d(t)),t), \\ x(\theta) = \phi(\theta),\ \forall\theta \in [-max\{d_m,\tau\},\ 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system. E, A, A_d and C are known matrices of appropriate dimensions, where E may be singular and we assume that $\operatorname{rank} E = r \leq n$. $\tau > 0$ is the constant neutral delay and $d(t)$ is the time-varying discrete delay satisfying

$$0 \leq d(t) \leq d_m, \quad \dot{d}(t) \leq \mu, \tag{2}$$

where d_m and μ are positive constants. $\phi(\cdot)$ is a continuous vector valued initial function. $f(x(t),t)$ and $g(x(t-d(t)),t)$

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are the nonlinear perturbations in the system model. They satisfy that $f(0,t)=0$ and $g(0,t)=0$. It is assumed that the nonlinear perturbations are bounded in magnitude, i.e.

$$\begin{aligned} &\|f(x(t),t)\| \le a\|x(t)\| \\ &\|g(x(t-d(t)),t)\| \le b\|x(t-d(t))\|, \ \forall t>0 \end{aligned} \tag{3}$$

where a and b are known positive scalars.

It should be noted that system (1) encompasses many natural models of time-delay systems and can be used to represent many important physical systems, for example, networks containing lossless transmission lines, vibrating massed attached to an elastic bar. In addition, if

$$\begin{aligned} &f(x(t),t) = \Delta A(t)x(t) \\ &g(x(t-d(t)),t) = \Delta A_d(t)x(t-d(t)) \end{aligned} \tag{4}$$

then the nonlinear perturbations are reduced to be the norm-bounded uncertainties that are well known in robust control of uncertain systems.

In order to simplify the treatment of the problem, the operator $\Im: \ \mathcal{C}([-\tau, \ 0], \ \mathbb{R}^n) \to \mathbb{R}^n$ is defined to be

$$\Im(x_t) = Ex(t) - Cx(t-\tau). \tag{5}$$

The stability of the operator $\Im$ is defined as follows.

Definition 1: [23] The operator $\Im$ is said to be stable if the zero solution of the homogeneous difference equation

$$\Im(x_t)=0, \ t\ge 0, \ x_0=\varphi\in\{\psi\in\mathcal{C}([-\tau, \ 0]:\Im\psi=0\},$$

is uniformly asymptotically stable.

If $\operatorname{rank} E = r < n$, then there must exist nonsingular constant matrices U and V, such that

$$UEV = \begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix}, \ UCV = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \tag{6}$$

where $E_1 \in \mathbb{R}^{r\times r}$ is a nonsingular matrix, $C_1 \in \mathbb{R}^{r\times r}$ and $C_2 \in \mathbb{R}^{(n-r)\times(n-r)}$ are constant matrices, respectively.

Lemma 1: [19] The operator $\Im$ is stable if $\|E_1^{-1}C_1\| < 1$ and $\|C_2\| \neq 0$, where E_1, C_1, C_2 are defined as in (6).

Definition 2: [15]–[17]

1) The pair (E,A) is said to be regular if $det(sE-A)$ is not identically zero.

2) The pair (E,A) is said to be impulse-free if $deg(det(sE-A)) = \operatorname{rank} \ E$.

3) The neutral singular system (1) is said to be regular and impulse-free if the pair (E,A) is regular and impulse-free.

4) The neutral singular system (1) is said to be asymptotically stable for any nonlinear perturbation (3) if for any $\epsilon>0$, there exists a scalar $\delta(\epsilon)>0$ such that for any compatible initial conditions $\phi(t)$ satisfying $\sup\limits_{-d(t)\le t\le 0}\|\phi(t)\|\le\delta(\epsilon)$, the solution $x(t)$ of the system (1) satisfies $\|x(t)\|\le\epsilon$ for $t\ge 0$. Furthermore, $\lim\limits_{t\to\infty} x(t) = 0$.

Definition 3: The neutral singular system (1) is said to be robustly stable if the system is regular, impulse-free and asymptotically stable for any nonlinear perturbations satisfying (3).

In this paper, the objective is to obtain some delay-dependent criteria to check the robust stability of the neutral singular system (1) with nonlinear perturbations (3).

We conclude this section by introducing the following lemmas that will be used in the proof of our main results.

Lemma 2: [24] Consider the function $\varphi: \ R^+ \to R$, if $\dot\varphi$ is bounded on $[0,\infty)$, that is, there exists a scalar $\alpha>0$ such that $\mid \dot\varphi(t) \mid \le \alpha$ for all $t\in[0,\infty)$, then $\varphi(t)$ is uniformly continuous on $[0,\infty)$.

Lemma 3: (Barbalat's Lemma) [24] Consider the function $\varphi: \ R^+ \to R$, if φ is uniformly continuous and $\int_0^\infty \varphi(s)ds < \infty$, then $\lim\limits_{t\to\infty}\varphi(t) = 0$.

Lemma 4: (S-procedure) [25] Let $F_i = F_i^T \in \mathbb{R}^{n\times n}, \ i = 0,1,2,\cdots,p$, Then the following statement is true

$$\xi^T F_0 \xi > 0, \quad for \ \ all \ \ \xi\neq 0 \ \ satisfying \ \ \xi^T F_i \xi \ge 0,$$

if there exist real scalars $\epsilon_i \ge 0, \ i=0,1,2,\cdots,p$ such that

$$F_0 - \sum_{i=1}^{p}\epsilon_i F_i > 0.$$

For $p=1$, these two statements are equivalent.

Lemma 5: [2] For any constant matrix $X\in\mathbb{R}^{n\times n}, \ X = X^T > 0$, scalar $r>0$, and vector function $\dot x: [-r, \ 0]\to\mathbb{R}^n$ such that the following integration is well defined, then

$$\begin{aligned} -r\int_{t-r}^{t}\dot x^T(s)X\dot x(s)ds \le &\begin{bmatrix} x^T(t) & x^T(t-r) \end{bmatrix} \\ &\times \begin{bmatrix} -X & X \\ X & -X \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-r) \end{bmatrix}. \end{aligned} \tag{7}$$

Lemma 6: [22] For any constant matrix $Y\in\mathbb{R}^{n\times n}, \ Y = Y^T > 0$, scalar $0\le d(t)\le d_m$, and vector function $\dot x: [-d_m, \ 0]\to\mathbb{R}^n$ such that the following integration is well defined, then

$$\begin{aligned} -d_m\int_{t-d(t)}^{t}\dot x^T(s)X\dot x(s)ds \le &\begin{bmatrix} x^T(t) & x^T(t-d(t)) \end{bmatrix} \\ &\times \begin{bmatrix} -Y & Y \\ Y & -Y \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}. \end{aligned} \tag{8}$$

III. MAIN RESULTS

In this section, we give some criteria of robust stability for the neutral singular system (1). The necessary condition for the stability of the system (1) is that the operator $\Im$ is stable [23]. The following theorem presents a solution to the stability analysis problem of the singular system (1) with the nonlinear perturbations (3).

Theorem 1: For given scalars $a>0$, $b>0$, $d_m>0$ and $\tau>0$, the neutral singular system (1) is robustly stable if the operator $\Im$ is stable and there exist scalars $\varepsilon_1>0$, $\varepsilon_2>0$, real matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $W_1>0$, $W_2>0$ and matrix S of appropriate dimensions such that

$$
\left[\begin{array}{ccccccccc}
(1,1) & (1,2) & E^TW_2E & (1,4) & (1,5) & (1,6) & A^TQ_3 & d_mA^TW_1 & \tau A^TW_2 \\
* & (2,2) & 0 & 0 & 0 & 0 & A_d^TQ_3 & d_mA_d^TW_1 & \tau A_d^TW_2 \\
* & * & (3,3) & 0 & (0 & 0 & 0 & 0 & 0 \\
* & * & * & -E^TQ_3E & 0 & 0 & C^TQ_3 & d_mC^TW_1 & \tau C^TW_2 \\
* & * & * & * & -\varepsilon_1 I & 0 & Q_3 & d_mW_1 & \tau W_2 \\
* & * & * & * & * & -\varepsilon_2 I & Q_3 & d_mW_1 & \tau W_2 \\
* & * & * & * & * & * & -Q_3 & 0 & 0 \\
* & * & * & * & * & * & * & -W_1 & 0 \\
* & * & * & * & * & * & * & * & -W_2
\end{array}\right] < 0 \tag{9}
$$

where

$$
\begin{aligned}
(1,1) =& A^T(PE+RS^T)+(E^TP+SR^T)A+Q_1+Q_2 \\
& -E^TW_1E-E^TW_2E+\varepsilon_1 a^2 I \\
(1,2) =& (E^TP+SR^T)A_d+E^TW_1E \\
(1,4) =& (E^TP+SR^T)C, \\
(1,5) =& (1,6)=E^TP+SR^T, \\
(2,2) =& -(1-\mu)Q_1-E^TW_1E+\varepsilon_2 b^2 I \\
(3,3) =& -Q_2-E^TW_2E
\end{aligned}
$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR=0$.

Proof: Choose a Lyapunov-Krasovskii function candidate as

$$
\begin{aligned}
V(t,x_t) =& x^T(t)E^TPEx(t)+\int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds \\
&+\int_{t-\tau}^{t} x^T(s)Q_2x(s)ds \\
&+\int_{t-\tau}^{t} \dot{x}^T(s)E^TQ_3E\dot{x}(s)ds \\
&+\int_{t-d_m}^{t} (d_m-t+s)\dot{x}^T(s)(d_mE^TW_1E)\dot{x}(s)ds \\
&+\int_{t-\tau}^{t} (\tau-t+s)\dot{x}^T(s)(\tau E^TW_2E)\dot{x}(s)ds,
\end{aligned} \tag{10}
$$

where $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $W_1>0$ and $W_2>0$. Theorem 1 can be proved by following the similar procedure as the proof of Theorem 1 in [26]. ∎

If $E=I$, the neutral singular system (1) transforms into the following neutral system,

$$
\begin{cases}
\dot{x}(t)-C\dot{x}(t-\tau) = Ax(t)+A_dx(t-d(t))+f(x(t),t) \\
\qquad\qquad +g(x(t-d(t)),t), \\
x(\theta) = \phi(\theta),\ \forall\theta\in[-max\{d_m,\tau\},\ 0],
\end{cases} \tag{11}
$$

It follows from $E^TR=0$ that $R=0$. Therefore, it is easy to obtain the following results for neutral system (11).

Corollary 1: When $E=I$, for given scalars scalars $a>0$, $b>0$, $d_m>0$ and $\tau>0$, the system (11) with the nonlinear perturbations satisfying (3) is robustly stable if $\|C\|<1$ and there exist scalars $\varepsilon_1>0$, $\varepsilon_2>0$, real matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $W_1>0$ and $W_2>0$ of appropriate dimensions such that

$$
\left[\begin{array}{ccccccccc}
(1,1) & (1,2) & W_2 & PC & P & P & A^TQ_3 & d_mA^TW_1 & \tau A^TW_2 \\
* & (2,2) & 0 & 0 & 0 & 0 & A_d^TQ_3 & d_mA_d^TW_1 & \tau A_d^TW_2 \\
* & * & (3,3) & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_3 & 0 & 0 & C^TQ_3 & d_mC^TW_1 & \tau C^TW_2 \\
* & * & * & * & -\varepsilon_1 I & 0 & Q_3 & d_mW_1 & \tau D^TW_2 \\
* & * & * & * & 0 & -\varepsilon_2 I & Q_3 & d_mW_1 & \tau D^TW_2 \\
* & * & * & * & * & * & -Q_3 & 0 & 0 \\
* & * & * & * & * & * & * & -W_1 & 0 \\
* & * & * & * & * & * & * & * & -W_2
\end{array}\right] < 0 \tag{12}
$$

where

$$
\begin{aligned}
(1,1) &= A^TP+PA+Q_1+Q_2-W_1-W_2+\varepsilon_1 a^2 I \\
(1,2) &= PA_d+W_1 \\
(2,2) &= -(1-\mu)Q_1-W_1+\varepsilon_2 b^2 I \\
(3,3) &= -Q_2-W_2.
\end{aligned}
$$

Remark 1: Theorem 1 and Corollary 1 provide a both **discrete-delay-dependent** and **neutral-delay-dependent** absolute stability for the system (1) and system (11), it is less conservative than some existing results of the absolute stability for the system with mixed discrete and neutral delays. If we set $W_1=0$ and $W_2=0$, the Lyapunov-Krasovskii function (10) reduces to $\tilde{V}(t,x_t)=x^T(t)E^TPEx(t)+\int_{t-d(t)}^{t}x^T(s)Q_1x(s)ds+\int_{t-\tau}^{t}x^T(s)Q_2x(s)ds+\int_{t-\tau}^{t}\dot{x}^T(s)E^TQ_3E\dot{x}(s)ds$. Similar to the proof of Theorem 1, using $\tilde{V}(t,x_t)$ we can obtain a **delay-independent** absolute stability condition for system (1). We will show the obtained **delay-independent** absolute stability condition for this case in Corollary 2.

Corollary 2: For given scalars $a>0$, $b>0$, $d_m>0$ and $\tau>0$, the singular system (1) with the nonlinear perturbations satisfying (3) is robustly stable if there exist scalars $\varepsilon_1>0$, $\varepsilon_2>0$, real matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$ and matrix S of appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & 0 & (1,4) & (1,5) & (1,6) & (1,7) \\ * & (2,2) & 0 & 0 & 0 & 0 & (2,7) \\ * & * & (3,3) & 0 & 0 & 0 & 0 \\ * & * & * & (4,4) & 0 & 0 & (3,7) \\ * & * & * & * & (5,5) & 0 & Q_3 \\ * & * & * & * & 0 & (6,6) & Q_3 \\ * & * & * & * & * & * & -Q_3 \end{bmatrix} < 0 \tag{13}$$

where

$$\begin{aligned} (1,1) &= A^T(PE + RS^T) + (E^T P + SR^T)A + Q_1 + Q_2 \\ &\quad + \varepsilon_1 a^2 I, (1,2) = (E^T P + SR^T)A_d \\ (1,4) &= (E^T P + SR^T)C, (1,5) = (1,6) = E^T P + SR^T, \\ (1,7) &= A^T Q_3, (2,2) = -(1-\mu)Q_1 + \varepsilon_2 b^2 I, \\ (2,7) &= A_d^T Q_3, (3,3) = -Q_2, (3,7) = C^T Q_3 \\ (4,4) &= -E^T Q_3 E, (5,5) = -\varepsilon_1 I, (6,6) = -\varepsilon_2 I \end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

Remark 2: Similar to Remark 1, if only set $W_2 = 0$, we can also obtain a **discrete-delay-dependent** and **neutral-delay-independent** absolute stability criterion, which is extensively studied by various different methods for neutral systems with mixed delays.

Furthermore, if $d(t) = \tau = d$ is constant, then system (1) becomes

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-d) + C\dot{x}(t-d) \\ \qquad\qquad + f(x(t),t) + g(x(t-d(t)),t), \\ x(\theta) = \phi(\theta), \ \forall\theta \in [-d, \ 0], \end{cases} \tag{14}$$

The corresponding Lyapunov-Krasovskii functional candidate is

$$\begin{aligned} \hat{V}(t,x_t) =& x^T(t)E^T PEx(t) + \int_{t-d}^{t} x^T(s)Q_1 x(s)ds \\ &+ \int_{t-d}^{t} \dot{x}^T(s)E^T Q_3 E\dot{x}(s)ds \\ &+ \int_{t-d}^{t} (d-t+s)\dot{x}^T(s)(dE^T W_1 E)\dot{x}(s)ds \end{aligned} \tag{15}$$

Then we have the following result.

Corollary 3: When $d(t) = \tau = d$ is constant, for given scalars $a > 0$, $b > 0$ and $d > 0$, the singular system (14) with the nonlinear perturbations satisfying (3) is robustly stable if the operator $\hat{\Im}$ is stable and there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, real matrices $P > 0$, $Q_1 > 0$, $Q_3 > 0$, $W_1 > 0$ and matrix S of appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & A^T Q_3 & dA^T W_1 \\ * & (2,2) & 0 & 0 & 0 & A_d^T Q_3 & dA_d^T W_1 \\ * & * & -E^T Q_3 E & 0 & 0 & C^T Q_3 & dC^T W_1 \\ * & * & * & -\varepsilon_1 I & 0 & Q_3 & dW_1 \\ * & * & * & * & -\varepsilon_2 I & Q_3 & dW_1 \\ * & * & * & * & * & -Q_3 & 0 \\ * & * & * & * & * & * & -W_1 \end{bmatrix} < 0 \tag{16}$$

where

$$\begin{aligned} (1,1) &= A^T(PE + RS^T) + (E^T P + SR^T)A + Q_1 \\ &\quad - E^T W_1 E + \varepsilon_1 a^2 I \\ (1,2) &= (E^T P + SR^T)A_d + E^T W_1 E \\ (1,3) &= (E^T P + SR^T)C \\ (1,4) &= (1,5) = E^T P + SR^T \\ (2,2) &= -Q_1 - E^T W_1 E + \varepsilon_2 b^2 I \end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

If we do not consider the nonlinear perturbations, systems (1) reduces to the following nominal neutral singular system

$$E\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_d x(t-d(t)) \tag{17}$$

Then similar to the proof of Theorem 1, we have the following result.

Corollary 4: When the system (1) has no nonlinear perturbations, for given scalars $d_m > 0$ and $\tau > 0$, the nominal neutral singular system (17) is robustly stable if the operator $\Im$ is stable and there exist scalars real matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $W_1 > 0$, $W_2 > 0$ and matrix S of appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & E^T W_2 E & (1,4) & A^T Q_3 & d_m A^T W_1 & \tau A^T W_2 \\ * & (2,2) & 0 & 0 & A_d^T Q_3 & d_m A_d^T W_1 & \tau A_d^T W_2 \\ * & * & (3,3) & 0 & 0 & 0 & 0 \\ * & * & * & -E^T Q_3 E & C^T Q_3 & d_m C^T W_1 & \tau C^T W_2 \\ * & * & * & * & -Q_3 & 0 & 0 \\ * & * & * & * & * & -W_1 & 0 \\ * & * & * & * & * & * & -W_2 \end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned}(1,1) =& A^T(PE+RS^T)+(E^TP+SR^T)A+Q_1+Q_2\\ &-E^TW_1E-E^TW_2E\\ (1,2) =&(E^TP+SR^T)A_d+E^TW_1E\\ (1,4) =&(E^TP+SR^T)C\\ (2,2) =&-(1-\mu)Q_1-E^TW_1E\\ (3,3) =&-Q_2-E^TW_2E\end{aligned}$$

and $R\in\mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR=0$.

If $C=0$, the neutral singular system reduces to the following retarded-type singular system:

$$\begin{cases} E\dot{x}(t) =Ax(t)+A_dx(t-d(t))+f(x(t),t)\\ \qquad\qquad +g(x(t-d(t)),t),\\ x(\theta) =\phi(\theta),\ \forall\theta\in[d_m,\ 0].\end{cases} \tag{19}$$

Take the Lyapunov-Krasovskii functional as

$$\begin{aligned}V(t,x_t) =& x^T(t)E^TPEx(t)+\int_{t-d(t)}^{t}x^T(s)Q_1x(s)ds\\ &+\int_{t-d_m}^{t}(d_m-t+s)\dot{x}^T(s)(d_mE^TW_1E)\dot{x}(s)ds\end{aligned} \tag{20}$$

then similar to the proof of Theorem 1, we have the following result.

Corollary 5: For given scalars $a>0$, $b>0$ and $d_m>0$, the singular system (19) is robustly stable if there exist scalars $\varepsilon_1>0$, $\varepsilon_2>0$, real matrices $P>0$, $Q_1>0$, $W_1>0$ and matrix S of appropriate dimensions such that

$$\begin{bmatrix}(1,1) & (1,2) & (1,3) & (1,4) & d_mA^TW_1\\ * & (2,2) & 0 & 0 & d_mA_d^TW_1\\ * & * & -\varepsilon_1 I & 0 & d_mW_1\\ * & * & * & -\varepsilon_2 I & d_mW_1\\ * & * & * & * & -W_1\end{bmatrix}<0 \tag{21}$$

where

$$\begin{aligned}(1,1) =& A^T(PE+RS^T)+(E^TP+SR^T)A+Q_1\\ &-E^TW_1E+\varepsilon_1a^2I\\ (1,2) =&(E^TP+SR^T)A_d+E^TW_1E\\ (1,3) =&(1,4)=E^TP+SR^T\\ (2,2) =&-(1-\mu)Q_1-E^TW_1E+\varepsilon_2b^2I\end{aligned}$$

and $R\in\mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR=0$.

Remark 3: The norm-bounded uncertainties can be treated as a special case of nonlinear perturbations. Therefore, the stability criterion for system (1) with norm-bounded uncertainties can be obtained by following a similar line as in Theorem 1.

IV. NUMERICAL EXAMPLE

The following numerical examples are presented to illustrate the usefulness of the proposed theoretical results.

Example 1: Consider the nonlinear neutral singular system described by (1) with

$$E=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},A=\begin{bmatrix}-0.5 & 0.1\\ 0.2 & -1\end{bmatrix},A_d=\begin{bmatrix}-1.1 & 1\\ 0 & 0.5\end{bmatrix}$$

$$C=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ a=0.05,\ b=0.1.$$

In this example, we choose $R=\begin{bmatrix}0 & 1\end{bmatrix}^T$. According to Theorem 1, by solving the feasibility problem of LMI (9), Table I shows that this system is robustly stable for maximum allowed time-delay bound d_m, when $\tau=0.2$.

Example 2 When $E=I$ and $d(t)=\tau=d$ is constant, consider the nominal neutral system (14) with

$$A=\begin{bmatrix}-0.9 & 0\\ 0.1 & -0.9\end{bmatrix},\ A_d=\begin{bmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{bmatrix},$$

$$C=\begin{bmatrix}-0.2 & 0\\ 0.2 & -0.1\end{bmatrix}$$

For this system, [5], [10], [27], [28] gave a **discrete-delay-dependent** and **neutral-delay-independent** robust stability criterion. From the above results in this paper, we can obtain a both **discrete-delay-dependent** and **neutral-delay-dependent** robust stability criterion. For comparison, Table II gives the maximum allowed delay d_m for various methods when $\tau=0.1$. This example demonstrates that the delay-dependent robust stability criterion in this paper gives a less conservative result than those in [4], [5], [27], [28].

Example 3 When $E=I$ and $C=0$, consider the nonlinear retarded-type system (19) with

$$A=\begin{bmatrix}-1.2 & 0.1\\ -0.1 & -1\end{bmatrix},\ A_d=\begin{bmatrix}-0.6 & 0.7\\ -1 & -0.8\end{bmatrix}$$

By applying Corollary 1 and Corollary 5 to the above system, we can obtain a discrete-delay-dependent robust stability criterion. For comparison, Table III lists the maximum allowed delay d_m for various methods.

It can be conclude that, for this time-delay system, if the delay is time-invariant (i.e. $\mu=0$), then the method in this paper can obtain the same maximum allowable time-delay d_m as that in [13], [14]. When $\mu<1$, the method in this paper and the result of [13] have the same conservatism, and both of them are much less conservative than the other methods. Moreover, the method in this paper is very simple due to no free matrices being involved. Furthermore, the method is also effective to the case of $\mu\geq 1$, while the other methods failed to obtain any results.

V. CONCLUSION

The problem of delay-dependent robust stability analysis for neutral singular systems with time-varying delays and nonlinear perturbations has been addressed. Some both discrete-delay-dependent and neutral-delay-dependent stability criteria have been proposed. By introducing integral

TABLE I: Maximum allowed time-delay d_m of Example 1

μ	0	0.2	0.5	≥ 1
d_m	1.5560	1.4292	1.3787	1.1918

TABLE II: Maximum allowed time-delay d_m of Example 2

	Park [4]	Liu [27]	Zhao [28]	Kwon [5]	This paper
d_m	1.3718	1.7844	1.7856	1.8266	1.9021

TABLE III: Maximum allowed time-delay d_m of Example 3

	$a = 0, \ b = 0.1$			$a = 0.1, \ b = 0.1$		
	$\mu = 0$	$\mu = 0.5$	$\mu \geq 1$	$\mu = 0$	$\mu = 0.5$	$\mu \geq 1$
d_m by Cao [11]	0.6811	0.5467	–	0.6129	0.4950	–
d_m by Han [12]	1.3279	0.6743	–	1.2503	0.5716	–
d_m by Han [14]	2.7424	1.1365	–	1.8753	0.9952	–
d_m by Zou [13]	2.7422	1.1424	–	1.8753	1.0097	–
d_m by this paper	2.7422	1.1424	0.7355	1.8753	1.0097	0.7147

inequalities, which avoid using both model transformation and bounding technique for cross terms, some less conservative stability criteria were obtained. Numerical examples have shown the effectiveness and improvements over some existing results.

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