

Fast Time-Frequency Domain Reflectometry Based on the AR Coefficient Estimation of a Chirp Signal

Seung Ho Doo, Won-Sang Ra, Tae Sung Yoon and Jin Bae Park

Abstract—In this paper, a novel reflectometry, which is characterized by a simple autoregressive(AR) modeling of a chirp signal and a weighted robust least squares(WRLS) AR coefficient estimator, is proposed. In spite of its superior fault detection performance over the conventional reflectometries, the recently developed time-frequency domain reflectometry(TFDR) might not be suitable for real-time implementation because it requires heavy computational burden. In order to solve this critical limitation, in our method, the time-frequency analysis is performed based on the estimated time-varying AR coefficient of a chirp signal. To do this, a new chirp signal model which contains a single time-varying AR coefficient is suggested. In addition, to ensure the noise insensitivity, the WRLS estimator is used to estimate the time-varying AR coefficient. As a result, the proposed reflectometry method can drastically reduce the computational complexity and provide the satisfactory fault detection performance even in noisy environments. To evaluate the fault detection performance of the proposed method, simulations and experiments are carried out. The results demonstrate that the proposed algorithm could be an excellent choice for the real-time reflectometry.

I. INTRODUCTION

Over the past few decades, an electrical communication wire is widely used in many fields including the Internet communication, aircraft, and etc. The detection and localization of faults with high accuracy have been required for diagnosis and maintenance of the wire, since in 1990's, the faults on electrical wires has been known for the main cause of a number of aircraft crashes[1], [2]. A chapter of these accidents has strongly motivated many researchers to develop the smart wiring technique, the so-called reflectometry[3]–[5].

The reflectometry is the fault detection methodology from the reflected signal which is produced at the impedance mismatching point on a wire. A general configuration of the reflectometry system is shown in Fig. 1. Using velocity of propagation(VOP) on a wire and the time delay between the transmitted reference signal and the reflected signal, the fault distance is calculated. The existing techniques can be categorized as time domain reflectometry(TDR), frequency domain reflectometry(FDR), and TFDR. Each methodology

is denominated by the domain for analyzing a signal. The TDR measures the reflected signal along the wire caused by the traveling of a step pulse with a fast rising time. It detects the fault by calculating the traveling time and magnitude of all reflected signals returning back from the wire[6]. The FDR sends a set of stepped-frequency sine waves down the wire. These waves from the source travel to the end of the wire and are reflected back to the source. Although the TDR and FDR have been applied for a few cases, their resolution and accuracy of the fault detection could be limited by the rising/falling time and frequency sweep bandwidth, respectively[2]. This is the reason why the TFDR has been devised in recent. In order to gain the enhanced fault detection performance, the TFDR analyzes the time-frequency domain cross correlation between the reference signal and the reflected signal[2]. Despite of its accurate and reliable fault detection performance, the heavy computational burden required by the TFDR might be a fatal deficiency for on-line fault detection applications.

For reducing the computational complexity of the TFDR methodology, the parameter estimates of the reflected chirp signal can be used. The estimation of the chirp signal parameters has been of interest for a long time. Most of the methods that have been suggested in the literature yield maximum likelihood estimates. A requisite condition for these methods is a high signal-to-noise ratio(SNR). P. M. Djurić and S. M. Kay employed the phase unwrapping method[7]. This method can provide the good chirp parameter estimation performance in low SNR signal. On the contrary, its performance might be degraded in high SNR. The random walk Metropolis-Hastings(MH) algorithm, one of the useful Markov Chain Monte Carlo methods, has been applied to estimate the chirp signal parameter[8], [9]. The random walk MH algorithm shows good estimation for the phase parameters. However, slow convergence problem of the algorithm could often arise[10].

In this paper, we proposed a practical TFDR algorithm which is based on the chirp parameter estimated from the noisy reflected signal. To do this, first we newly model the chirp signal using the AR relation. The newly derived AR model has just a single unknown time-varying coefficient. In order to handle the time-varying nature of the AR coefficient and effectively estimate it, the state-space model with a state variable is derived. Since the resultant state-space model contains the stochastic parameter uncertainty in its measurement matrix, the AR coefficient estimation problem can be cast into the WRLS filtering problem. The fault is detected by analyzing the correlation between the AR coefficient of the

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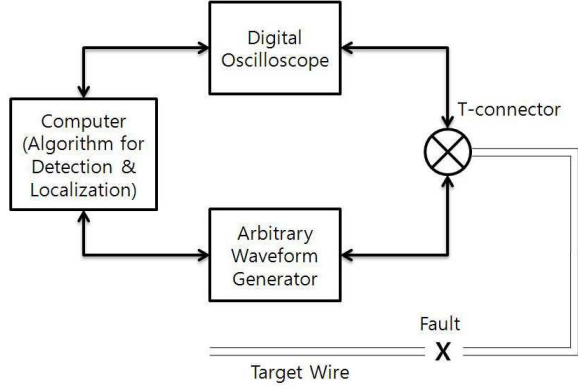


Fig. 1. A general reflectometry system

transmitted reference signal and the estimated one of the measured reflected signal. The proposed TFDR scheme is suitable for real-time implementation because it does not require the time-consuming FFT contrary to the conventional TFDR. With the help of the WRLS AR coefficient estimator, it can provide the acceptable fault detection performance even in the noisy environment. To demonstrate the validity of the proposed method, various simulations and experiments are carried out. From the results, it is shown that the proposed method is computationally efficient and can provide the almost same fault localization performance with the conventional TFDR.

II. CHIRP SIGNAL MODELING FOR REFLECTOMETRY

In this section, a simple 2nd order AR model for the chirp signal is introduced. The proposed 2nd order model reduces the computational burden largely in reflectometry. One of the AR coefficient of the suggested model is constant and the other is time-varying.

A. AR Modeling of a Chirp Signal

The general chirp signal is described as

$$\begin{aligned}
 s_k &\triangleq M e^{j(\frac{1}{2}\beta(T_s k)^2 + \omega_0(T_s k) - \frac{\pi}{2})} \\
 &= M \left\{ \cos \left(\frac{1}{2}\beta(T_s k)^2 + \omega_0(T_s k) - \frac{\pi}{2} \right) \right. \\
 &\quad \left. + j \sin \left(\frac{1}{2}\beta(T_s k)^2 + \omega_0(T_s k) - \frac{\pi}{2} \right) \right\}, \quad (1)
 \end{aligned}$$

where M is the magnitude of the chirp signal, β is the frequency sweep rate, T_s is the sampling period and ω_0 is the initial frequency. For notational convenience, the following definitions will be used.

$$\begin{aligned}
 A_k &\triangleq \frac{1}{2}\beta(T_s(k-1))^2 + \omega_0 T_s(k-1) + \frac{1}{2}\beta T_s^2 - \frac{\pi}{2}, \quad (2) \\
 B_k &\triangleq -\beta(T_s^2 k) + \beta T_s^2 - \omega_0 T_s, \\
 C_k &\triangleq \frac{1}{2}\beta(T_s(k-1))^2 + \omega_0 T_s(k-1) - \frac{\pi}{2} \\
 &= A_k - \frac{1}{2}\beta T_s^2,
 \end{aligned}$$

$$\begin{aligned}
 D &\triangleq \cos \left(\frac{1}{2}\beta T_s^2 \right), \\
 E &\triangleq \sin \left(\frac{1}{2}\beta T_s^2 \right).
 \end{aligned}$$

Using the above definitions, we can derive the equations as follows:

$$s_k = M \{ \cos(A_k - B_k) + j \sin(A_k - B_k) \} \quad (3)$$

$$s_{k-1} = M \{ \cos(C_k) + j \sin(C_k) \} \quad (4)$$

$$s_{k-2} = M \{ \cos(A_k + B_k) + j \sin(A_k + B_k) \}. \quad (5)$$

From (3)~(5), one gets

$$s_k + s_{k-2} = 2M \cos(B_k) \{ \cos(A_k) + j \sin(A_k) \} \quad (6)$$

Using the definition in (2) and the trigonometric identities, (6) can be rewritten as (8).

$$\begin{aligned}
 s_k + s_{k-2} &= 2M \cos(B_k) \{ \cos(C_k + \frac{1}{2}\beta T_s^2) \\
 &\quad + j \sin(C_k + \frac{1}{2}\beta T_s^2) \} \\
 &= 2M \cos(B_k) \{ D(\cos(C_k) + j \sin(C_k)) \\
 &\quad + j E(\cos(C_k) + j \sin(C_k)) \}. \quad (8)
 \end{aligned}$$

Substituting (4) into (8) yields

$$s_k = 2 \cos(B_k) \{ D + jE \} s_{k-1} - s_{k-2}. \quad (9)$$

In real situation, since the imaginary part of the chirp signal cannot be acquired from the oscilloscope, it is necessary to model the chirp signal only with its real part. To take this situation into signal modeling, first we redefine the chirp signal (1) as

$$s_k \triangleq a_k + j b_k, \quad (10)$$

where

$$\begin{aligned}
 a_k &\triangleq M \cos \left(\frac{1}{2}\beta(T_s k)^2 + \omega_0(T_s k) - \frac{\pi}{2} \right), \\
 b_k &\triangleq M \sin \left(\frac{1}{2}\beta(T_s k)^2 + \omega_0(T_s k) - \frac{\pi}{2} \right).
 \end{aligned}$$

Assumption 1: In a reflectometry system, the high frequency digital oscilloscope is used usually, hence it can be assumed that $\beta T_s^2 \approx 0$ without loss of generality. That is,

$$D = \cos \left(\frac{1}{2}\beta T_s^2 \right) \approx 1, \quad E = \sin \left(\frac{1}{2}\beta T_s^2 \right) \approx 0.$$

Using the Assumption 1, we can get (11).

$$\begin{aligned}
 \begin{bmatrix} a_k \\ b_k \end{bmatrix} &= 2 \cos B_k \begin{bmatrix} D & -E \\ E & D \end{bmatrix} \begin{bmatrix} a_{k-1} \\ b_{k-1} \end{bmatrix} - \begin{bmatrix} a_{k-2} \\ b_{k-2} \end{bmatrix} \\
 &\approx 2 \cos B_k \begin{bmatrix} a_{k-1} \\ b_{k-1} \end{bmatrix} - \begin{bmatrix} a_{k-2} \\ b_{k-2} \end{bmatrix} \quad (11)
 \end{aligned}$$

Proposition 1: (Approximate 2nd order AR model of the chirp signal) Under the Assumption 1, the 2nd order AR model of the chirp signal can be written as follows:

$$s_k \approx 2 \cos(B_k) s_{k-1} - s_{k-2}.$$

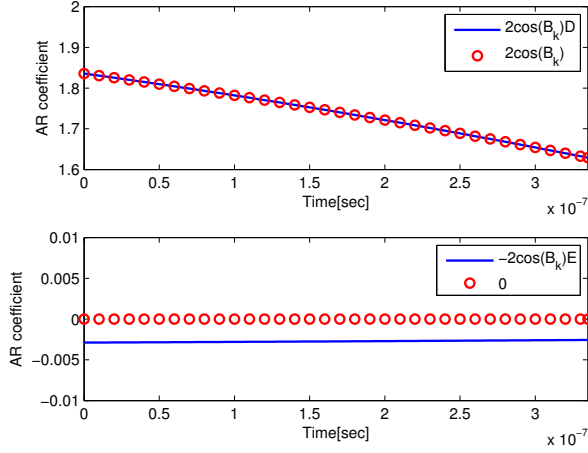


Fig. 2. AR coefficient comparison between the true chirp signal and the proposed model

B. Verification of the Proposed Model

For reflectometry, the chirp signal which has linearly increasing frequency is generated. The frequency range of the chirp signal is 13 ~ 19.7MHz, the magnitude is $6V_{p-p}$, and the time duration is 340nsec. In (11), it is assumed that $2\cos(B_k)D$ and $-2\cos(B_k)E$ can be approximated to $2\cos(B_k)$ and 0, respectively. To check the validity of these approximations, in Fig. 2 each term of the real parts of the proposed model and the true signal AR coefficient are plotted. From the results, we can see that the approximate AR model in Proposition 1 is proper to represent the chirp signal.

III. DETECTION AND LOCALIZATION OF A FAULT

Since the noise free measurement can not be obtained in real system for detection and localization of a fault in a coaxial cable, precise AR coefficient estimator should be designed. In this paper, the WRLS estimator which is recently developed is applied. The WRLS estimator successfully eliminates the scale factor error and the bias error of nominal weighted least squares(WLS) estimator which are caused by stochastic uncertainties of the system[11]. For fault localization, the cross-correlation of true AR coefficient of transmitted signal and the estimated AR coefficient of received signals are considered. And then, based on the cross-correlation results, a time delay between these signals is calculated.

A. State-Space Model for a Time-Varying AR Coefficient Estimation of a Chirp Signal

Since the time duration of the reference signal is 340nsec, the AR coefficient of the reference signal is also has same time duration. The 1st order AR coefficient $2\cos(B_k)$ is defined as a state variable x_k as shown in Fig. 3. The state variable x_k of the reference signal is decreased from 1.8355 to 1.6290 during 340nsec. Therefore, we can decide the

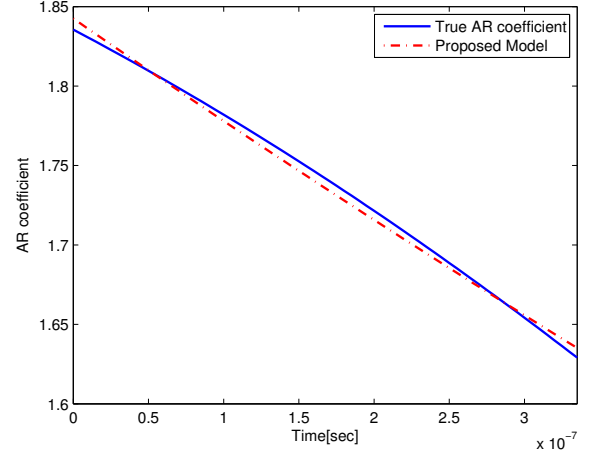


Fig. 3. Proposed transition matrix F_k

F_k as 0.99822 by dividing the variation of the 1st order AR coefficient by the time duration. As a result, the state transition equation becomes

$$x_{k+1} = F_k x_k + u_k, \quad (12)$$

where the model error u_k is assumed that it is the zero mean white noise.

$$x_k \triangleq 2\cos(B_k), \quad F_k = 0.99822.$$

From the Proposition 1, we get

$$a_k + a_{k-2} \approx 2\cos(B_k)a_{k-1}. \quad (13)$$

Since the acquired signal \tilde{a}_k is corrupted by the measurement noise \bar{v}_k in general, we can define that

$$\tilde{a}_k = a_k + \bar{v}_k, \quad (14)$$

where \bar{v}_k is assumed as the zero-mean white noise with a known covariance \bar{R}_k . Therefore, using (13), (14) can be rewritten as

$$\tilde{a}_k + \tilde{a}_{k-2} = (\tilde{a}_{k-1} - \bar{v}_{k-1})(2\cos(B_k)) + (\bar{v}_k + \bar{v}_{k-2}). \quad (15)$$

Therefore, we can set the measurement equations as follows:

$$y_k = [\tilde{H}_k - \Delta H_k]x_k + v_k, \quad (16)$$

where

$$y_k \triangleq \tilde{a}_k + \tilde{a}_{k-2}, \quad v_k \triangleq \bar{v}_k + \bar{v}_{k-2}, \\ \tilde{H}_k \triangleq \tilde{a}_{k-1}, \quad \Delta H_k \triangleq \bar{v}_{k-1},$$

and, since $cov(\bar{v}_k) = \bar{R}_k$ we can get

$$E[\Delta H_k^T \Delta H_k] = \bar{R}_k.$$

Gathering (12) and (16), we can get a state-space equation as follows:

$$\begin{cases} x_{k+1} = F_k x_k + u_k \\ y_k = [\tilde{H}_k - \Delta H_k]x_k + v_k \end{cases} \quad (17)$$

TABLE I
WRLS ESTIMATOR[12]

<ul style="list-style-type: none"> State-space system model $\begin{cases} x_{k+1} = F_k x_k + u_k \\ y_k = [H_k - \Delta H_k] x_k + v_k \end{cases}$ Known statistical information $E[\Delta H_k^T \Delta H_k] \triangleq W_k, E[\Delta H_k^T v_k] \triangleq 0.$ $E[\Delta H_k^T u_k] = 0, E[u_k^T v_k] = 0,$ $E[\Delta H_k] = 0, E[u_k] = 0, E[v_k] = 0$ WRLS estimator $\mathcal{P}_{k k}^{-1} = \lambda \mathcal{P}_{k k-1}^{-1} + \tilde{H}_k^T \tilde{H}_k - W_k,$ $\hat{x}_{k k} = (I + \mathcal{P}_{k k} W_k) \hat{x}_{k k-1} + \mathcal{P}_{k k} \tilde{H}_k^T (y_k - \tilde{H}_k \hat{x}_{k k-1}),$ $\mathcal{P}_{k+1 k} = F_k \mathcal{P}_{k k} F_k^T,$ $\hat{x}_{k+1 k} = F_k \hat{x}_{k k}$

Therefore, the AR coefficient estimation problem can be interpreted as a special case of the standard WRLS estimation problem[12]. By applying the WRLS estimation equation summarized in Table I for the state-space model (17), we can readily design the time-varying AR coefficient estimator of the chirp signal.

Assumption 2: For designing the WRLS AR coefficient estimator, we assume that ΔH_k is stationary, and ΔH_k and v_k are mutually uncorrelated as follows:

$$E[\Delta H_k^T \Delta H_k] \triangleq W_k, E[\Delta H_k v_k] \triangleq 0.$$

The forgetting factor of the WRLS estimator is defined as 0.948, since the state variable x_k is nonstationary.

B. AR Coefficient Cross-Correlation Function

The time delay between the reference signal and the reflected signal is directly related to the fault distance of a coaxial cable. In general, the magnitude of the reflected signal is attenuated as the fault distance is increased. In the proposed chirp signal model, the AR coefficient is not related to the magnitude of the signal. Therefore, very similar AR coefficients can be obtained from the reference signal and the reflected signal even though the magnitudes of those signals are not similar. The cross-correlation function which is adopted in this paper is shown in (18).

$$C_{RS}[k] = \frac{1}{E_s} \sum_{n=0}^N R[n] S[n+k], \quad (18)$$

$$E_s = \sum_{n=0}^N s[n]^2,$$

where $R[n]$ is the reference signal, N is number of samples of the reference signal and $S[n]$ is the reflected signal.

IV. RESULTS

For evaluating the estimation performance of the proposed method, the computer simulations are executed using the

TABLE II
SIMULATION CONDITIONS

Computer Specification	CPU: AMD Athlon 64 X2 Dual 3800+
	RAM: 3GB
	OS: MS Windows XP Pro. Simulation Language: Matlab R2006a
Common Factor	VOP: $2.502 \times 10m/s$
	Attenuation Rate: 0.45%/m
WRLS	F_k : 0.99822
	W_k : $0 \sim 0.115$
	V_k : 0
	λ : 0.948
	x_0 : 1.45
LMS	\mathcal{P}_0 : 0.08
	μ : 0.19 x_0 : 1.45

TABLE III
COMPUTATIONAL TIME

Methodology	Average Single Computational Time(sec)
Conventional TFDR	5.39
Proposed Method	0.0293

chirp signals with various noise levels. To verify the proposed method, several experiments are carried out with 10C-FBT coaxial cable which has a fault at 100.08m point. The simulation conditions of this section are in Table II.

A. Simulation Results

In this section, it is assumed that there is a fault on a coaxial cable at 100.08m point and five reflected signals are obtained. The simulation conditions of the WRLS AR coefficient estimator are shown in Table II. Simulation is carried out on various SNR. From the proposed model, the 1st order AR coefficient is estimated from the WRLS estimator. The AR coefficient estimation result of the 1st order AR coefficient on noise free case is in Fig. 4. Using the estimated AR coefficient, we can get a cross-correlation result. The peak point interval of the cross-correlation result means the time delay between the reference signal and the reflected signal. Therefore, clearness of the peak points in the cross-correlation result is directly relate to

TABLE IV
SIMULATION RESULTS

SNR (dB)	Proposed method error(%)	Conventional method error(%)
Noise free	0.000	0.000
75	0.006	0.002
70	0.006	0.006
65	0.006	0.008
60	0.011	0.011
55	0.014	0.015
50	0.017	0.019
45	0.029	0.022
40	0.036	0.025
35	0.055	0.028

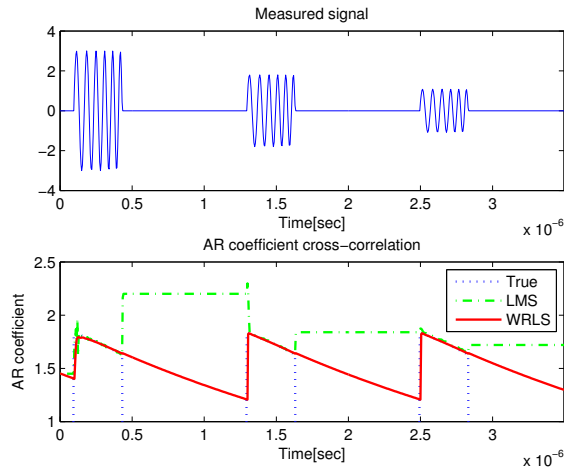


Fig. 4. Chirp signal AR coefficient estimation: noise free

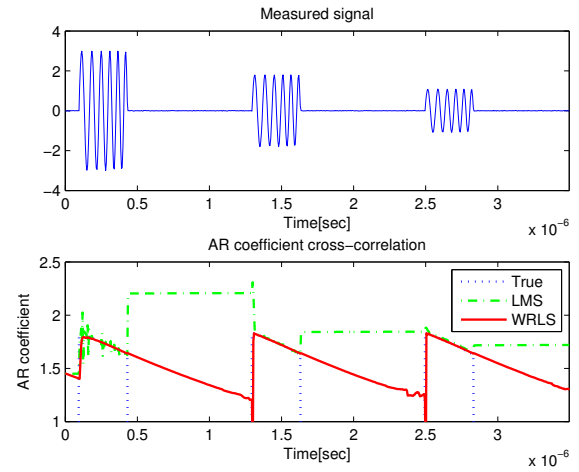


Fig. 6. Chirp signal AR coefficient estimation: SNR = 65dB

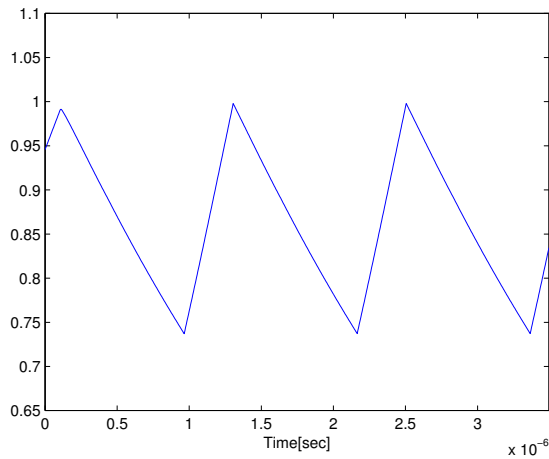


Fig. 5. Chirp signal AR coefficient cross-correlation: noise free

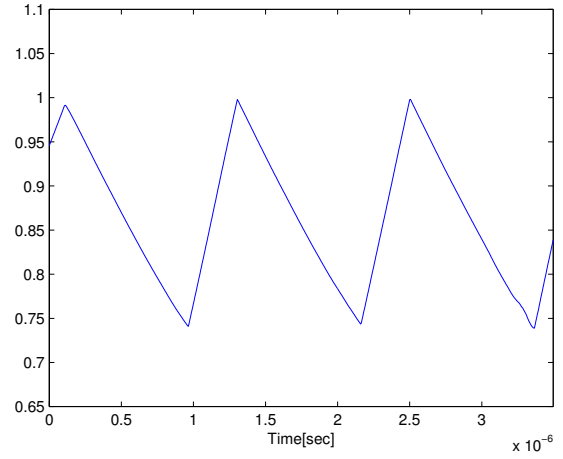


Fig. 7. Chirp signal AR coefficient cross-correlation: SNR = 65dB

the performance of the system. The cross-correlation result of the AR coefficient on noise free case is shown in Fig. 5. The AR coefficient estimation result of the noise contaminated signal ($SNR = 65dB$) is shown in Fig. 6 and the cross-correlation result of the noise contaminated signal ($SNR = 65dB$) is shown in Fig. 7. On noise free case, estimation performance of the conventional LMS estimator [13] is similar to the WRLS estimator. However, AR coefficient estimation performance of the conventional LMS estimator is degraded when the signal is noise contaminated. The computational time and fault distance estimation results using the proposed method and the conventional TFDR are shown in Table III and Table IV, respectively. All the results of the Table III and Table IV have been obtained by 100 independent Monte-Carlo runs. It is shown that the performance of the proposed method is almost similar to the conventional TFDR even though it has very short computational time.

Remark 1: The proposed method successfully estimates

the fault distance of the target wire. The error of the proposed method is less than 0.055% under the various SNR situations.

Remark 2: Under the various noise situations, the proposed method is superior to the conventional method in the system computation time. The average single computational time of the proposed method is 0.0293sec while the conventional method is 5.392sec. The system computation time is improved approximately 185 times than that of the conventional TFDR method.

B. Experimental Results

The system for experiments consists of an arbitrary waveform generator (National Instrument (NI) PXI-5422), a digital oscilloscope (NI PXI-5124), and a controller (NI PXI-8105). The target wire is 10C-FBT 100.08m coaxial cable. The 10C-FBT coaxial cable is widely used in many industrial field. The measured signal and the AR coefficient estimation result of the proposed method are

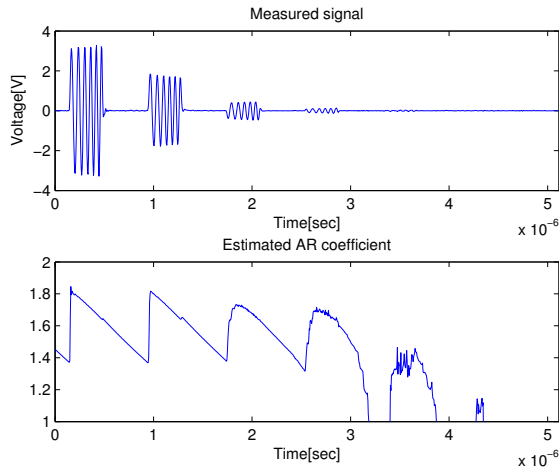


Fig. 8. AR coefficient estimation result of experiment

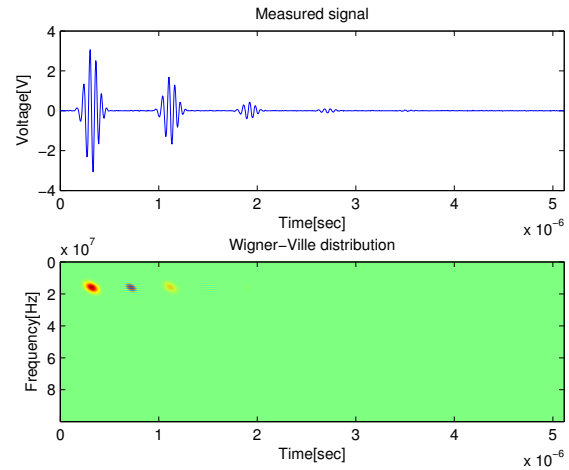


Fig. 9. Wigner-Ville time-frequency distribution of experiment

shown in Fig. 8. The measured signal and the Wigner-Ville time-frequency distribution result of the conventional TFDR method are shown in Fig. 9. The experiments are carried out 10 times. The error of the proposed method is 0.004% and the error of the conventional TFDR is 0.021% on the average. Experimental results of the proposed method and conventional TFDR show each method has excellent performance. However, the error of the proposed method is slightly low.

Remark 3: In the experiments, the proposed method shows good performance to estimate the fault distance compared with the conventional TFDR, though the computational time of the proposed one is very short.

V. CONCLUSION

In this paper, a novel reflectometry which adopts a simple AR modeling of the chirp signal and the WRLS AR coefficient estimator is proposed. The conventional TFDR is known for state-of-art technique and very accurate method. However, the computational complexity of the method restricts the real-time implementation of the wire fault detection system. Therefore, we propose a novel reflectometry which adopts simple AR model of the chirp signal and the WRLS AR coefficient estimator for robust estimation and reducing the computational complexity. From the simulation results, it is shown that the proposed simple AR model of the chirp signal is proper model for the chirp signal, and the WRLS AR coefficient estimator also has excellent performance in estimating time-varying AR coefficient of the chirp signal. The fault distance estimation performance of the proposed method is excellent. For evaluating the performance of the proposed method, simulations and experiments are carried out. From the simulation and experimental results, we can validate the performance of the proposed method.

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