2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Distributed Robust Adaptive Tracking Control with Lossy Interconnection Links and Bounded Disturbances

Xiao-Zheng Jin and Guang-Hong Yang

Abstract— **In this paper, direct adaptive-state feedback control schemes are developed to solve the problem of asymptotic tracking and disturbance rejection for a class of distributed large-scale systems with faulty and perturbed interconnection links. Adaptation laws are proposed to update controller parameters on-line when all interconnected fault factors, the upper bounds of perturbations in interconnection links and external disturbances on subsystems are unknown. Then a class of distributed state feedback controllers is constructed to automatically compensate the fault and perturbation effects, and reject the disturbances simultaneously based on the information from adaptive schemes. The proposed adaptive robust tracking controllers can guarantee that the resulting adaptive closed-loop distributed system stable and each subsystem can asymptotically-output track the corresponding reference signal. The proposed design technique is finally evaluated in the light of a simulation example.**

I. INTRODUCTION

A large class of practical control systems, such as chemical processes, vehicular platoons and Microelectromechanical system (MEMS), can be considered as large-scale systems composed with a large number of spatially interconnected units. And many approaches have been developed to synthesize some types of distributed controllers for guaranteeing the dynamic large scale system well-posed, stable, and contractive (see, e.g., $[1]-[11]$, and the references therein) in recent. Using networks, communications among subsystems play a very important role in distributed systems. Thus, many issues which always exist in communications, such as single attenuations [1], bandwidth limitations (bit rate limitations) [5], time delays $[6]-[8]$ and perturbations [9], are addressed by some researchers. LMI methods are adopted to deal with these issues for guaranteeing the well-posedness, stability, and contractiveness of the system in above works. However, to the best of the authors' knowledge, the problem of asymptotic tracking for distributed control systems with faulty and perturbed interconnection links has not yet been investigated by using adaptive method.

The asymptotic tracking problem is more challenging in the presence of unknown time-varying disturbances. Continuous robust adaptive control laws in the presence of bounded disturbances can generally ensure closed-loop signal boundedness and convergence of the tracking error to a residual bounded set with size of the order of the disturbance magnitude, but not asymptotic tracking [20], [21]. Recently, the problem of disturbance rejection has received considerable attention, and robust adaptive controllers have been developed in [15], [17], [18] and [19] to guarantee asymptotic tracking for systems. In this paper, a new method is proposed to deal with disturbance rejection problem of guaranteeing asymptotic tracking for distributed systems.

In this paper, a general fault model for signal attenuation and perturbations in interconnection channels is considered. Each signal attenuation factor and upper bound of perturbations are assumed to be unknown. We also assume that the unknown external disturbances exist on the subsystems all the time. A direct adaptive method is proposed to solve the robust tracking problem for developing some distributed state feedback controllers. For this purpose, we first propose some adaptation laws to update the controller parameters. Then, the distributed controllers are constructed by using the updated values of these estimations. Based on the Lyapunov stability theory, the adaptive closed-loop large-scale system can be guaranteed to be stable and each subsystem can asymptotically-output track the corresponding reference signal in the presence of faults and perturbations in interconnection channels, and external disturbances.

II. PRELIMINARIES AND PROBLEM STATEMENT

Notations: R stands for the set of real numbers, and for a real matrix E, $\lambda_{\max}(E)$ represents the largest eigenvalue of E. Given matrices $M_k, k=1,\ldots,n$, the notation $\mathrm{diag}_{k=1}^{n}[M_k]$ denotes the block-diagonal matrix with M_k along the diagonal and denoted $\mathrm{diag}_k[M_k]$ for brevity. For signals or vectors x_k, the notation $\mathrm{cat}_{k=1}^{n}x_k$ denotes the signal or vector $(x_1,x_2,\ldots,x_n)$ formed by concatenating x_k. This is also usually denoted $\mathrm{cat}_k x_k$ for brevity.

In this paper, we consider a large-scale system G composed of N interconnected linear time-invariant continuous time subsystems G_i, $i=1,2,\ldots,N$. Each subsystem is captured the following state-space equation:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} A^i_{TT} & A^i_{TS} & B^i_{Td} & B^i_{Tu} \\ A^i_{ST} & A^i_{SS} & B^i_{Sd} & B^i_{Su} \\ C^i_T & C^i_S & D^i_d & D^i_u \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i(t) \\ u_i(t) \end{bmatrix} \quad (1)$$

where $x_i(t)\in R^{n_i}$ is the state, $u_i(t)\in R^{m_i}$ is the control input, $y_i(t)\in R^{l_i}$ is the measured output, $d_i(t)\in R^{p_i}$ is the external

This work was supported in part by the Funds for Creative Research Groups of China (No. 60821063), the State Key Program of National Natural Science of China (Grant No. 60534010), National 973 Program of China (Grant No. 2009CB320604), the Funds of National Science of China (Grant No. 60674021), the 111 Project (B08015) and the Funds of PhD program of MOE, China (Grant No. 20060145019).

Xiao-Zheng Jin is with the College of Information Science and Engineering, Northeastern University, Shengyang, Liaoning, 110004, China jin445118@163.com

Guang-Hong Yang is with the College of Information Science and Engineering, Northeastern University, Shengyang, Liaoning, 110004, China yangguanghong@ise.neu.edu.cn

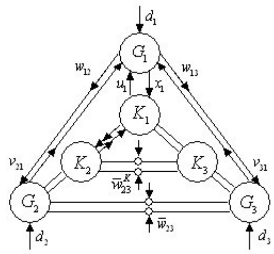

Fig. 1. Example of interconnected closed-loop system with N = 3 subsystems.

disturbance, and $v_i := \mathrm{cat}_j(v_{ij})$, $v_{ij} \in R^{q_{ji}}$ and $w_i := \mathrm{cat}_j(w_{ij})$, $w_{ij} \in R^{q_{ij}}$ $j = 1, 2, \ldots, N$ are the interconnection input to each subsystem and the interconnection output from each subsystem, respectively. All system matrices are known real constant matrices with appropriate dimensions.

In the normal case, once the relationships between the inputs and outputs at each subsystem have been defined, the distributed system can be described by closing all loops by imposing the constraints of interconnection with the interconnection condition such that

$$v_{ij}(t) = w_{ji}(t). \tag{2}$$

Here, we assume every subsystem is controllable and the states of each subsystem are available at every instant. Moreover, every state has its interconnection channel interconnected with other subsystems. We also assume the controller and plant use identical interconnection channels. Then, a state feedback controller with same interconnection structure for this system has controllers K_i given by

$$\begin{bmatrix} u_i(t) \\ w_i^K(t) \end{bmatrix} = \begin{bmatrix} K_{i11} & K_{i12} \\ K_{i21} & K_{i22} \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i^K(t) \end{bmatrix} \tag{3}$$

where $v_i^K(t) := \mathrm{cat}_j(v_{ij}^K)$, $w_i^K(t) := \mathrm{cat}_j(w_{ij}^K)$ and $v_{ij}^K(t)$, $w_{ji}(t) \in R^{q_{ji}^K}$ also have interconnection condition with

$$v_{ij}^K(t) = w_{ji}^K(t) \tag{4}$$

in the normal case. Then, the closed-loop system can be illustrated for example in Fig.1.

We make the following assumption on the subsystem matrices:

Assumption 1. The subsystems are interconnected only through their states, that means,

$$A_{SS}^i = 0, \quad B_{Su}^i = 0, \quad B_{Sd}^i = 0, \quad K_{i22} = 0. \tag{5}$$

In this paper, we formulate the faults including faulty interconnection links between communicating subsystems. Let $v_{ij}^{hF}(t)$ represent the signals from the jth communicating subsystem that have failed in the hth faulty mode. Then we denote the fault model as follows:

$$v_{ij}^{hF}(t) = \rho_{ji}^h(t) v_{ij}(t),\ i, j = 1, 2, \ldots, N,\ h = 1, 2, \ldots, L \tag{6}$$

where $\rho_{ji}^h(t)$ is unknown interconnected factor, the index h denotes the hth faulty mode and L is the total faulty modes. For every faulty mode, $\underline{\rho}_{ji}^h$ and $\bar{\rho}_{ji}^h$ represent the lower and upper bounds of $\rho_{ji}^h(t)$, respectively. Note the practical case, we have $0 \le \underline{\rho}_{ji}^h \le \rho_{ji}^h(t) \le \bar{\rho}_{ji}^h$, and when $\underline{\rho}_{ji}^h = \bar{\rho}_{ji}^h = I$, there are no faults for the jth interconnection links v_{ij}. when $\underline{\rho}_{ji}^h = \bar{\rho}_{ji}^h = 0$ jth interconnection link is complete disconnection. when $0 < \underline{\rho}_{ji}^h \le \bar{\rho}_{ji}^h < I$, that means the type of fault is loss of effectiveness.

Denote

$$v_{ij}^{hF}(t) = [v_{ij1}^{hF}(t), v_{ij2}^{hF}(t), \cdots, v_{ijq_{ji}}^{hF}(t)]^T = \rho_{ji}^h(t) v_{ij}(t) \tag{7}$$

where $\rho_{ji}^h(t) = \mathrm{diag}[\rho_{ji1}^h(t), \rho_{ji2}^h(t), \ldots, \rho_{jiq_{ji}}^h(t)]$, $\rho_{jik}^h(t) \in [\underline{\rho}_{jik}^h, \bar{\rho}_{jik}^h]$.

Then, the sets of operators with above structures are denoted by

$$\Delta_{\rho_{ji}^h} = \{\rho_{ji}^h(t) : \rho_{jik}^h(t) \in [\underline{\rho}_{jik}^h, \bar{\rho}_{jik}^h], k = 1, 2, \ldots, q_{ji}\}. \tag{8}$$

For convenience in the following sections, for all possible faulty modes L, the following uniform interconnection links fault model is exploited:

$$v_{ij}^F(t) = \rho_{ji}(t) v_{ij}(t), \quad \rho_{ji}(t) \in \{\rho_{ji}^1(t) \cdots \rho_{ji}^L(t)\}. \tag{9}$$

Here, we let $\bar{w}_{ji}(t) \in R^{q_{ji}}$ denote perturbation which combined by unknown time-varying parameter variation, noise, and nonlinearity of the transmission channel between ith and jth subsystems.

Then, based on the above description, the equation (2) can be represented by

$$v_{ij}(t) = \rho_{ji}(t) w_{ji}(t) + \bar{w}_{ji}(t) \tag{10}$$

for all $i, j = 1 \ldots N$, and the dynamics with faulty interconnection links (1) can be rewritten by

$$\begin{aligned} \dot{x}_i(t) = {} & A_{TT}^i x_i(t) + \sum_{j=1}^N A_{TS}^{ij} \rho_{ji}(t) A_{ST}^{ji} x_j(t) \\ & + \sum_{j=1}^N A_{TS}^{ij} \bar{w}_{ji}(t) + B_{Tu}^i u_i(t) + B_{Td}^i d_i(t), \end{aligned} \tag{11}$$

$$\begin{aligned} y_i(t) = {} & C_T^i x_i(t) + \sum_{j=1}^N C_S^{ij} \rho_{ji}(t) A_{ST}^{ji} x_j(t) \\ & + \sum_{j=1}^N C_S^{ij} \bar{w}_{ji}(t) + D_u^i u_i(t) + D_d^i d_i(t). \end{aligned} \tag{12}$$

Since we assume the controller and plant use identical interconnection channels, then we have

$$v_{ij}^K(t) = \rho_{ji}(t) w_{ji}^K(t) + \bar{w}_{ji}(t) \tag{13}$$

where $\rho_{ji}(t)$ and $\bar{w}_{ij}(t)$, $i, j = 1 \ldots N$ are denote as before. Then, in terms of (3) and Assumption 1, the controller form can be described as:

$$\begin{aligned} u_i(t) = {} & \hat{K}_{1i}(t) x_i(t) + \sum_{j=1}^N K_{2i} \rho_{ji}(t) \hat{K}_{3i}(t) x_j(t) \\ & + \sum_{j=1}^N K_{2i} \rho_{ji}(t) \bar{w}_{ji}(t) x_j(t) + K_{4i}(t) \end{aligned} \tag{14}$$

where $\rho_{ji}(t) \in \Delta_{\rho_{ji}^h}$; $\hat{K}_{1i}(t)$, $\hat{K}_{3i}(t)$ are the estimate of $K_{1i}(t)$ and $K_{3i}(t)$, respectively; K_{2i} is an appropriate dimensions matrix chosen by the system designer; $K_{i4}(t)$ is given by a function. All controller parameters will be designed in section 3 in detail.

Then, consider the large scale system described by (11) and (12) with interconnected faults and perturbations given by (10). The design problem under consideration is to find a direct adaptive state feedback controller (14) such that

1). During normal transmission, the closed-loop system is stable, and the output $Sy_i(t)$ tracks the reference signal $r_i(t)$ without steady-state error, that is $\lim_{t\to\infty} e_i(t) = 0$,

$$e_i(t) = r_i(t) - S_i y_i(t), \quad i = 1,2,\ldots,N \tag{15}$$

Where $S_i \in R^{s_i \times l_i}$ is a known constant matrix used to form the output required to track the reference signals.

2). In the event of faults and perturbations in interconnection channels, the closed-loop system is still stable, and the output $S_i y_i(t)$ tracks the reference signal $r_i(t)$ without steady-state error.

Combining equations (1) and (15), we have the following augmented subsystem $G_{ai}, i \in \{1,2,\ldots,N\}$

$$\dot{\xi}_i(t) = A_a^i \xi_i(t) + \sum_{j=1}^{N} A_a^{ij} \xi_j(t) + B_a^i u_i(t) + G_a^i z_i(t) + \sum_{j=1}^{N} \bar{A}_a^{ij} \bar{w}_{ji}(t) \tag{16}$$

where $\xi_i(t) = [\eta_i^T(t),\ x_i^T(t)]^T, \eta_i(t) = \int_0^t e_i(\tau)d\tau,\ z_i(t) = [r_i^T(t),\ d_i^T(t)]^T$, and

$$A_a^i = \begin{bmatrix} 0 & -S_i C_T^i \\ 0 & A_{TT}^i \end{bmatrix}, A_a^{ij} = \begin{bmatrix} 0 & -S_i \sum_{j=1}^{N} C_S^{ij} \rho_{ji} A_{ST}^{ji} \\ 0 & \sum_{j=1}^{N} A_{TS}^{ij} \rho_{ji} A_{ST}^{ji} \end{bmatrix},$$

$$B_a^i = \begin{bmatrix} -S_i D_u^i \\ B_{Tu}^i, \end{bmatrix}, \bar{A}_a^{ij} = \begin{bmatrix} -S_i \sum_{j=1}^{N} C_S^{ij} \\ \sum_{j=1}^{N} A_{TS}^{ij} \end{bmatrix}, G_a^i = \begin{bmatrix} I & -S_i D_d^i \\ 0 & B_{Td}^i \end{bmatrix}.$$

An assumption which is quite natural and common in the robust control literature introduced as follows:

Assumption 2: The perturbations, reference signals, and external disturbances are piecewise continuous bounded functions; that is there exist positive constants $\bar{\bar{w}}_{ji}$ and $\bar{z}_i$ such that

$$\|\bar{w}_{ji}(t)\| \le \bar{\bar{w}}_{ji}, \quad \|z_i(t)\| \le \bar{z}_i,$$

respectively.

Now, the main objective of this paper is to synthesize the distributed adaptive controller $u_i(t)$ given in (14) such that the state $\eta_i(t)$ in subsystem G_{ai} (16) can be guaranteed to converge to zero. Then the outputs of distributed subsystems can asymptotically track corresponding reference signals $r_i(t)$ in the presence of failures and perturbations in interconnection channels, external disturbances.

III. DISTRIBUTED ADAPTIVE TRACKING CONTROL SYSTEM DESIGN

Considering a large scale system G_{ai} described by (16) and controller model given by (14), the controller gain $\hat{K}_{1i}(t) = [\hat{K}_{1i,1}(t), \hat{K}_{1i,2}(t), \ldots, \hat{K}_{1i,m}(t)]^T \in R^{m_i \times n_i}$ updated by the following adaptive law: $i = 1,2,\ldots,N, k = 1,2,\ldots,m_i$

$$\frac{d\hat{K}_{1i,k}(t)}{dt} = -\Gamma_{1i,k} \xi_i \xi_i^T P_i b_{ak}^i \tag{17}$$

where $\Gamma_{1i,k}$ is any positive constant, $\hat{K}_{1i,k}(t_0)$ is finite, P_i is a positive symmetric matrix, and b_{ak}^i is the kth column of B_a^i; K_{2i} is an appropriate dimensions matrix chosen by the system designer; $\hat{K}_{3i}(t) = [\hat{K}_{3i,1}(t), \hat{K}_{3i,2}(t), \ldots, \hat{K}_{3i,m_i}(t)]^T \in R^{m_i \times n_i}$ updated according to the adaptive law: $i, j = 1,2,\ldots,N, k = 1,2,\ldots,m_i$

$$\frac{d\hat{K}_{3i,k}(t)}{dt} = -\Gamma_{3i,k} \xi_j \xi_i^T P_i B_a^i k_{2ik} \tag{18}$$

where Γ_{3k} is any positive constant, $\hat{K}_{3i,k}(t_0)$ is finite, k_{2ik} is the kth column of K_{2i}; $K_{4i}(t)$ is given by the following function:

$$K_{4i}(t) = \frac{-(\xi_i^T P_i B_a^i)^T \beta_i \parallel \xi_i^T P_i \parallel \hat{k}_{5i}(t)}{\parallel \xi_i^T P_i B_a^i \parallel^2 \alpha_i}, i = 1,2,\ldots,N \tag{19}$$

where α_i, β_i are suitable positive constants which satisfied:

$$\alpha_i \le \beta_i, \tag{20}$$

and $\hat{k}_5(t) \in R$ is updated by the following adaptive law:

$$\frac{d\hat{k}_{5i}(t)}{dt} = \gamma_i \parallel \xi_i^T P_i \parallel \quad i = 1,2,\ldots,N \tag{21}$$

where γ_i is any positive constant, $\hat{k}_{5i}(t_0)$ is finite, and from (21), we can see $\hat{k}_{5i}(t) \ge 0$ if $\hat{k}_{5i}(t_0) \ge 0$.

Then the large scale closed-loop system model can be written by

$$\begin{aligned}\dot{\xi}_i(t) = {} & (A_a^i + B_a^i \hat{K}_{1i})\xi_i(t) + \sum_{j=1}^{N} (A_a^{ij} + B_a^i K_{2i} \rho_{ji} \hat{K}_{3i})\xi_j(t) \\ & + B_a^i \sum_{j=1}^{N} K_{2i} \rho_{ji} \bar{w}_{ji} + B_a^i K_{4i} + \sum_{j=1}^{N} \bar{A}_a^{ij} \bar{w}_{ji} + G_a^i z_i.\end{aligned} \tag{22}$$

On the other hand, letting

$$\begin{aligned}\tilde{K}_{1i,k}(t) &= \hat{K}_{1i,k}(t) - K_{1i,k}, \\ \tilde{K}_{3i,k}(t) &= \hat{K}_{3i,k}(t) - K_{3i,k}, \\ \tilde{k}_{5i}(t) &= \hat{k}_{5i}(t) - k_{5i}\end{aligned} \tag{23}$$

where $i = 1,2,\ldots,N, k = 1,2,\ldots,m_i$.

Due to $K_{1i,k}$, $K_{3i,k}$, and k_{5i} are unknown constants, we can write the following error system

$$\begin{aligned}\frac{d\tilde{K}_{1i,k}(t)}{dt} &= -\Gamma_{1i,k} \xi_i \xi_i^T P_i b_{ak}^i, \\ \frac{d\tilde{K}_{3i,k}(t)}{dt} &= -\Gamma_{3i,k} \xi_j \xi_i^T P_i B_a^i k_{2ik}, \\ \frac{d\tilde{k}_{5i}(t)}{dt} &= \gamma_i \parallel \xi_i^T P_i \parallel\end{aligned} \tag{24}$$

where $i, j = 1,2,\ldots,N, k = 1,2,\ldots,m_i$.

Before giving our main result, the following Lemmas are introduced firstly:

Lemma 1. For appropriate dimension matrices X, Y, and any $\zeta > 0$, the following inequality holds true

$$X^T Y + Y^T X \le \zeta X^T X + \zeta^{-1} Y^T Y. \tag{25}$$

Lemma 2. ([25], [16]) Consider the following Riccati equation:

$$A^T P + PA + PRP + Q = 0. \tag{26}$$

If $R=R^T\geq 0$, $Q=Q^T>0$, A is Hurwitz, and the associated Hamiltonian matrix

$$H=\begin{bmatrix} A & R \\ -Q & -A^T \end{bmatrix}$$

has no eigenvalue on the imaginary axis, then there exists a $P=P^T>0$, which is the solution of (26).

In the following, by $(\xi,\tilde{K}_{1i},\tilde{K}_{3i},\tilde{k}_{5i})(t)$ we denote a solution of the closed-loop system and the error system. Then, the following theorem can be obtained which shows the globally boundedness of the solutions of the adaptive closed-loop system described by (22) and (24).

Theorem 1. Consider the adaptive closed-loop system described by (22) and (24). The closed-loop large scale system is uniformly bounded and the tracking error $e(t)$ converges asymptotically to zero for any $\rho(t)\in\Delta_{\rho_{ji}^h}$ if there exist a symmetric matrix P_i, and $\hat{K}_{1i,k}$, $\hat{K}_{3i,k}$, $\hat{k}_{5i}$ determined according to the adaptive laws (17), (18) and (21), and control gain function K_{4i} given by (19).

Proof: For the adaptive closed-loop large scale system described by (22), we first define a Lyapunov functional candidate as:

$$V=\sum_{i=1}^{N}\xi_i^T P_i\xi_i+\sum_{i=1}^{N}\sum_{k=1}^{m}\tilde{K}_{1i,k}^T\Gamma_{1i,k}^{-1}\tilde{K}_{1i,k} +\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{m}\rho_{ji}\tilde{K}_{3i,k}^T\Gamma_{3i,k}^{-1}\tilde{K}_{3i,k}+\sum_{i=1}^{N}\gamma^{-1}\tilde{k}_{5i}^2 \tag{27}$$

Then, according to equations (19) and (23), the time derivative of V for $t>0$ associated with a certain failure mode $\rho\in\Delta_{\rho_{ji}^h}$ can be derived:

$$\begin{aligned}\dot{V}(t) &=\sum_{i=1}^{N}\xi_i^T[(A_a^i+B_a^i\hat{K}_{1i})^TP_i+P_i(A_a^i+B_a^i\hat{K}_{1i})]\xi_i\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i^TP_i(A_a^{ij}+B_a^iK_{2i}\rho_{ji}K_{3i}\xi_j)\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_j^T(A_a^{ij}+B_a^iK_{2i}\rho_{ji}K_{3i})^TP_i\xi_i\\ &+2\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i^TP_iB_a^iK_{2i}\rho_{ji}\tilde{K}_{3i}\xi_j\\ &-\sum_{i=1}^{N}\frac{2\|\xi_i^TP_iB_a^i\|^2\beta_i\|\xi_i^TP_i\|\hat{k}_{5i}}{\|\xi_i^TP_iB_a^i\|^2\alpha_i}+\sum_{i=1}^{N}2\xi_i^TP_iB_a^i\sum_{j=1}^{N}K_{2i}\bar{w}_{ji}\\ &+\sum_{i=1}^{N}2\xi_i^TP_i\sum_{j=1}^{N}\bar{A}_a^{ij}\bar{w}_{ji}+\sum_{i=1}^{N}2\xi_i^TP_iG_a^iz_i\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{m}2\rho_{ji}\tilde{K}_{3i,k}^T\Gamma_{3i,k}^{-1}\dot{\tilde{K}}_{3i,k}\\ &+\sum_{i=1}^{N}\sum_{k=1}^{m}2\tilde{K}_{1i,k}^T\Gamma_{1i,k}^{-1}\dot{\tilde{K}}_{1i,k}+\sum_{i=1}^{N}2\gamma^{-1}\tilde{k}_{5i}\dot{\tilde{k}}_{5i}.\end{aligned} \tag{28}$$

Thus, by the light of Lemma 1, inequality (20), and Assumption 2, we choose adaptive laws (18) and rewrite (28) as

$$\begin{aligned}&\dot{V}(t)\\ &\leq\sum_{i=1}^{N}\xi_i^T[(A_a^i+B_a^i\hat{K}_{1i})^TP_i+P_i(A_a^i+B_a^i\hat{K}_{1i})+\zeta_iNP_i^2\\ &+\sum_{j=1}^{N}\zeta_i^{-1}(A_a^{ji}+B_a^jK_{2j}\rho_{ij}K_{3j})^T(A_a^{ji}+B_a^jK_{2j}\rho_{ij}K_{3j})]\xi_i\\ &+2\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i^TP_iB_a^iK_{2i}\rho_{ji}\tilde{K}_{3i}\xi_j\\ &-\sum_{i=1}^{N}2\parallel\xi_i^TP_i\parallel\hat{k}_{5i}+\sum_{i=1}^{N}2\parallel\xi_i^TP_i\parallel\parallel B_a^i\sum_{j=1}^{N}K_{2i}\parallel\bar{\bar{w}}_{ji}\\ &+\sum_{i=1}^{N}2\parallel\xi_i^TP_i\parallel\parallel\sum_{j=1}^{N}\bar{A}_a^{ij}\parallel\bar{\bar{w}}_{ji}\\ &+\sum_{i=1}^{N}2\parallel\xi_i^TP_i\parallel\parallel\sum_{j=1}^{N}G_a^i\parallel\bar{z}_i\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{m}2\rho_{ji}\tilde{K}_{3i,k}^T\Gamma_{3i,k}^{-1}\dot{\tilde{K}}_{3i,k}\\ &+\sum_{i=1}^{N}\sum_{k=1}^{m}2\tilde{K}_{1i,k}^T\Gamma_{1i,k}^{-1}\dot{\tilde{K}}_{1i,k}+\sum_{i=1}^{N}2\gamma^{-1}\tilde{k}_{5i}\dot{\tilde{k}}_{5i}.\end{aligned} \tag{29}$$

According to Lemma 2, we let

$$\iota_i=\lambda_{\max}(\sum_{j=1}^{N}\zeta_i^{-1}(A_a^{ji}+B_a^jK_{2j}\rho_{ij}K_{3j})^T(A_a^{ji}+B_a^jK_{2j}\rho_{ij}K_{3j}). \tag{30}$$

Hence, if there exist constants K_{1i}, K_{3j}, $i,j=1,2,\ldots,N$ let $A_a^i+B_a^iK_{1i}$ Hurwitz and Hamiltonian matrix H has no eigenvalue on the imaginary axis, then for any $\rho(t)\in\Delta_{\rho_{ji}^h}$, there exist a solution $P_i>0$ such that

$$(A_a^i+B_a^iK_{1i})^TP_i+P_i(A_a^i+B_a^iK_{1i})+\zeta_iNP_i^2+\zeta_i^{-1}(\iota_i+\varepsilon_i)I=0. \tag{31}$$

On the other hand, since $\bar{\bar{w}}_{ji}$ and $\bar{z}_i$ are unknown bounded positive constants, there always exists a constant k_{5i}, $i=1,2,\ldots,N$ let the following inequality holds true:

$$\begin{aligned}\parallel\xi_i^TP_i\parallel k_{5i}\geq&\parallel\xi_i^TP_i\parallel\parallel B_a^i\sum_{j=1}^{N}K_{2i}\parallel\bar{\bar{w}}_{ji}\\ &+\parallel\xi_i^TP_i\parallel\parallel\sum_{j=1}^{N}\bar{A}_a^{ij}\parallel\bar{\bar{w}}_{ji}+\parallel\xi_i^TP_i\parallel\parallel\sum_{j=1}^{N}G_a^i\parallel\bar{z}_i.\end{aligned} \tag{32}$$

Then, based on the above mention, definition (23), chosen the adaptive laws (17), (21), it follows from (29) that

$$\begin{aligned}&\dot{V}(t)\\ &\leq-\sum_{i=1}^{N}\zeta_i^{-1}\varepsilon_i\xi_i^T\xi_i-\sum_{i=1}^{N}2\parallel\xi_i^TP_i\parallel\tilde{k}_{5i}+\sum_{i=1}^{N}2\xi_i^TP_iB_a^i\tilde{K}_{1i}\xi_i\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}2\xi_i^TP_iB_a^iK_{2i}\rho_{ji}\tilde{K}_{3i}\xi_j+\sum_{i=1}^{N}\sum_{k=1}^{m}2\tilde{K}_{1i,k}^T\Gamma_{1i,k}^{-1}\dot{\tilde{K}}_{1i,k}\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{m}2\rho_{ji}\tilde{K}_{3i,k}^T\Gamma_{3i,k}^{-1}\dot{\tilde{K}}_{3i,k}+\sum_{i=1}^{N}2\gamma^{-1}\tilde{k}_{5i}\dot{\tilde{k}}_{5i}.\\ &=-\sum_{i=1}^{N}\zeta_i^{-1}\varepsilon_i\xi_i^T\xi_i\end{aligned} \tag{33}$$

Hence, it is easy to see that $\dot{V}(t)<0$ for any $\xi_i\neq 0$.

Equation (33) also implies

$$\int_0^t \sum_{i=1}^{N} ||\xi_i(\tau)||^2 d\tau \leq \frac{V(0)-V(t)}{\zeta_{\min}^{-1}\varepsilon_{\min}} \quad (34)$$

where $\zeta_{\min}^{-1} = \min(\zeta_i^{-1})$, $\varepsilon_{\min} = \min \xi_i$, $i = 1,2,\ldots,N$. Since the right hand side of (34) is bounds, following Barbalat lemma, it indicates $\lim_{t\to\infty} \xi_i(t) = 0$. Thus, the solutions of closed-loop distributed system are uniformly bounded, and the state $\xi_i(t)$ converges asymptotically to zero. ■

Thus, for large-scale system (16) with signal attenuations, perturbation and disturbance effects, from (18), (19), (21) and (24), we can obtain the distributed adaptive controllers (14), by which the solutions of the resulting adaptive closed-loop large-scale system can be guaranteed to be stable, and the output of each subsystem is uniformly asymptotically track the reference signal with disturbance rejection.

IV. SIMULATION EXAMPLE

In this section, an example of robust tracking control system design is given to demonstrate the proposed method. A large-scale dynamical system is composed of two dynamical subsystems borrowed from [14] with interconnection input, interconnection output and measured output added:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_{ij}(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} A_{TT}^i & A_{TS}^{ij} & B_{Td}^i & B_{Tu}^i \\ A_{ST}^{ji} & 0 & 0 & 0 \\ C_T^i & C_S^i & D_d^i & D_u^i \end{bmatrix} \begin{bmatrix} x_i(t) \\ \rho_{ji} v_{ij}(t) \\ d_i(t) \\ u_i(t) \end{bmatrix} \quad (35)$$

where $i = 1, j = 2$ or $i = 2, j = 1$ and
$A_{TT}^1 = \begin{bmatrix} 3 & 0 \\ -2 & -1 \end{bmatrix}$, $A_{TT}^2 = \begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix}$, $A_{TS}^{21} = \begin{bmatrix} 2 & -2 \\ 0.5 & -1 \end{bmatrix}$,
$A_{TS}^{12} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$, $A_{ST}^{21} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$, $A_{ST}^{12} = \begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix}$,
$B_{Tu}^1 = \begin{bmatrix} 0.1 & 0.5 \\ -1 & -1 \end{bmatrix}$, $B_{Tu}^2 = \begin{bmatrix} 0.2 & 1 \\ -1.5 & -1 \end{bmatrix}$, $D_d^1 = \begin{bmatrix} -2 & 0.5 \\ 0.7 & 1 \end{bmatrix}$,
$B_{Td}^1 = \begin{bmatrix} 1 & -0.5 \\ -0.1 & 1 \end{bmatrix}$, $B_{Td}^2 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, $D_d^2 = \begin{bmatrix} 1 & 0.5 \\ 1 & 2 \end{bmatrix}$,
$C_T^1 = \begin{bmatrix} -1 & 2 \\ 0.5 & -1 \end{bmatrix}$, $C_T^2 = \begin{bmatrix} 1 & 2 \\ 0.5 & -1 \end{bmatrix}$, $C_S^1 = \begin{bmatrix} 2 & -3 \\ -1.5 & 1 \end{bmatrix}$,
$C_S^2 = \begin{bmatrix} 2 & -2 \\ -1.5 & 2 \end{bmatrix}$, $D_u^1 = \begin{bmatrix} 0.2 & 1 \\ -1 & 0.6 \end{bmatrix}$, $D_u^2 = \begin{bmatrix} 0.7 & 1 \\ -1 & 1 \end{bmatrix}$.

Here, the dimension of interconnection signal $q_{ii} = 0, i = 1,2$, which means the subsystem is not fed back into itself. We let C_{ij}^s denote the sth interconnection channel with the signals transmit from ith subsystem to jth subsystem. We assume each of the four interconnection channels may lose its effectiveness and consider the following three possible faulty modes:

Normal mode 1: Both of the two subsystems interconnection channels are normal, that is, $\rho_{11}^1(t) = \rho_{12}^1(t) = \rho_{21}^1(t) = \rho_{22}^1(t) = 1$.

Faulty mode 2: Interconnection channels C_{12}^1 and C_{21}^1 are completely disconnected, the other channels may be normal or attenuations, that is, $\rho_{11}^2(t) = \rho_{21}^2(t) = 0$ and $a_2 < \rho_{12}^2(t) \leq 1, b_2 < \rho_{12}^2(t) \leq 1, a_2 = 0.3, b_2 = 0.5$.

Faulty mode 3: Interconnection channels C_{12}^2 and C_{21}^2 are completely disconnected, the other channels may be normal or attenuations, that is, $\rho_{12}^3(t) = \rho_{22}^3(t) = 0$ and $a_3 < \rho_{11}^3(t) \leq 1, b_3 < \rho_{21}^3(t) \leq 1, a_3 = 0.5, b_3 = 0.3$.

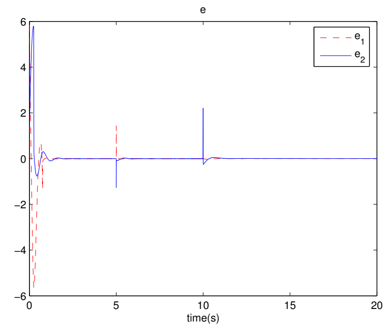

Fig. 2. Response curves of the tracking errors with distributed controllers.

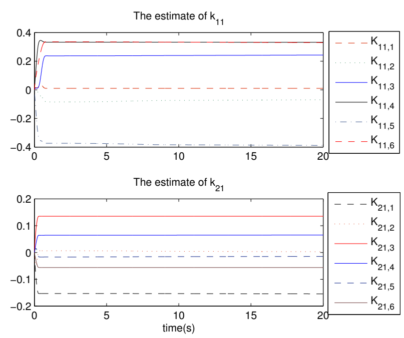

Fig. 3. Response curves of the estimates of controller parameters $K_{1i}, i = 1,2$.

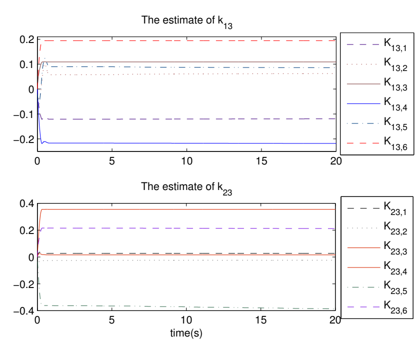

Fig. 4. Response curves of the estimates of controller parameters $K_{3i}, i = 1,2$.

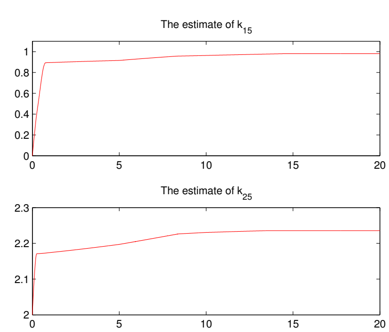

Fig. 5. Response curves of the estimates of controller parameters $K_{5i}, i = 1,2$.

To verify the effectiveness of the proposed adaptive method, the simulations are given with the following parameters and initial conditions:

$$\Gamma_{1i,k}=I_3,\Gamma_{3i,k}=I_3,\gamma_1=1.5,\gamma_2=1,\alpha_i=1,\beta_i=3,$$
$$\hat{K}_{1i,k}(0)=0,\hat{K}_{3i,k}(0)=0,k_{51}(0)=0,k_{52}(0)=2,$$
$$S_1=S_2=[1\ 1],\zeta_i=1,i=1,2,k=1,2,$$
$$K_{21}=\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix},K_{22}=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}.$$

Here, the reference signals $r_1(t)$ and $r_2(t)$ are denoted by

$$r_1(t)=\begin{cases}1.5 & 0\le t\le 5\\ 3 & t>3\end{cases},r_2(t)=\begin{cases}-1.5 & 0\le t\le 5\\ -3 & t>3\end{cases},$$

and the following faulty case is considered in the simulations, that is, before 10 second, the interconnected systems operate in normal case, and the external disturbances $d_1(t)=[-0.5,\ 0.5sin(t)]^T$ and $d_2(t)=[-0.5,\ 0.5sin(0.2t)]^T$ enter into the subsystems G_1 and G_2 at the beginning ($t\ge 0$), respectively. At 10 second, some faults in interconnection channels have occurs, described by $\rho_{12}=\rho_{21}=\text{diag}[0,1]$, and at the same time, the first channels of G_{12} and G_{21} have enter perturbations $\bar{w}_{21}(t)=[-0.5,\ 0.5+0.1sin(0.3t)]^T$ and $\bar{w}_{12}(t)=[-0.6,\ 0.5cos(0.2t)]^T$.

Fig.2 is the tracking error curves of two subsystems. It can be observed from Figs.2 that the resulting distributed adaptive state feedback controllers can get satisfied tracking results with faulty and perturbed interconnected links, and external disturbances. Fig.3-Fig.5 are the response curves of the estimations of controller parameters $\hat{K}_{1i}$, $\hat{K}_{3i}$ and $\hat{k}_{5i}$, $i=1,2$, respectively. It is easy to see the estimations can converge and all signals are uniformly bounded.

V. CONCLUSIONS

This paper has shown a direct adaptive design method to solve the asymptotic tracking control and disturbance rejection problem for distributed large scale systems with faulty and perturbed interconnection links. For the sake of automatically compensating the effects of single attenuation and unknown perturbations in interconnection links and external disturbances on subsystems, the distributed state feedback controllers are constructed by the adaptive schemes, which are based on update adaptation laws to estimate the unknown controller parameters on-line. On the basis of Lyapunov stability theory, it has shown that the resulting adaptive closed-loop large-scale system can be guaranteed to be stable and each subsystem can asymptotically-output track the corresponding reference signal under the influence of faults and perturbations in interconnection links, and external disturbances. A numerical example has shown the effectiveness of the proposed method.

REFERENCES

[1] C. Langbort, R. S. Chandra, and R. DAndrea, Distributed control design for systems interconnected over an arbitrary graph, *IEEE Transactions on Automatic Control.*, vol 49(9), 2004. pp. 1502-1519.

[2] R. DAndrea and G. E. Dullerud, Distributed control design for spatially interconnected systems, *IEEE Transactions on Automatic Control.*, vol. 48(9), 2003, pp. 1478-1495.

[3] B. Bamieh, F. Paganini, and M. Dahleh, Distributed control of spatially invariant systems, *IEEE Transactions on Automatic Control.*, vol 47(7), 2002, pp. 1091-1118.

[4] G. E. Dullerud and R. DAndrea, Distributed control of heterogeneous systems, *IEEE Transactions on Automatic Control.*, vol. 49(12), 2004, pp. 2113-2128.

[5] F. Paganini. A recursive information flow system for distributed control arrays, *in Proceeding of the American Control Conference.*, San Diego, California, USA, 1999, pp. 3821-3825.

[6] R. S. Chandra, C. Langbort, and R. DAndrea, Distributed control design with robustness to small time delays, *in Proceeding of the 2005 American Control Conference.*, Portland, OR, USA, 2005, pp. 4850-4855.

[7] B Bamieh, P. G. Voulgaris, A convex characterization of distributed control problems in spatially invariant systems with communication constraints, *Systems & Control Letters.*, vol. 54, 2005, pp. 575-583.

[8] J. K. Yook, D. M. Tilbury, and N. R. Soparkar. Trading computation for bandwidth: reducing communication in distributed control systems using state estimators, *IEEE Transactions on Control Systems Technology.*, vol. 10(4), 2002, pp. 503-518.

[9] S. Jiang, P. G. Voulgaris, Design of distributed controllers with constrained and noisy links, *in Proceeding of the 2006 Conference on Decision and Control.*, San Diego, CA, USA, 2006, pp. 3622-3627.

[10] Q. Xin, M. V. Salapaka, P. G. Voulgaris, M. Khammash, Structured optimal and robust control with multiple criteria: a convex solution, *IEEE Transactions on Automatic Control.*, Vol. 49(10), 2004, pp. 1623-1640.

[11] F. Wu, Distributed control for interconnected linear parameter-dependent systems, *IEE Proc.-Control Theory Appl.*, vol. 150(5), 2003, pp. 518-527.

[12] L. Xiao, M. Johanssonm, H. Hindi, S. Boyd and A. Goldsmith, Joint optimization of communication rates and linear systems, *IEEE Transactions on Automatic Control.*, vol. 48(1), 2003, pp. 148-153.

[13] M. Rotkowitz, S. Lall, A Characterization of Convex Problems in Decentralized Control, *IEEE Transactions on Automatic Control.*, vol. 51(2), 2006, pp. 274-286.

[14] H. Wu, Decentralized adaptive robust control for a class of large-scale systems including delayed state perturbations in the interconnections, *IEEE Transactions on Automatic Control.*, vol. 47(10), 2002, pp. 1745-1751.

[15] C. H. Chou, C. C. Cheng, A decentralized model reference adaptive variable structure controller for large-scale time-varying delay systems, *IEEE Transactions on Automatic Control.*, vol. 48(7), 2003, pp. 1213-1217.

[16] P. R. Pagilla, R. V. Dwivedula,N. B. Siraskar. A decentralized model reference adaptive controller for large-scale systems. *IEEE/ASME Transactions on mechatronics*, vol. 12(2), 2007, pp. 154-163.

[17] R. Matino and P. Tomei. Adaptive tracking and disturbance rejection for uncertain nonlinear systems. *IEEE Transactions on Automatic Control*, vol. 50(1), 2005, pp. 90-95.

[18] V. Stepanyan and N. Hovakimyan. Robust adaptive observer design for uncertain systems with bounded disturbances. *IEEE Transactions on neural networks*, vol. 18(5), 2007, pp. 1392-1403.

[19] Z. Cai, M. S. Queiroz, D. M. Dawson. Robust adaptive asymptotic tracking of nonlinear systems with additive disturbance. *IEEE Transactions on Automatic Control*, vol 51(3), 2006, pp. 524-529.

[20] D. Ye, G. H. Yang. Adaptive Fault-Tolerant Tracking Control Against Actuator Faults With Application to Flight Control. *IEEE Transactions on Control Systems Technology*, vol 14(6), 2006, pp. 1088-1096.

[21] G. Tao, S. M. Joshi, X. L. Ma. Adaptive state feedback and tracking control of systems with actuator failures. *IEEE Transactions on Automatic Control*, vol. 46(1), 2001, pp. 78-95.

[22] Z. S. Duan, J. Z. Wang, L. Huang. Special decentralized control problems in discrete-time interconnected systems composed of two subsystems. *Systems & Control Letters*, vol. 56, 2007, pp. 206-214.

[23] D. Swaroop, J. K. Hedrick. Constant spacing strategies for platooning in automated highway systems. *J. Dyna. Syst., Measure., Control*, vol. 121, 1999, pp. 462-470.

[24] D. Laughlin, M. Morari, R. D. Braatz. Robust performance of cross-directional basis-weight control in paper machines. *Automatica*, vol. 29, 1993, pp. 1395-1410.

[25] M. A. Shayman. Geometry of the algebraic Riccati equation, Part I. *SIAM J. Control Optim*, vol. 21, 1983, pp. 375-394.

[26] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Englewood Cliffs, NJ: Prentice-Hall, 1996.