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Delay-dependent absolute stability criteria for uncertain Lur'e singular systems with time-varying delay

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***Abstract*— The problem of absolute stability for Lur'e singular systems with time-varying delay is presented. Two cases of time-varying delays - one being continuous-uniformly bounded and the other being differentiable-uniformly bounded with the derivative of the delay bounded by a constant are considered. Based on a new integral inequality, which avoids employing both model transformation and bounding technique for cross terms, some delay-dependent absolute stability criteria are obtained and formulated in the form of linear matrix inequalities. Numerical examples are also given to show the effectiveness of the obtained results.**

I. INTRODUCTION

Time-delays are frequently encountered in various systems, such as nuclear reactors, chemical engineering systems, biological systems and population dynamics models [1]–[3]. Recently, many efforts have been made to obtain less conservative delay-dependent criteria, the important index of measuring the conservatism of the criteria derived is the upper bound of delays which are titled as maximum allowable delay bound in the following.

Since the introduction of absolute stability by Lur'e [4], the problem of absolute stability for Lur'e control systems has been a topic of recurring interest over the past decades [5]–[14]. It should be pointed out that the existing results can be classified into two categories: delay-independent and delay-dependent cases. When the time-delay is small, delay-independent results are often overly conservative, especially, they are not applicable to closed-loop systems which are open-loop unstable and are stabilized using delayed inputs. In order to derive some delay-dependent absolute stability criteria, one usually employs model transformation to transform the original system to a distributed system and then uses both bounding technique for cross terms [10] or uses the slack variable method [14], [15].

On the other hand, singular systems have been extensively studied in the past years due to the fact that singular systems better described physical systems than state-space ones. Depending on the area application, these models are also called descriptor systems, semi-state systems, differential-algebraic systems or generalized state-space systems [16], [17]. Therefore, the study of absolute stability problem for Lur'e singular system with time-varying delay is of theoretical and practical importance.

It should be pointed out that when the absolute stability problem for singular systems is investigated, the regularity and absence of impulses (for continuous systems) and causality (for discrete systems) are required to be considered simultaneously [18]–[20]. Hence, the absolute stability problem for Lur'e singular time-delay systems is much more complicated than that for state-space ones. Under the admissibility and strict positive real (SPR) assumption, Lee and Chen considered the absolute stability of Lur'e-type discrete-time descriptor systems [21]. Based on the concept of generalized absolute stability, Yang et.al studied the generalised absolute stability problem for Lur'e type descriptor systems [22]. However, they all have not considered the time-delay. Lu et. al has studied the delay-dependent robust H_∞ control for uncertain Lur'e singular system [23]. Model transformation is employed to derive some sufficient conditions [23], but the delay-dependent criterion is relevant to the derivative of the delay ($\dot{d}(t)$), which could not obtain the maximum allowable delay bound.

To the best of the authors' knowledge, the absolute stability problem for Lur'e singular systems with time-varying delay has not been fully investigated, which motivates this paper. The robust absolute stability analysis for Lur'e singular systems with parameter uncertainties and time-varying delay is discussed in this paper. Since model transformation and bounding techniques for cross terms appearing in the derivative of corresponding Lyapunov functional may introduce additional conservativeness [24], neither model transformation nor bounding technique for cross terms is applied in analyzing the considered systems which may yield a less conservative absolute stability condition. By employing an integral inequality [12], some delay-independent and delay-dependent stability conditions in terms of linear matrix inequalities are obtained. Numerical examples illustrate the effectiveness of the obtained results.

II. PROBLEM FORMULATION

Consider the following singular system:

$$\begin{cases} E\dot{x}(t) =Ax(t) + A_d x(t-d(t)) + D\omega(t), \\ z(t) =Mx(t) + Nx(t-d(t)), \\ \omega(t) =-\varphi(t, z(t)), \\ x(\theta) =\phi(\theta),\ \forall\theta \in [-d_M,\ 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $\omega(t) \in \mathbb{R}^m$ and $z(t) \in \mathbb{R}^m$ are the state vector, input vector and output vector of the system, respectively. $\phi(\cdot)$ is a continuous vector valued initial function. E,

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A, A_d, D, M and N are known matrices of appropriate dimensions, where E may be singular and we assume that $\operatorname{rank} E = r \leq n$. $\varphi(t, z(t)) : [0, \infty) \times \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a memoryless, possibly time-varying, nonlinear vector valued function which is piecewise continuous in t, globally Lipchitz in $z(t)$, $\varphi(t, 0) = 0$, and satisfies the following sector condition for $\forall t \geq 0, \forall z(t) \in \mathbb{R}^m$:

$$[\varphi(t, z(t)) - K_1 z(t)]^T [\varphi(t, z(t)) - K_2 z(t)] \leq 0. \quad (2)$$

where K_1 and K_2 are constant real matrices of appropriate dimensions and $K = K_1 - K_2$ is a symmetric positive definite matrix. It is customary that such a nonlinear function $\varphi(t, z(t))$ is said to belong to a sector $[K_1,\ K_2]$ [25]. The time-delay $d(t)$ is a time-varying delay satisfying $0 \leq d(t) \leq d_M < \infty, \forall t \geq 0$.

Definition 1: [16]–[18]
1) The pair (E, A) is said to be regular if $det(sE - A)$ is not identically zero.
2) The pair (E, A) is said to be impulse-free if $deg(det(sE - A)) = \text{rank}\ E$.

Definition 2: [18], [23]
1) The Lur'e singular system (1) is said to be regular and impulse-free if the pair (E, A) is regular and impulse-free.
2) The Lur'e singular system (1) is said to be globally uniformly asymptotically stable for any nonlinear function $\varphi(t, z(t))$ satisfying (2) if for any $\epsilon > 0$, there exists a scalar $\delta(\epsilon) > 0$ such that for any compatible initial conditions $\phi(t)$ satisfying $\sup\limits_{-d(t) \leq t \leq 0} \|\phi(t)\| \leq \delta(\epsilon)$, the solution $x(t)$ of the system (1) satisfies $\|x(t)\| \leq \epsilon$ for $t \geq 0$. Furthermore, $\lim\limits_{t \rightarrow \infty} x(t) = 0$.

Definition 3: The Lur'e singular system (1) is said to be absolutely stable in the sector $[K_1,\ K_2]$ if the system is regular, impulse-free and globally uniformly asymptotically stable for any nonlinear function $\varphi(t, z(t))$ satisfying (2).

In this paper, the objective is to obtain some criteria to check the absolute stability of the Lur'e singular system described by (1) and (2). Throughout this paper, we will deal with the following two cases of the time-varying delay $d(t)$

Case 1: $d(t)$ is continuous function satisfying

$$0 \leq d(t) \leq d_M < \infty, \quad \forall t \geq 0 \quad (3)$$

Case 2: $d(t)$ is a differentiable function satisfying

$$0 \leq d(t) \leq d_M < \infty, \quad \dot{d}(t) \leq d_\tau < \infty \quad \forall t \geq 0 \quad (4)$$

where d_M and d_τ are constant.

We conclude this section by introducing the following lemmas that will be used in the proof of the main results.

Lemma 1: [26] Consider the function $\varphi :\ R^+ \rightarrow R$, if $\dot{\varphi}$ is bounded on $[0, \infty)$, that is, there exists a scalar $\alpha > 0$ such that $|\ \dot{\varphi}(t)\ | \leq \alpha$ for all $t \in [0, \infty)$, then $\varphi(t)$ is uniformly continuous on $[0, \infty)$.

Lemma 2: (Barbalat's Lemma) [26] Consider the function $\varphi :\ R^+ \rightarrow R$, if φ is uniformly continuous and $\int_0^\infty \varphi(s) ds < \infty$, then $\lim\limits_{t \rightarrow \infty} \varphi(t) = 0$.

Lemma 3: (S-procedure) [27] Let $F_i = F_i^T \in \mathbb{R}^{n \times n},\ i = 0, 1, 2, \cdots, p$, Then the following statement is true

$$\xi^T F_0 \xi > 0, \quad for \quad all \quad \xi \neq 0 \quad satisfying \quad \xi^T F_i \xi \geq 0,$$

if there exist real scalars $\epsilon_i \geq 0$ $(i = 0, 1, 2, \cdots, p)$ such that

$$F_0 - \sum_{i=1}^{p} \epsilon_i F_i > 0.$$

For $p = 1$, these two statements are equivalent.

Lemma 4: [12] For any constant matrix $X \in \mathbb{R}^{n \times n}$, $X = X^T \geq 0$, scalar $0 \leq d(t) \leq d_M$, and vector function $\dot{x} : [-d_M,\ 0] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, then

$$-d_M \int_{t-d(t)}^{t} \dot{x}^T(s) X \dot{x}(s) ds \leq \left[\begin{array}{cc} x^T(t) & x^T(t - d(t)) \end{array} \right] \left[\begin{array}{cc} -X & X \\ X & -X \end{array} \right] \left[\begin{array}{c} x(t) \\ x(t - d(t)) \end{array} \right]. \quad (5)$$

III. MAIN RESULTS

First, we give some criteria of absolute stability for the nonlinear singular system (1).

A. Absolute stability Analysis

We first consider the case when the nonlinear function $\varphi(t, z(t))$ belongs to a sector $[0,\ K]$, that is, $\varphi(t, z(t))$ satisfies

$$\varphi(t, z(t))^T [\varphi(t, z(t)) - K z(t)] \leq 0. \quad (6)$$

For Case 1, the following theorem presents a solution to the stability analysis problem of the singular system (1) with the nonlinear function (6).

Theorem 1: Under Case 1, the singular system (1) with the nonlinear function satisfying (6) is absolutely stable, if there exist a scalar $\varepsilon > 0$, real matrices $P > 0$, $W > 0$ and matrix S with appropriate dimensions such that

$$\left[\begin{array}{cccc} (1,1) & (1,2) & (1,3) & d_M A^T W \\ * & -E^T W E & -\varepsilon N^T K^T & d_M A_d^T W \\ * & * & -2\varepsilon I & d_M D^T W \\ * & * & * & -W \end{array} \right] < 0 \quad (7)$$

where

$$(1,1) = A^T (PE + RS^T) + (E^T P + SR^T) A - E^T W E$$
$$(1,2) = (E^T P + SR^T) A_d + E^T W E$$
$$(1,3) = (E^T P + SR^T) D - \varepsilon M^T K^T$$

and $R \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

Proof: We first show the nonlinear singular system (1) is regular and impulse-free.

Since $\text{rank} E = r \leq n$, there must exist two invertible matrices G and $H \in \mathbb{R}^{n \times n}$ such that

$$\bar{E} = GEH = \left[\begin{array}{cc} I_r & 0 \\ 0 & 0 \end{array} \right] \quad (8)$$

Then, R can be parameterized as $R = G^T \begin{bmatrix} 0 \\ \bar{\Phi} \end{bmatrix}$, where $\bar{\Phi} \in \mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.

Similar to (8), we define

$$\bar{A} = GAH = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$$
$$\bar{P} = G^{-T}PG^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}$$
$$\bar{W} = G^{-T}WG^{-1} = \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} \\ \bar{W}_{21} & \bar{W}_{22} \end{bmatrix}$$
$$\bar{S} = H^T S = \begin{bmatrix} \bar{S}_{11} \\ \bar{S}_{21} \end{bmatrix}, \bar{R} = G^{-T}R = \begin{bmatrix} 0 \\ \bar{\Phi} \end{bmatrix}$$

Since $(1,1) < 0$, we can formulate the following inequality easily,

$$\Psi = A^T(PE + RS^T) + (E^TP + SR^T)A - E^TWE < 0.$$

Pre- and post-multiplying $\Psi < 0$ by H^T and H, respectively, yields

$$\begin{aligned}\bar{\Psi} =& H^T\Psi H \\ =& \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} \\ * & \bar{A}_{22}^T\bar{\Phi}\bar{S}_{21}^T + \bar{S}_{21}\bar{\Phi}^T\bar{A}_{22} \end{bmatrix} < 0\end{aligned} \tag{9}$$

Since $\bar{\Psi}_{11}$ and $\bar{\Psi}_{12}$ are irrelevant to the results of the following discussion, the real expression of these two variables are omitted here. From (9), it is easy to see that

$$\bar{A}_{22}^T\bar{\Phi}\bar{S}_{21}^T + \bar{S}_{21}\bar{\Phi}^T\bar{A}_{22} < 0 \tag{10}$$

and thus $\bar{A}_{22}$ is nonsingular. Otherwise, supposing $\bar{A}_{22}$ is singular, there must exist a non-zero vector $\zeta \in \mathbb{R}^{n-r}$, which ensures $\bar{A}_{22}\zeta = 0$. And then we can conclude that $\zeta^T(\bar{A}_{22}^T\bar{\Phi}\bar{S}_{21}^T + \bar{S}_{21}\bar{\Phi}^T\bar{A}_{22})\zeta = 0$, and this contradicts (10). So $\bar{A}_{22}$ is nonsingular, which implies that det$(sE - A)$ is not identically zero and deg(det$(sE - A)) = r =$rank E. Then, the pair of (E, A) is regular and impulse-free, which implies that the system (1) is regular and impulse-free.

In the following, we will prove that the system (1) is also globally uniformly asymptotically stable.

Choose a Lyapunov-Krasovskii function candidate as

$$\begin{aligned}V(t, x_t) =& x^T(t)E^TPEx(t) \\ &+ \int_{-d_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)(d_M E^TWE)\dot{x}(s)dsd\theta,\end{aligned} \tag{11}$$

where $P > 0$ and $W > 0$. Taking the derivation of $V(t, x_t)$ with respect to t along the trajectory of (1) yields

$$\begin{aligned}\dot{V}(t, x_t) =& x^T(t)(A^TPE + E^TPA)x(t) \\ &+ 2x^T(t)E^TPA_d x(t - d(t)) \\ &+ 2x^T(t)E^TPD\omega(t) + d_M^2\dot{x}^T(t)E^TWE\dot{x}(t) \\ &- \int_{t-d_M}^{t} \dot{x}^T(s)(d_M E^TWE)\dot{x}(s)ds\end{aligned}$$

Noting $E^TR = 0$, we can deduce

$$0 = 2\dot{x}^T(t)E^TRS^Tx(t), \tag{12}$$

where S is any matrix with appropriate dimensions.

According to (1), the following holds

$$\begin{aligned}d_M^2\dot{x}^T(t)E^TWE\dot{x}(t) =& \xi^T(t)\begin{bmatrix} A^T \\ A_d^T \\ D^T \end{bmatrix} d_M^2 W \\ &\times \begin{bmatrix} A & A_d & D \end{bmatrix}\xi(t)\end{aligned}$$

where $\xi^T(t) = \begin{bmatrix} x^T(t) & x^T(t - d(t)) & \omega^T(t) \end{bmatrix}$. Then according to Lemma 4 and combining with (12), we have

$$\dot{V}(t, x_t) \le \xi^T(t)\Omega\xi(t)$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & \Omega_{23} \\ * & * & \Omega_{33} \end{bmatrix}$$

with

$$\begin{aligned}\Omega_{11} =& A^T(PE + RS^T) + (E^TP + SR^T)A - E^TWE \\ &+ A^T(d_M^2W)A \\ \Omega_{12} =& (E^TP + SR^T)A_d + E^TWE + A^T(d_M^2W)A_d \\ \Omega_{13} =& (E^TP + SR^T)D + A^T(d_M^2W)D \\ \Omega_{22} =& -E^TWE + A_d^T(d_M^2W)A_d \\ \Omega_{23} =& A_d^T(d_M^2W)D \\ \Omega_{33} =& D^T(d_M^2W)D.\end{aligned}$$

A sufficient condition for absolute stability of the nonlinear singular system (1) is that there exist a scalar $\varepsilon \ge 0$, real matrices $P > 0$, $W > 0$ and matrix S such that

$$\dot{V}(t, x_t) \le \xi^T(t)\Omega\xi(t) < 0 \tag{13}$$

for all $\xi(t) \ne 0$ satisfying (6). From (1) and (6), we have

$$\omega^T(t)\omega(t) + \omega^T(t)K(Mx(t) + Nx(t - d(t))) \le 0. \tag{14}$$

Then according to Lemma 3, one can see that (13) is implied by the existence of a scalar $\varepsilon \ge 0$ such that

$$\begin{aligned}&\xi^T(t)\Omega\xi(t) - 2\varepsilon\omega^T(t)\omega(t) \\ &\quad - 2\varepsilon\omega^T(t)K(Mx(t) + Nx(t - d(t))) < 0,\end{aligned} \tag{15}$$

for all $\xi(t) \ne 0$. Rewrite (15) as

$$\xi^T(t)\Xi\xi(t) < 0, \tag{16}$$

where

$$\Xi = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} - \varepsilon M^TK^T \\ * & \Omega_{22} & \Omega_{23} - \varepsilon N^TK^T \\ * & * & \Omega_{33} - 2\varepsilon I \end{bmatrix}$$

Using Schur complement, (7) implies $\Xi < 0$, and then $\dot{V}(t, x_t) < 0$ holds.

Now, let $\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} = H^{-1}x(t)$, where $\bar{x}_1(t) \in \mathbb{R}^r, \bar{x}_2(t) \in \mathbb{R}^{n-r}$, the singular system (1) can be written as:

$$\bar{E}\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_d\bar{x}(t - d(t)) + D\omega(t) \tag{17}$$

Then we have

$$\begin{aligned}\lambda_1\|\bar{x}_1(t)\|^2 - V(\bar{x}(0)) &\le \bar{x}_1^T(t)\bar{P}_{11}\bar{x}_1(t) - V(\bar{x}(0)) \\ &\le -\lambda_2\int_0^t \|\bar{x}_1(s)\|^2 ds < 0\end{aligned} \tag{18}$$

where $\lambda_1 = \lambda_{min}(\bar{P}_{11}) > 0, \quad \lambda_2 = -\lambda_{max}(H^T\Xi H) > 0$.

Taking into account (18), we can deduce that

$$\lambda_1\|\bar{x}_1(t)\|^2 + \lambda_2\int_0^t \|\bar{x}_1(s)\|^2 ds \le V(\bar{x}(0))$$

Therefore

$$0 < \|\bar{x}_1(t)\|^2 \le \frac{1}{\lambda_1}V(\bar{x}(0)),$$

$$0 < \int_0^t \|\bar{x}_1(s)\|^2 ds \le \frac{1}{\lambda_2}V(\bar{x}(0)).$$

Thus, $\|\bar{x}_1(t)\|$ and $\int_0^t \|\bar{x}_1(s)\|ds$ are bounded. Similarly, we have that $\|\dot{\bar{x}}_1(t)\|$ is bounded. By Lemma 1, we obtain $\|\bar{x}_1(t)\|^2$ is uniformly continuous. Noting that $\int_0^t \|\bar{x}_1(s)\|^2 ds$ is bounded, and using Lemma 2, we have

$$\lim_{t\to\infty} \bar{x}_1(t) = 0.$$

Using the same method in [18], we also have $\lim_{t\to\infty} \bar{x}_2(t) = 0$. Then $\lim_{t\to\infty} x(t) = 0$ holds. According to Definition 2, the singular delay system (1) is globally uniformly asymptotically stable for any nonlinear function $\varphi(t, z(t))$ satisfying (6). As a whole, in view of Definition 3, the singular system (1) with the nonlinear function satisfying (6) is absolutely stable. This completes the proof. ∎

Remark 1: If $E = I$, it follows from $E^TR = 0$ that $R = 0$. Therefore, it is easy to show that Theorem 1 reduces to Proposition 1 in [12].

For Case 2, since $d(t)$ is a differentiable function, by making use of this additional information, we can choose a Lyapunov-Krasovskii functional candidate as

$$\tilde{V}(t, x_t) = V(t, x_t) + \int_{t-d(t)}^t x^T(s)Qx(s)ds \tag{19}$$

where $V(t, x_t)$ is defined in (11). Then, similar to the proof of Theorem 1, we can obtain the following result.

Theorem 2: Under Case 2, the singular system (1) with the nonlinear function satisfying (6) is absolutely stable, if there exist a scalar $\varepsilon > 0$, real matrices $P > 0$, $Q > 0$, $W > 0$ and matrix S with appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & d_M A^T W \\ * & (2,2) & -\varepsilon N^T K^T & d_M A_d^T W \\ * & * & -2\varepsilon I & d_M D^T W \\ * & * & * & -W \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned}(1,1) =& A^T(PE + RS^T) + (E^TP + SR^T)A \\ & - E^TWE + Q \\ (1,2) =& (E^TP + SR^T)A_d + E^TWE \\ (1,3) =& (E^TP + SR^T)D - \varepsilon M^T K^T \\ (2,2) =& -(1 - d_\tau)Q - E^TWE\end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR = 0$.

Remark 2: Clearly, Theorem 1 and Theorem 2 provide delay-dependent sufficient conditions for Case 1 and Case 2, respectively, which can guarantee the absolute stability of the Lur'e singular system (1) with a nonlinear connection function satisfying (6). Choosing Theorem 1 or Theorem 2 will depend on the situation concerning the information of delay $d(t)$. If $d(t)$ is only a continuous function, but not a differentiable function for all $t \ge 0$, we can only use Theorem 1. If $d(t)$ is a differentiable, both Theorem 1 and Theorem 2 can be employed. For this situation, in the case of $d_\tau < 1$, Theorem 2 supplies a less conservative result than Theorem 1, which is proved in Proposition 3 of [12].

For the nonlinearity $\varphi(t, z(t))$ satisfying the more general sector condition (2), we can conclude that the absolute stability of system (1) in the sector $[K_1,\ K_2]$ is equivalent to the absolute stability of the following system:

$$\begin{cases} E\dot{x}(t) = (A - DK_1M)x(t) + (A_d - DK_1N)x(t - d(t)) \\ \qquad\qquad + D\tilde{\omega}(t), \\ z(t) = Mx(t) + Nx(t - d(t)), \\ \tilde{\omega}(t) = -\tilde{\varphi}(t, z(t)), \end{cases} \tag{21}$$

in the sector $[0,\ K_2 - K_1]$, that is, $\tilde{\varphi}(t, z(t))$ satisfies for $\forall t \ge 0, \forall z(t) \in \mathbb{R}^m$,

$$\tilde{\varphi}(t, z(t))^T[\tilde{\varphi}(t, z(t)) - (K_2 - K_1)z(t)] \le 0. \tag{22}$$

According to Theorem 1, we have the following result.

Theorem 3: Under Case 1, the singular system (1) with the nonlinear function satisfying (2) is absolutely stable, if there exist a scalar $\varepsilon > 0$, real matrices $P > 0$, $W > 0$ and matrix S with appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) \\ * & -E^TWE & (2,3) & (2,4) \\ * & * & -2\varepsilon I & d_M D^T W \\ * & * & * & -W \end{bmatrix} < 0 \tag{23}$$

where

$$\begin{aligned}(1,1) &= (A - DK_1M)^T(PE + RS^T) \\ &\quad + (E^TP + SR^T)(A - DK_1M) - E^TWE \\ (1,2) &= (E^TP + SR^T)(A_d - DK_1N) + E^TWE \\ (1,3) &= (E^TP + SR^T)D - \varepsilon M^T(K_2 - K_1)^T \\ (1,4) &= d_M(A - DK_1M)^TW \\ (2,3) &= -\varepsilon N^T(K_2 - K_1)^T \\ (2,4) &= d_M(A_d - DK_1N)^TW\end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR = 0$.

Theorem 4: Under Case 2, the singular system (1) with the nonlinear function satisfying (2) is absolutely stable, if there exist a scalar $\varepsilon > 0$, real matrices $P > 0$, $Q > 0$,

$W > 0$ and matrix S with appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) \\ * & (2,2) & (2,3) & (2,4) \\ * & * & -2\varepsilon I & d_M D^T W \\ * & * & * & -W \end{bmatrix} < 0 \quad (24)$$

where

$$\begin{aligned} (1,1) =& (A - DK_1M)^T(PE + RS^T) \\ &+ (E^TP + SR^T)(A - DK_1M) - E^TWE + Q \\ (1,2) =& (E^TP + SR^T)(A_d - DK_1N) + E^TWE \\ (1,3) =& (E^TP + SR^T)D - \varepsilon M^T(K_2 - K_1)^T \\ (1,4) =& d_M(A - DK_1M)^TW \\ (2,2) =& -(1 - d_\tau)Q - E^TWE \\ (2,3) =& -\varepsilon N^T(K_2 - K_1)^T \\ (2,4) =& d_M(A_d - DK_1N)^TW \end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR = 0$.

B. Robust Absolute Stability

If there exist norm-bounded uncertainties in system's matrices A and B, system becomes

$$\begin{cases} E\dot{x}(t) =(A + LF(t)E_a)x(t) \\ \qquad\quad + (A_d + LF(t)E_d)x(t - d(t)) + D\tilde{\omega}(t), \\ z(t) =Mx(t) + Nx(t - d(t)), \\ \tilde{\omega}(t) = -\tilde{\varphi}(t, z(t)), \end{cases} \quad (25)$$

where L, E_a, E_d are known real constant matrices of appropriate dimensions, and $F(t)$ is an unknown matrix function satisfying $F^T(t)F(t) \le I$.

Applying the routine method in [28], we can obtain a more general result.

Theorem 5: Under Case 1, the uncertain singular system (25) with the nonlinear function satisfying (2) is absolutely stable, if there exist a scalars $\varepsilon > 0$, $\mu > 0$, real matrices $P > 0$, $W > 0$ and matrix S with appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & \mu E_a^T \\ * & (2,2) & (2,3) & (2,4) & 0 & \mu E_b^T \\ * & * & -2\varepsilon I & (3,4) & 0 & 0 \\ * & * & * & -W & d_M W^T L & 0 \\ * & * & * & * & -\mu I & 0 \\ * & * & * & * & * & -\mu I \end{bmatrix} < 0 \quad (26)$$

where

$$\begin{aligned} (1,1) =& (A - DK_1M)^T(PE + RS^T) \\ &+ (E^TP + SR^T)(A - DK_1M) - E^TWE \\ (1,2) =& (E^TP + SR^T)(A_d - DK_1N) + E^TWE \\ (1,3) =& (E^TP + SR^T)D - \varepsilon M^T(K_2 - K_1)^T \\ (1,4) =& d_M(A - DK_1M)^TW \\ (1,5) =& (E^TP + SR^T)L, \ (2,2) = -E^TWE \\ (2,3) =& -\varepsilon N^T(K_2 - K_1)^T \\ (2,4) =& d_M(A_d - DK_1N)^TW, \ (3,4) = d_M D^T W \end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR = 0$.

Theorem 6: Under Case 2, the uncertain singular system (25) with the nonlinear function satisfying (2) is absolutely stable, if there exist a scalars $\varepsilon > 0$, $\mu > 0$, real matrices $P > 0$, $Q > 0$, $W > 0$ and matrix S with appropriate dimensions such that

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & \mu E_a^T \\ * & (2,2) & (2,3) & (2,4) & 0 & \mu E_b^T \\ * & * & -2\varepsilon I & d_M D^T W & 0 & 0 \\ * & * & * & -W & (4,5) & 0 \\ * & * & * & * & -\mu I & 0 \\ * & * & * & * & * & -\mu I \end{bmatrix} < 0 \quad (27)$$

where

$$\begin{aligned} (1,1) =& (A - DK_1M)^T(PE + RS^T) \\ &+ (E^TP + SR^T)(A - DK_1M) + Q - E^TWE \\ (1,2) =& (E^TP + SR^T)(A_d - DK_1N) + E^TWE \\ (1,3) =& (E^TP + SR^T)D - \varepsilon M^T(K_2 - K_1)^T \\ (1,4) =& d_M(A - DK_1M)^TW \\ (1,5) =& (E^TP + SR^T)L \\ (2,2) =& -(1 - d_\tau)Q - E^TWE \\ (2,3) =& -\varepsilon N^T(K_2 - K_1)^T \\ (2,4) =& d_M(A_d - DK_1N)^TW, \ (4,5) = d_M W^T L \end{aligned}$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR = 0$.

IV. NUMERICAL EXAMPLE

The following numerical examples are presented to illustrate the usefulness of the proposed theoretical results.

Example 1: (Absolute Stability) Consider the singular system described by (21) and (22) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix}, A_d = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, M = \begin{bmatrix} 0.6 & 0.8 \end{bmatrix}, N = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$

$$K_1 = 0.1, K_2 = 0.5$$

In this example, we choose $R = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. For Case 1, using Theorem 3 yields the maximum allowable delay bound as $d_M = 0.5123$. For Case 2, applying Theorem 4, Table 1 lists the maximum allowable delay bound d_M for absolute stability. From this table, one can see that when $d_\tau < 1$, the results obtained using Case 2 is less conservative than those using Case 1, which further verifies Remark 2.

Example 2 (Robust Absolute Stability) Consider the sin-

TABLE I: Maximum allowed delay bound d_M for different d_τ

d_τ	0	0.3	0.6	0.7	1
d_M	1.7036	1.4647	1.2091	1.0906	0.5123

TABLE II: Maximum allowable delay bound d_M for different uncertainties

λ	0.15	0.5	1	1.5
d_M	0.6402	0.5675	0.4743	0.3931

gular system described by (25) and (22) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -2 & 0 \\ 0.5 & 1 \end{bmatrix}, A_d = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, M = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$N = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, K_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, E_a = 0.1I$$
$$K_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, L = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} (\lambda > 0), E_b = 0.1I$$

In this example, we choose $R = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. Table II gives the maximum allowable delay d_{max} for various parameter λ when $d_\tau = 0.5$ according to Theorem 6.

V. CONCLUSION

The problem of absolute stability for Lur'e singular systems with time-varying delay has been presented. Full consideration two cases of time-varying delays - one being continuous-uniformly bounded and the other being differentiable-uniformly bounded with the derivative of the delay bounded by a constant. By employing a new integral inequality, which avoids employing both model transformation and bounding technique for cross terms, some new linear matrix inequalities(LMIs) based delay-dependent absolute stability criteria are obtained. Numerical examples has also been given to illustrate the effectiveness of the obtained results.

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