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# Robust Tracking Control of a Class of Nonlinear Switched Systems: An Average Dwell-Time Method

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*Abstract*— **This paper is concerned with the output tracking control problem for a class of nonlinear switched cascade systems with external disturbances under some average dwell-time based switching laws. The problem is solved based on the variable structure control technique and the characteristic of the system. The variable structure controllers and the average dwell-time are designed under which the output of the closed-loop switched system can follow the desired output exactly after a finite time internal and all the states remain globally bounded. And the effectiveness of the proposed design approach is illustrated with simulation results.**

## I. INTRODUCTION

Variable structure control with sliding model control has developed into a general design method being examined for a wide spectrum of system types, which is characterized by a discontinuous control that changes structure on reaching a sliding surface. And the objective of variable structure control has been greatly extended from stabilization to other control functions. This control method can make the system completely insensitive to parametric uncertainty and external disturbances. Today, research and development continue to apply variable structure control to a wide variety of engineering systems.

On the other hand, a large class of natural and man-made systems are often governed by several dynamical modes. The interchange between modes is often determined by time $t$ and state $x$ or based on some environmental factors that are not predicted a priori. Such a system is called a switched system, which is a special kind of hybrid system that consists of a family of continuous time or discrete-time dynamical systems and a rule called the switching signal to control the switching between modes. An important qualitative property of such systems is stability [1-10]. The challenge to analyze the stability of switched systems lies partly in the fact that even if the individual systems are stable, the switched system might be unstable. [2] showed that when all modes are exponentially stable the entire switched system is exponentially stable under any switching signal if the time between two successive switchings, called the dwell time, is sufficiently large. Later, [3] extended the dwell time approach to the concept of average-dwell time. Then, [4] used this average-dwell time approach to achieve the same stability result where the family of modes was enlarged to include unstable modes. [5] used the concept to analyze the stability of general switched nonlinear systems. Besides this method, many other methods had been reported, like common Lyapunov function method [6,7], multiple Lyapunov function method [8], switched Lyapunov function method [9], convex combination method [10], and so on. And all these methods are summarized in the books [11, 12].

In addition to the stability analysis problem for switched systems, many other problems such as controllability and reachability problems [13, 14], robust $H_\infty$ control problems [15, 16], passivity [17] have been extensively investigated. However, as a valuable problem which have great practical significance, little attention has been paid to tracking control problem for switched systems.

In fact, switched systems are a certain kind of variable structure systems. Therefore, sliding modes may exist on the switching surfaces, which we usually prefer them not to happen. But, there still have existing results enforcing sliding mode occurring for switched systems such that the system possess certain desirable properties such as insensitivity to parameter variations and external disturbances [18, 19].

In this paper, we consider the robust output tracking control problem via variable structure control strategy for a class of disturbed nonlinear switched cascade systems under some average-dwell time based switching laws. Sufficient conditions for the solvability of the problem are given in the paper. The piecewise Lyapunov functions and a common

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sliding surface are constructed. Based on the characteristic of the switched system, the switched variable structure controller and the average dwell-time are designed, which guarantees that the states of the corresponding disturbed closed-loop system remain globally bounded and the output of the system can follow the desired signal exactly after a finite time interval.

**Notation:** We use standard notations throughout this paper. Given a real matrix $M$, $M^T$ denotes the transpose of $M$. $I$ is an identity matrix whose dimension is implied from context. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of $P$. $\|\cdot\|$ denotes the Euclidean norm. $R^n$ denotes the $n$-dimensional real Euclidean space. $R^{m\times n}$ is the set of all real $m \times n$ matrix.

## II. PROBLEM STATEMENT

We consider the nonlinear switched cascade systems described by

$$\begin{cases} \dot{z} = f_\sigma(z,\xi), \\ \dot{\xi} = A_\sigma \xi + B\big[G_\sigma(z,\xi)u_\sigma + F_\sigma(z,\xi)\big], \\ y = C\xi, \end{cases} \tag{1}$$

where $z \in R^{n-d}$, $\xi \in R^d$ are the states; $y \in R^m$ is the measurable output; $\sigma : [0,\infty] \to I_N = \{1,\dots,N\}$ is the switching signal which will be determined later, and $\sigma(t) = i$ means that the $i$th subsystem is activated; $u_i(t) \in R^m$ is the control input; $F_i(z,\xi)$ represents the external disturbances; $A_i$, $B_i$ are known matrices; $f_i(z,\xi)$, $G_i(z,\xi)$ are known smooth vector fields with appropriate dimensions. Further, $\det(G_i(z,\xi)) \neq 0$ for $\forall\ [z^T,\xi^T]^T \in R^n$, $f_i(0,0) = 0$.

In this paper, we need the following assumptions.

**Assumption 1:** Matrix $B$ is of full column rank, and $m < d$.

**Assumption 2:** The Matrix $CB$ is nonsingular.

**Assumption 3:** $\|F_i(z,\xi)\| \le \rho_i(t)$, $i \in I_N$ for some known continuous and uniformly bounded functions $\rho_i(t)$.

The basic assumption on the reference trajectory $y_d$ is as follows.

**Assumption 4:** The reference trajectory $y_d$ is piecewise differentiable. Additionally, there exist known constants $Y_1$ and $Y_2$ such that

$$\|y_d(t)\| \le Y_1, \quad \|\dot{y}_d(t)\| \le Y_2, \quad \forall t \in [0,\infty).$$

**Remark 1.** Assumptions 1∼3 are assumptions that are usually made in the variable structure control. Similarly, Assumption 4 is the usual assumption made when the tracking control problem is considered.

We now state the output tracking control problem for switched system (1).

**The output tracking control problem:** Find, if possible a switching law $\sigma$ and an output feedback controller $u_\sigma = \alpha_\sigma(z,\xi)$ such that the following two facts are true:
(i) $\lim_{t\to+\infty}(y(t) - y_d(t)) = 0$;
(ii) the state $(z,\xi)$ of the closed-loop system (1) is globally bounded.

Our objective is to solve the output tracking control problem with variable structure output feedback.

The following Definition and lemma will be used in the development of our results.

Consider the general nonlinear system

$$\dot{x} = f(x,u), \tag{2}$$

where $x \in R^n$ is the state, $u \in R^p$ is the input, $f(x,u)$ is a smooth vector and satisfies $f(0,0) = 0$. Denote $|||\cdot|||$ as the essential supremum norm in the functional space $L^p_\infty$, i.e.,

$$|||\ u\ ||| = \sup\{\|u(t)\|, t \ge 0\} < \infty.$$

**Definition 1** [20]: System (2) is input-to-state stable if and only if there exist a proper, positive definite and radially unbounded function $V(x)$ such that for some class $\mathcal{K}_\infty$ functions $\gamma$, $\eta$ we have

$$\frac{\partial V(x)}{\partial x} f(x,u) \le -\eta(\|x\|) + \gamma(\|u\|), \qquad \forall x,\ u.$$

Consider the nonlinear switched system

$$\dot{x} = f_\sigma(x,v), \tag{3}$$

where $\sigma$ is the switching signal as given in the description of system (1), $f_i(x,v)$ are smooth vector fields, the set of measurable functions $v :\ [0,\ \infty) \to R^l$ is the input.

**Lemma 1** [21]: Suppose that there exist continuous differentiable functions $V_p :\ R^n \to [0,\infty)$, $p \in I_N$, class $\mathcal{K}_\infty$ functions $\alpha_1$, $\alpha_2$, $\gamma$, and numbers $\lambda_0$, $\mu \ge 1$ such that $\forall x \in R^n,\ v \in R^l$, and $\forall p,\ q \in I_N$, we have

$$\alpha_1(\|x\|) \le V_p(x) \le \alpha_2(\|x\|), \tag{4}$$

$$\frac{\partial V_p}{\partial x} f_p(x,v) \le -\lambda_0 V_p(x) + \gamma(\|v\|), \tag{5}$$

$$V_p(x) \le \mu V_q(x), \tag{6}$$

Let $\sigma$ be a switching signal having average dwell-time $\tau_a$. Then, the switched nonlinear system (3) is input-to-state stable if $\tau_a > \frac{\ln\mu}{\lambda_0}$.

## III. MAIN RESULTS

For a vector $\epsilon = [\epsilon_1,\cdots,\epsilon_d] \in R^d$, define

$$\text{sgn}\ \epsilon = [\text{sgn}\epsilon_1, \text{sgn}\epsilon_2, \dots, \text{sgn}\epsilon_d]^T.$$

Let

$$\{(t_k, i_k), | i_k \in I_N;\ k = 1,\ 2, \ldots, N_\sigma(0,t);$$
$$0 = t_1 \le t_2 \le \cdots \le t_{N_\sigma(0,t)} \le T\}$$

be the switching sequence in the interval $[0,\ T)$ that is generated by the switching signal $\sigma$, and $N_\sigma(0,T)$ is the number of switchings that occur during $[0,T)$.

**Theorem 1:** Suppose Assumption 1∼4 are satisfied and that (i) there exist smooth positive definite functions $G_i,\ i \in I_N$, class $\mathcal{K}_\infty$ functions $\beta_1,\ \beta_2,\ \gamma_i$, and positive numbers $\alpha_{1i}$, $\alpha_{2i}$, $\alpha_3$, $\lambda_{0i}$, $\mu_z \ge 1$ such that for $\forall z \in R^{n-d},\ \xi \in R^d$, and $\forall i,\ j \in I_N$, we have

$$\beta_1(\|z\|) \le G_i(z) \le \beta_2(\|z\|), \tag{7}$$
$$\frac{\partial G_i(z)}{\partial z} f_i(z,\xi) \ \le\ -\lambda_{0i} U_i(z) + \gamma_i(\|\xi\|), \tag{8}$$
$$G_i(z) \ \le\ \mu_z G_j(z). \tag{9}$$

(ii) there exist positive definite matrix $Q$, a matrix $N$ and a positive scalar $\vartheta$ such that the following inequalities

$$A_i Q + Q A_i^T + BN + N^T B^T + \vartheta Q + I \ <\ 0, \tag{10}$$
$$B^T Q^{-1} = C, \tag{11}$$

hold.

Let

$$e(t) = (CB)^{-1}[y(t) - y_d(t)]. \tag{12}$$

Then, under an arbitrary switching law satisfying the average dwell-time

$$\tau_a \ge \tau_a^* = \frac{\ln \mu_z}{\lambda} \text{ and } \ t_2 \ge t_d, \tag{13}$$

where $\lambda \in (0, \lambda_0)$, $\lambda_0 = \min\{\lambda_{0i} \mid i = 1, 2, \cdots, N\}$, $t_d$ is a certain positive constant that can be calculated later. The variable structure controller

$$\begin{aligned} u_i \ =\ & -G_i^{-1}(z,\xi)\big[(CB)^{-1} C A_i \xi + \kappa_1 e + (\kappa_2 + \rho(t))\text{sgn}e \\ & -(CB)^{-1}\dot{y}_d(t)\big], \end{aligned} \tag{14}$$

where $\kappa_1$ and $\kappa_2$ are two positive constants, will solve the output tracking problem for the corresponding closed-loop system (1).

**Proof:** The proof is divided into two parts. First of all, we will show that the output $y(t)$ of (1) can follow exactly the desired signal $y_d(t)$ after a finite time interval. Then, we will show that the state of (1) is globally bounded under the average-dwell time based switching laws.

We first give the proof for the first part. Let $\rho(t) = \max\{\rho_i(t) \mid i = 1, 2, \cdots, N\}$. The derivative of $e(t)$ along the trajectory of system (1) with (15) is

$$\begin{aligned} \dot{e}(t) \ &=\ (CB)^{-1}[\dot{y}(t) - \dot{y}_d(t)] \\ &=\ (CB)^{-1}[C\dot{\xi} - \dot{y}_d(t)] \\ &=\ (CB)^{-1} C A_i \xi + \big[G_i(z,\xi)u_i + F_i(z,\xi)\big] \\ &\quad -(CB)^{-1}\dot{y}_d(t) \\ &=\ -\kappa_1 e - (\kappa_2 + \rho(t))\text{sgn}e + F_i(z,\xi). \end{aligned}$$

For any $p = 1, 2, \cdots, m$. When $e_p > 0$ we can get

$$\begin{aligned} \dot{e}_p(t) \ &=\ -\kappa_1 e_p - (\kappa_2 + \rho(t))\text{sgn}e_p + \big(F_i(z,\xi)\big)_p \\ &\le\ -\kappa_1 e_p - \kappa_2 - \rho(t) + \|\big(F_i(z,\xi)\big)_p\| \\ &\le\ -\kappa_1 e_p - \kappa_2 - \rho(t) + \|\big(F_i(z,\xi)\big)\| \\ &\le\ -\kappa_1 e_p - \kappa_2. \end{aligned} \tag{15}$$

Similarly, when $e_p < 0$, we have

$$\begin{aligned} \dot{e}_p(t) \ &=\ -\kappa_1 e_p - (\kappa_2 + \rho(t))\text{sgn}e_p + \big(F_i(z,\xi)\big)_p \\ &\ge\ -\kappa_1 e_p + \kappa_2 + \rho(t) - \|\big(F_i(z,\xi)\big)_p\| \\ &\ge\ -\kappa_1 e_p + \kappa_2 + \rho(t) - \|\big(F_i(z,\xi)\big)\| \\ &\ge\ -\kappa_1 e_p + \kappa_2. \end{aligned} \tag{16}$$

It can be seen from (15) and (16) that all $e_p$, $p = 1, 2, \cdots, m$, will arrive at zero in finite time interval and be kept here thereafter. Denote the time instant that all $e_p$ hit zero as $t_d$.

Now, we proceed to prove the second part. Firstly, we will prove that the state $\xi$ of the second part subsystem of system (1), i.e.

$$\dot{\xi} = A_\sigma \xi + B\big[G_\sigma(z,\xi)u_\sigma + F_\sigma(z,\xi)\big], \tag{17}$$

is globally bounded under switching laws satisfying the average dwell-time (13).

View $y_{nd} = [y_d^T,\ \dot{y}_d^T]^T$ as the new input for the closed-loop system (14), (17), let $P = Q^{-1}$, $K = NP$, it is easy to verify that (10) is equivalent to

$$PA_i + A_i^T P + PBK + K^T B^T P + \vartheta P + P^2 < 0. \tag{18}$$

Choose

$$V(\xi) = \xi^T P \xi \tag{19}$$

as the common Lyapunov function candidate for system (17). Where $P$ is the common solution $P$ of (18).

Then, based on (11) and (18), when $\sigma = i$, the derivative of $V(\xi)$ along the trajectory of (17) is

$$\begin{aligned} \dot{V} \ &=\ \xi^T (A_i^T P + PA_i)\xi + 2\xi^T PB[G_i(z,\xi)u_i + F_i(z,\xi)] \\ &=\ \xi^T (A_i^T P + PA_i + PBK + K^T B^T P)\xi \\ &\quad +2\xi^T C^T [G_i(z,\xi)u_i + F_i(z,\xi) - K\xi] \\ &\le\ -\vartheta \xi^T P \xi - \xi^T P^2 \xi + 2y^T [G_i(z,\xi)u_i \\ &\quad +F_i(z,\xi) - K\xi]. \end{aligned}$$

Let $\delta = \lambda_{\min}(P^2)$, then, from (14), we have

$$\dot{V} \leq -\vartheta V(\xi) - \delta\|\xi\|^2 + 2y^T[-(CB)^{-1}CA_i\xi - \kappa_1 e - (\kappa_2 + \rho(t))\text{sgn}e + (CB)^{-1}\dot{y}_d(t) + F_i(z,\xi) - K\xi].$$

Based on Assumption 3, $\|y_d\| \leq \|y_{nd}\|$, $\|\dot{y}_d\| \leq \|y_{nd}\|$ and $y = y_d$ when $t \geq t_d$, we can find positive constants $\varpi_1$, $\varpi_2$, $\varpi_3$, such that

$$\begin{aligned}\dot{V} \leq & -\vartheta V(\xi) - \delta\|\xi\|^2 + \varpi_1\|y_{nd}\|\|\xi\| + \varpi_2\|y_{nd}\|^2 \\ & +\varpi_3\|y_{nd}\| \\ \leq & -\vartheta V(\xi) - \delta\big(\|\xi\| - \frac{\varpi_1}{2\delta}\|y_{nd}\|\big)^2 + \big(\varpi_2 + \frac{\varpi_1^2}{4\delta}\big)\|y_{nd}\|^2 \\ & +\varpi_3\|y_{nd}\| \\ \leq & -\vartheta V(\xi) + \big(\varpi_2 + \frac{\varpi_1^2}{4\delta}\big)\|y_{nd}\|^2 + \varpi_3\|y_{nd}\|, \quad \forall t \geq t_d.\end{aligned}$$

Let $\chi(\|y_{nd}\|) = \big(\varpi_2 + \frac{\varpi_1^2}{4\delta}\big)\|y_{nd}\|^2 + \varpi_3\|y_{nd}\|$, we know that $V(\xi)$, $\chi(\|y_{nd}\|)$ are class $\mathcal{K}_\infty$ functions. It is easy to know that the closed-loop system (14), (17) is input-to-state stable with respect to the new input $y_{nd}$ when $t \geq t_d$ under arbitrary switching laws. So, under the switching laws satisfying the average dwell-time (13), it is also input-to-state stable with respect to the new input $y_{nd}$ when $t \geq t_d$. By virtue of Assumption 4, we can conclude that $y_{nd}$ is bounded. Thus, the global boundedness of $\xi$ under the switching laws satisfying the average dwell-time (13) when $t \geq t_d$ follows from the property of input-to-state stability.

When $t \leq t_d$, noticing that all the switched subsystems of closed-loop system (14), (17) are globally Lipschitz for a given bounded $y_{nd}$, therefore, the state $\xi$ has no finite escape time when $t \in [0,\ t_d]$ for arbitrary switched subsystems. So, when $t \in [0,\ t_d]$, the state $\xi$ has no finite escape time either under arbitrary switching laws satisfying the average dwell time (13) for the closed-loop system (14), (17) no matter which subsystem is activated firstly.

The above analysis shows that the state $\xi$ is globally bounded for the closed-loop switched subsystem (14), (17) under an arbitrary switching law satisfying the average dwell time (13).

From (7)$\sim$(9) and Lemma 1, it is easy to see that the $z$-part of system (1) is input-to-state stable with regard to $\xi$ under arbitrary switching laws satisfying the average-dwell time

$$\tau_{a_1} \geq \tau_{a_1}^* = \frac{\ln \mu_z}{\lambda_0}, \quad \lambda_0 = \min\{\lambda_{0i} \mid i = 1, 2, \cdots, N\}. \tag{20}$$

As the designed average dwell-time (13) is a special case of (20), it is easy to know that the $z$-part of system (1) is also input-to-state stable with regard to $\xi$ under arbitrary switching laws satisfying the average-dwell time (13). Similarly, from the input-to-state stable theory, we know that the state $z$ is also globally bounded under the designed switching law (13).

**Remark 2:** It is pointed out in the proof of Theorem 1 that the state $\xi$ has no finite escape time for arbitrary switched subsystems of system (1) when $t \leq t_d$. However, if switchings occurs during $0 \leq t \leq t_d$, the state $\xi$ of the switched system (1) would have finite escape time. So, we let $t_2 \geq t_d$ in order to guarantee the boundedness of the state $\xi$ of the switched system (1).

## IV. EXAMPLE

Consider the switched system:

$$\begin{cases} \dot{z} = f_\sigma(z,\xi), \\ \dot{\xi} = A_\sigma\xi + B\big[G_\sigma(z,\xi)u_\sigma + F_\sigma(z,\xi)\big], \\ y = C\xi, \end{cases} \tag{21}$$

where

$$f_1 = -z^5 - 2z + \xi_2,\ f_2 = -z^3 - z + \xi_1,\ G_1 = 6 + \xi_1^2,$$

$$G_2 = 2 + \xi_2^2,\ F_1 = 0,\ F_2 = 0.06\cos t,$$

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -4 & 0 \\ -2 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 4 \end{bmatrix},$$

Let $\vartheta = 1$. Solving the matrix inequality (10), we can obtain

$$P = \begin{pmatrix} 0.2291 & 0.6872 \\ 0.6872 & 2.7490 \end{pmatrix}, \quad N = [0,\ -13.205].$$

Then, from (11), we know that $C = [2.7490,\ 10.9959]$. Thus, we have $y = 2.7490\xi_1 + 10.9959\xi_2$.

Choosing $y_d = \cos 10t$, $\rho_1(t) = 0$, $\rho_2(t) = 0.06$, we can calculate that $e = -\xi_1 + \xi_2 + y_d$, $t_d = 1.23$.

For the $z$-part of system (21), select $G_1(z) = \frac{1}{2}z^2$, $G_2(z) = z^2$. Simple calculation shows that $\lambda_{01} = \lambda_{02} = 1.5$, $\gamma_{01}(\|\xi\|) = \xi_2^2$, $\gamma_{02}(\|\xi\|) = \xi_1^2$, $\mu_z = 2$. Choose $\lambda = 1 < \lambda_0 = 1.5$, the average-dwell time that the switching laws satisfied is

$$\tau_a \geq \tau_a^* = 0.69. \tag{22}$$

Design the switched variable structure controller for system (21) as

$$\begin{aligned} u_1 &= -\frac{1}{6 + \xi_1^2}[-0.125\xi_1 + 0.25\xi_2 + 0.0227e \\ &\quad +0.01\text{sgn}e + 0.227\sin 10t], \\ u_2 &= -\frac{1}{2 + \xi_2^2}[-0.75\xi_1 - 1.25\xi_2 + 0.0227e \\ &\quad +0.07\text{sgn}e + 0.227\sin 10t], \end{aligned}$$

where $\kappa_1 = 1$, $\kappa_2 = 0.01$.

Let the initial state be $[1,\ 0.8,\ 0]$ and $t_2 \geq 1.23$. Fig. 1, Fig. 2 and Fig. 3. show the simulation results using the proposed method. Fig. 1 and Fig. 2 indicate that the output of system (21) with the designed variable controller can track the desired signal $y_d$ reasonably and all the states of system (21) remain uniformly bounded under the designed average-dwell time based switching law. Fig. 3. shows that the length of the first switching time interval of the designed switching law is bigger than 1.23, which is in accordance with (13).

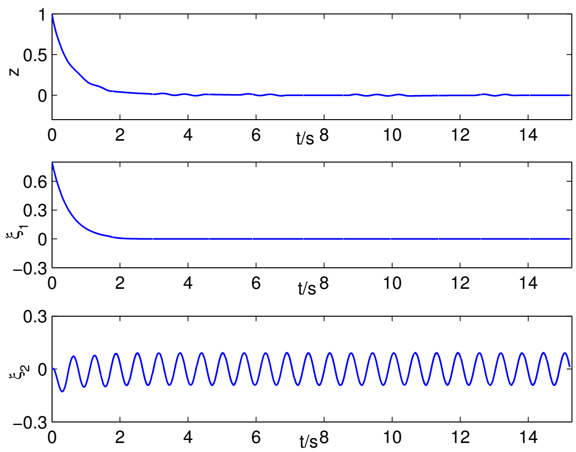


Fig.1. The state response of the closed–loop switched system.

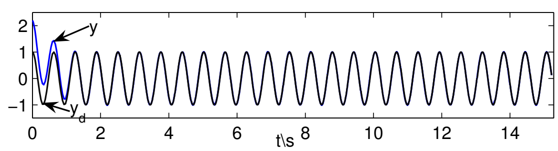


Fig. 2. The output of the system and the reference trajectory.

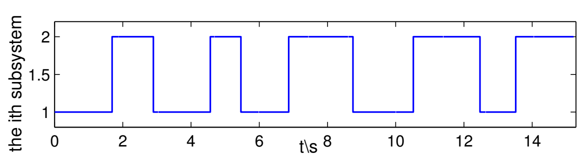


Fig. 3. The switching signal.

## V. CONCLUSIONS

In this paper, we have studied the robust output tracking control problem for a class of nonlinear switched cascade systems with external disturbances under some average-dwell time based switching laws. Based on the structural characteristic of the system, a sliding surface and a switched variable structure controller are designed. The states of the corresponding disturbed closed-loop system remain globally bounded and the output of the system can follow the desired signal exactly after a finite time interval under the designed switching law. An example is given out to demonstrate the design procedure of our approach. Simulation results show that the goal of output tracking can be achieved by the approach.

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