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Stabilization of Switched Nonlinear Systems Using Multiple Lyapunov Function Method

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Abstract— **The stabilization of a class of single input switched nonlinear systems is investigated in the paper. The systems concerned are of switched upper-triangular structure. The stabilization of the switched system under some switching law is investigated. Sufficient conditions are given under which the globally asymptotically stabilization problem is solvable. We exploit the structural characteristics of the switched nonlinear systems to construct the Lyapunov functions. The switching law and a nonlinear switched state feedback controller are explicitly designed. The relevant result for the linear switched system with the same structure is particularized.**

I. INTRODUCTION

A switched system is a dynamical system described by a family of continuous time subsystems and a rule that governs the switching between them. In recent years, the study of switched systems has received more and more attention. The motivation for studying switched systems comes partly form the fact that many practical systems are inherently multimodel in the sense that several dynamical systems are required to describe their behavior which may depend on various environmental factors, and from the fact that the methods of intelligent control design are based on the idea of switching between different controllers. For example, chemical processes, transportation systems, computer controlled systems and communication industries can be modeled as switched systems. Stability issues have been a major focus in studying switched systems. There has been increasing interest in the stability analysis and design methodology recently in the literature about switched systems [1-5]. Because of the interaction between continuous dynamics and discrete dynamics, switched systems may have very complicated behaviors. For example, switching between stable subsystems may lead to instability, whereas switching between unstable subsystems can give rise to stability. Stability under arbitrary switching law is a desirable property which can be assured by a common Lyapunov function [6, 7]. However, when the switched systems fail to have a common Lyapunov function, they still may be asymptotically stable under some properly chosen switching law. In this case, multiple Lyapunov function method and single Lyapunov function method are generally used [8-11]. Many other methods such as programming method [4], dwelled-time method [12], conic switching method [13] and so on are derived to discuss the stability of switched systems. Among these methods, the multiple Lyapunov function method is relatively more preferable. Since the switching law can be explicitly constructed when this method is employed, and the constructed switching law can either dependent on the whole or partial state of the switched systems.

Compared with the existing results on asymptotic stability of switched systems, little attention has been paid to the study of switched systems with continuous control variables. This is because more difficulties arise from the interaction between continuous control variables and discrete switching signal. [3] and [14] analyzed the stabilization of switched linear systems. [15] studied the stabilization of a class of cascaded switched nonlinear systems. [16] discussed the stabilization of switched nonlinear systems in compatible Byrnes-Isidori canonical form. Common Lyapunov functions were constructed in these papers to guarantee the stabilization of the switched systems they studied under arbitrary switching law. Multiple Lyapunov functions were used in [17, 18] to solve the stabilization problem of switched nonlinear systems with input constrains. The state feedback controller and output feedback controller and their corresponding switching laws were designed respectively in these two papers.

In this paper, we discuss the stabilization problem of a

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class of upper-triangular switched nonlinear systems with multiple Lyapunov function method. The switched system we studied consists of two parts, one of which is an autonomous switched system, while the other is a switched system with control input. Under the assumption that the autonomous part is uniformly globally quadratically stable, sufficient conditions are given, which guarantee the globally asymptotically stabilizbility of the switched system. A nonlinear switched state feedback and the switching law are constructed based on the structure characteristics of the switched system. And the switching law constructed only dependents on partial state of the switched system. Finally, a corollary is given to illustrate the applicability of the method for the linear switched system with the same structure.

The paper is organized as follows: Section 2 includes the description of the switched nonlinear system we study and the preparative definitions. Section 3 is our main result, and an example is worked out to illustrate the feasibility of our results in section 4. A brief conclusion is given in section 5.

Notation: R^+ is the set of nonnegative real numbers, R^n is an n-dimensional real vector space, $|\cdot|$ is the Euclidean vector norm.

II. PRELIMINARIES AND SYSTEM DESCRIPTION

Consider a switched nonlinear control system described as

$$\dot{\xi} = f_{\sigma(t)}(\xi) + g_{\sigma(t)}(\xi)u_{\sigma(t)}, \qquad u_{\sigma(t)} \in R, \tag{1}$$

where $\xi \in R^n$ is the state; $\sigma(t) : [0, +\infty) \rightarrow \underline{P} = \{1, \cdots, N\}$ is the switching signal which is assumed to be a piecewise right continuous function of time, implying that only a finite number of switches is allowed on any finite interval of time. The variable $\sigma(t)$, which takes values in the finite index set $\underline{P}$, is a discrete state that indexes the vector fields $f_i(\xi)$, $g_i(\xi)$, and the control input u_i, which altogether determine $\dot{\xi}(t)$. The vector fields $f_i(\xi)$, $g_i(\xi)$, $i = 1, \cdots, N$, are smooth, and $f_i(0) = 0$. We assume that the state of the switched system does not jump at the switching instants, i.e. the trajectory is everywhere continuous, is made all through the paper.

Define the distribution

$$G = \text{Span}\{g_1, \cdots, g_N\}.$$

The following proposition tells when system (1) can be transformed into a switched upp-triangular form under some geometry conditions [19].

Proposition 1: If there exists a nonsingular involutive distribution $\triangle$ of dimension d for all ξ in R^n, which is invariant under $f_i(\xi)$, $g_i(\xi)$, $\forall\ i \in \underline{P}$. In addition, $G \in \triangle$. Then, system (1) can be transformed into the following switched triangular form

$$\begin{cases} \dot{\xi}_1 = f_{1\sigma(t)}(\xi_1, \xi_2) + g_{1\sigma(t)}(\xi_1, \xi_2)u_{\sigma(t)}, \\ \dot{\xi}_2 = f_{2\sigma(t)}(\xi_2), \end{cases} \tag{2}$$

where $\xi_1 \in R^{n-d}$, $\xi_2 \in R^d$.

Remark 1: For the linear switched system

$$\dot{\xi} = A_{\sigma(t)}\xi + B_{\sigma(t)}u_{\sigma(t)}, \quad \xi \in R^n,\ u_{\sigma(t)} \in R. \tag{3}$$

if $\triangle$ is invariant under A_i and $\text{Im}\{B_i\} \subset \triangle$, for each $i \in \underline{P}$, then the linear switched system (3) can be transformed into

$$\begin{cases} \dot{\xi}_1 = A_{11\sigma(t)}\xi_1 + A_{12\sigma(t)}\xi_2 + B_{11\sigma(t)}u_{\sigma(t)}, \\ \dot{\xi}_2 = A_{22\sigma(t)}\xi_2, \end{cases} \tag{4}$$

As the coordinate transformation is a diffeomorphism, we will mainly investigate the globally stabilization problem for switched system (2).

III. MAIN RESULTS

This section gives the globally stabilization of switched system (2). The Lyapunov functions are recursively constructed, a nonlinear switched feedback controller and the switching law are also explicitly formulated simultaneously. We first write the functions $g_{1i}(\xi_1, \xi_2)$ in the form

$$g_{1i}(\xi_1, \xi_2) = g_{1i}(\xi_1, 0) + \hat{g}_{1i}(\xi_1, \xi_2)\xi_2, \qquad i \in \underline{P}. \tag{5}$$

For switched system (2), we make the following assumptions:

Assumption 1: There exist smooth, positive definite and radially unbounded functions $U_i(\xi_1)$, functions $\beta_{ij}(\xi_1) \leq 0$, $i, j = 1, \cdots, N$, such that

$$\frac{\partial U_i(\xi_1)}{\partial \xi_1} f_{1i}(\xi_1, 0) + \frac{1}{4\varepsilon_i^2}\left[\frac{\partial U_i(\xi_1)}{\partial \xi_1} g_{1i}(\xi_1, 0)\right]^2 + \gamma_i|\xi_1|^2 + \sum_{j=1}^{N} \beta_{ij}(\xi_1)(U_j(\xi_1) - U_i(\xi_1)) \leq 0, \tag{6}$$

$$\left|\frac{\partial U_i(\xi_1)}{\partial \xi_1}\right| \leq \alpha_i|\xi_1|, \quad \forall i \in \underline{P}, \quad \forall \xi_1 \in R^{n-d} \text{ and } \xi_1 \neq 0, \tag{7}$$

for some positive constants ε_i, γ_i, α_i.

Assumption 2: The vector fields $f_{1i}(\xi_1, \xi_2)$, $i \in \underline{P}$ are globally Lipschitz continuous. i.e. for each $i \in \underline{P}$, there exist $l_i > 0$, such that

$$|f_{1i}(\xi_1, \xi_2) - f_{1i}(\xi_1, 0)| \leq l_i|\xi_2|, \quad \forall \xi_1 \in R^{n-d},\ \xi_2 \in R^d. \tag{8}$$

Assumption 3: There exist a proper positive definite, and radially unbounded function $Q(\xi_2)$ such that

$$\frac{\partial Q(\xi_2)}{\partial \xi_2} f_{2i}(\xi_2) \leq -\gamma_0|\xi_2|^2, \quad \forall i \in \underline{P},\ \xi_2 \in R^d, \tag{9}$$

for some positive constant $\gamma_0 > 0$.

Theorem 1: Suppose that the switched nonlinear system (2) satisfies Assumption 1-3, then the switching law

$$\sigma(t) = \min\{i | i = \arg\max_{i\in \underline{P}}\{U_i\}\}, \tag{10}$$

and the nonlinear switched feedback controller

$$u_i = -\frac{1}{2\varepsilon_i^2}\frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g}_{1i}(\xi_1,\xi_2)\xi_2, \tag{11}$$

solve the globally asymptotically stabilization problem for switched system (2).

Proof: For system (2), we define the following Lyapunov function candidate

$$W(\xi) = W_{\sigma(t)}(\xi) = U_{\sigma(t)}(\xi_1) + k_{\sigma(t)}Q(\xi_2), \tag{12}$$

positive constants k_i, $i = 1, \cdots, N$, will be defined later.

When $\sigma(t) = i$, i.e. the ith subsystem is activated, from (6) we have

$$\frac{\partial U_i(\xi_1)}{\partial \xi_1}f_{1i}(\xi_1,0)+\frac{1}{4\varepsilon_i^2}\left[\frac{\partial U_i(\xi_1)}{\partial \xi_1}g_{1i}(\xi_1,0)\right]^2+\gamma_i|\xi_1|^2 \le 0, \tag{13}$$

and the time derivative of $W(\xi)$ along the trajectory of the switched system (2) is

$$\begin{aligned}
\dot{W}(\xi) &= \frac{\partial U_i(\xi_1)}{\partial \xi_1}f_{1i}(\xi_1,\xi_2) + \frac{\partial U_i(\xi_1)}{\partial \xi_1}g_{1i}(\xi_1,\xi_2)u_i \\
&\quad +k_i\frac{\partial Q(\xi_2)}{\partial \xi_2}f_{2i}(\xi_2) \\
&= \frac{\partial U_i(\xi_1)}{\partial \xi_1}f_{1i}(\xi_1,0) + \frac{\partial U_i(\xi_1)}{\partial \xi_1}\big(f_{1i}(\xi_1,\xi_2) \\
&\quad -f_{1i}(\xi_1,0)\big) + \frac{\partial U_i(\xi_1)}{\partial \xi_1}g_{1i}(\xi_1,0)u_i \\
&\quad +\frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g}_{1i}(\xi_1,\xi_2)\xi_2 u_i \\
&\le \frac{\partial U_i(\xi_1)}{\partial \xi_1}f_{1i}(\xi_1,0) + \left|\frac{\partial U_i(\xi_1)}{\partial \xi_1}\right| |f_{1i}(\xi_1,\xi_2) \\
&\quad -f_{1i}(\xi_1,0)| + \frac{1}{4\varepsilon_i^2}\left[\frac{\partial U_i(\xi_1)}{\partial \xi_1}g_{1i}(\xi_1,0)\right]^2 \\
&\quad +\varepsilon_i^2u_i^2 + \frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g}_{1i}(\xi_1,\xi_2)\xi_2 u_i \\
&\quad +k_i\frac{\partial Q(\xi_2)}{\partial \xi_2}f_{2i}(\xi_2).
\end{aligned}$$

From (13), Assumption 2 and Assumption 3, we obtain

$$\begin{aligned}
\dot{W}(\xi) &\le -\gamma_i|\xi_1|^2 + \alpha_i l_i|\xi_1||\xi_2| + \varepsilon_i^2u_i^2 \\
&\quad +\frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g}_{1i}(\xi_1,\xi_2)\xi_2 u_i - k_i\gamma_0|\xi_2|^2 \\
&\le -\gamma_i|\xi_1|^2 + \alpha_i l_i|\xi_1||\xi_2| - k_i\gamma_0|\xi_2|^2 \\
&\quad +\left[\varepsilon_i u_i + \frac{1}{2\varepsilon_i}\frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g_{1i}}(\xi_1,\xi_2)\xi_2\right]^2 \\
&\quad -\frac{1}{4\varepsilon_i^2}\left[\frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g_{1i}}(\xi_1,\xi_2)\xi_2\right]^2.
\end{aligned} \tag{14}$$

Substituting (11) into (14), then, we get

$$\begin{aligned}
\dot{W}(\xi) &\le -\gamma_i|\xi_1|^2 + \alpha_i l_i|\xi_1||\xi_2| - k_i\gamma_0|\xi_2|^2 \\
&\quad -\frac{1}{4\varepsilon_i^2}\left[\frac{\partial U_i(\xi_1)}{\partial \xi_1}\hat{g_{1i}}(\xi_1,\xi_2)\xi_2\right]^2 \\
&\le -\gamma_i|\xi_1|^2 + \alpha_i l_i|\xi_1||\xi_2| - k_i\gamma_0|\xi_2|^2 \\
&\le -\gamma_i|\xi_1|^2 + \frac{\gamma_i}{2}|\xi_1|^2 + \frac{\alpha_i^2 l_i^2}{2\gamma_i}|\xi_2|^2 - k_i\gamma_0|\xi_2|^2 \\
&\le -\frac{\gamma_i}{2}|\xi_1|^2 - \left(k_i\gamma_0 - \frac{\alpha_i^2 l_i^2}{2\gamma_i}\right)|\xi_2|^2 \\
&\le -\min\left(\frac{\gamma_i}{2},\ k_i\gamma_0 - \frac{\alpha_i^2 l_i^2}{2\gamma_i}\right)\left(|\xi_1|^2 + |\xi_2|^2\right).
\end{aligned}$$

For each $i \in \underline{P}$, choose $k_i > \frac{\alpha_i^2 l_i^2}{2\gamma_0\gamma_i}$, The globally asymptotically stabilization problem for switched system (2) under the switching law (10) follows.

Remark 2: Since the second part of the switched system (2) has a lower dimension, its Lyapunov function is relatively easier to find than that of the whole switched system. There are methods available for finding the common quadratic Lyapunov function for such switched systems [4, 6].

In the following, we examine the linear switched system (4) for which the result of theorem 1 above applies. First of all, the following assumptions are made

Assumption 4: For the linear switched system

$$\dot{\xi}_1(t) = A_{11\sigma(t)}\xi_1(t) + B_{11\sigma(t)}u_{\sigma(t)}, \tag{15}$$

there exist positive definite, quadratic functions $V_1(\xi_1) = \xi_1^T P_i\xi_1$, functions $\hat{\beta}_{ij}(\xi_1) \le 0$, $i, j = 1, \cdots, N$, such that

$$\begin{aligned}
&A_{11i}^T P_i + P_i A_{11i} + \frac{1}{4\hat{\varepsilon}_i^2}P_i B_{11i}B_{11i}^T P_i \\
&\quad + \sum_{j=1}^N \hat{\beta}_{ij}(\xi)(P_j - P_i) + \hat{\gamma}_i I \quad \le 0,
\end{aligned} \tag{16}$$

holds for some positive constants $\hat{\gamma}_i$, $i = 1, \cdots, N$.

Assumption 5: For the switched system $\dot{\xi}_2(t) = A_{22\sigma(t)}\xi_2(t)$, there exists positive definite, quadratic function $V_2(\xi_2) = \xi_2^T\hat{Q}\xi_2$, such that

$$A_{22i}^T P_i + P_i A_{22i} \le -\hat{\gamma}_0 I, \tag{17}$$

holds for some positive constant $\hat{\gamma}_0$.

Corollary 1: Suppose that the linear switched system (4) satisfies Assumption 4-5, then the switching law

$$\sigma(t) = \min\{i | i = \arg\max_{i\in \underline{P}}\{P_i\}\}, \tag{18}$$

and the linear switched feedback controller

$$u_i = -\frac{1}{8\hat{\varepsilon}_i^2}B_{11i}^T P_i\xi_1, \tag{19}$$

solve the globally asymptotically stabilization problem for switched system (4).

Remark 3 : As the Lyapunov functions for subsystems of the first part of system (2) in corollary 1 are in quadratic form, the condition (7) in theorem 1 are automatically satisfied.

IV. EXAMPE

Consider the nonlinear switched system with the following structure

$$\begin{cases} \dot{\xi} = f_{1\sigma(t)}(\xi, z) + g_{1\sigma(t)}(\xi, z)u_{\sigma(t)}, \\ \dot{z} = f_{2\sigma(t)}(z), \end{cases} \tag{20}$$

in which

$$f_{11}(\xi, z) = -\xi + z_1 \sin z_2, \quad g_{11}(\xi, z) = \xi z_1,$$

$$f_{12}(\xi, z) = -4\xi\sqrt{1+4\xi^2} + z_2 \sin z_1, \quad g_{12}(\xi, z) = \xi + z_1 z_2,$$

$$f_{22}(z) = \begin{pmatrix} -7z_1\sqrt{1+z_1^2}(1+\sin^2 z_2) \\ -5z_2 \end{pmatrix},$$

$$f_{21}(z) = \begin{pmatrix} -4z_1\sqrt{1+z_1^2} \\ -z_2(1+z_1^2) \end{pmatrix}, \quad \sigma(t) = \{1, 2\}.$$

we can get

$$f_{11}(\xi, 0) = -\xi,\ g_{11}(\xi, 0) = 0,\ \hat{g}_{11}(\xi, z) = [\xi, 0],$$

$$f_{12}(\xi, 0) = -4\xi\sqrt{1+4\xi^2},\ g_{12}(\xi, 0) = \xi,$$

$$\hat{g}_{12}(\xi, z) = [\frac{1}{2}z_2, \frac{1}{2}z_1].$$

Choosing $U_1(\xi) = 2\xi^2$, $U_2(\xi) = \frac{1}{2}\sqrt{1+4\xi^2}$, $Q(z) = \sqrt{1+z_1^2} + z_2^2$, $\varepsilon_1 = \varepsilon_2 = 1$, $\gamma_1 = 1$, $\gamma_2 = \frac{7}{4}$, $\beta_{12}(\xi) = -1$, $\beta_{21}(\xi) = -\frac{1}{2}\xi^2(1+4\xi^2)^{-\frac{1}{2}}$, we can calculate that

$$\begin{aligned} & \frac{\partial U_1(\xi_1)}{\partial \xi_1} f_{11}(\xi_1, 0) + \frac{1}{4}\left[\frac{\partial U_1(\xi_1)}{\partial \xi_1} g_{11}(\xi_1, 0)\right]^2 \\ & +\gamma_1|\xi_1|^2 + \beta_{12}(\xi_1)(U_2(\xi_1) - U_1(\xi_1)) \\ \leq\ & -\xi^2 - \frac{1}{2}\sqrt{1+4\xi^2} \leq 0, \\ & \frac{\partial U_2(\xi_1)}{\partial \xi_1} f_{12}(\xi_1, 0) + \frac{1}{4}\left[\frac{\partial U_2(\xi_1)}{\partial \xi_1} g_{12}(\xi_1, 0)\right]^2 \\ & +\gamma_2|\xi_1|^2 + \beta_{21}(\xi_1)(U_1(\xi_1) - U_2(\xi_1)) \\ \leq\ & -6\xi^2 - \frac{\xi^4(\sqrt{1+4\xi^2} - 1)}{(1+4\xi^2)} \leq 0, \end{aligned}$$

$$\left|\frac{\partial U_1(\xi)}{\partial \xi}\right| = 4\xi, \quad \left|\frac{\partial U_2(\xi)}{\partial \xi}\right| = \frac{2\xi}{\sqrt{1+4\xi^2}} \leq 2\xi,$$

$$|f_{11}(\xi, z) - f_{11}(\xi, 0)| \leq |z_1|, \quad |f_{12}(\xi, z) - f_{12}(\xi, 0)| \leq |z_2|,$$

$$\frac{\partial Q(z)}{\partial z} f_{21}(z) = -4z_1^2 - 2z_2^2(1+z_1^2) \leq -z_1^2 - z_2^2,$$

$$\frac{\partial Q(z)}{\partial z} f_{22}(z) = -7z_1^2(1+\sin^2 z_2) - 10z_2^2 \leq -z_1^2 - z_2^2.$$

Let $\alpha_1 = 5$, $\alpha_2 = 2$, $l_1 = l_2 = 1$, γ_0=1, we can see that Assumption 1, Assumption 2, and Assumption 3 are satisfied.

Choose $k_1 = 13$, $k_2 = 2$, the lyapunov function for system (20) is

$$W(\xi, z) = \begin{cases} \xi^2 + 13\left(\sqrt{1+z_1^2} + z_2^2\right), & \text{if } \sigma(t) = 1, \\ \sqrt{1+\xi^2} + 2\left(\sqrt{1+z_1^2} + z_2^2\right), & \text{if } \sigma(t) = 2, \end{cases}$$

Construct the switching as

$$\sigma(t) = \begin{cases} 1, & \text{if } U_1(\xi) \geq U_2(\xi), \\ 2, & \text{otherwise}, \end{cases} \tag{21}$$

According to (11) we can obtain the switched state feedback controller as,

$$u_i = \begin{cases} -2\xi^2 z_1, & i = 1, \\ -2\xi z_1 z_2(1+4\xi^2)^{-\frac{1}{2}} & i = 2, \end{cases} \tag{22}$$

Let the initial state $(\xi_0^T, z_0^T)^T = [3, -2, 3]^T$, Figure 1 shows the the state response of the closed-loop switched system (20) with the designed state feedback (22) under the constructed switching law (21), which indicate that the feasibility of our result.

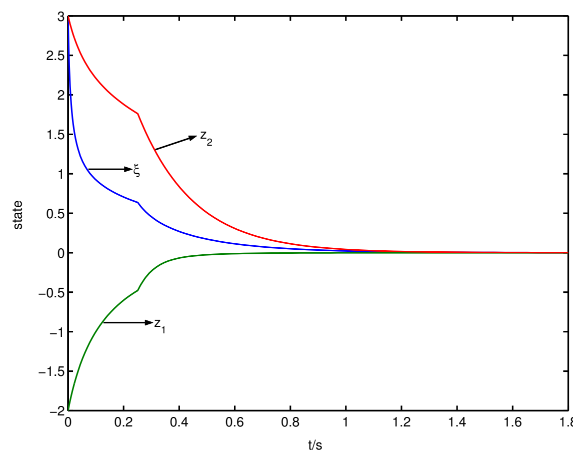

Fig. 1. The state response of the switched system (37)

V. CONCLUSIONS

The paper has considered the globally stabilization of a class of switched nonlinear systems with triangular structure under some switching law. Using the multiple Lyapunov functions of the first part with single control input and the common Lyapunov function of the second part without control input the piecewise continuous Lyapunov functions for the whole switched system are constructed. The partial state dependent switching law and the switched state feedback controller for the switched system are also explicitly formulated. A numerical example has been given out to illustrate the feasibility of our methods. The globally stabilization of this kind of switched systems under other conditions or using other methods remains for further study. Other problems of switched systems with the same structure can be considered, such as robust control, tracking control problems.

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