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Relative GPS Carrier-Phase Positioning Using Particle Filters with Position Samples

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Abstract— **In this paper, the problem of precise relative positioning using GPS (Global Positioning System) Carrier-Phase (CP) information is addressed. The unknown cycle ambiguity between GPS satellites and antennas at the moment of receiving the CP signal should be resolved for precise navigation. The sequential Monte Carlo filter approach, called particle filter (PF), is applied to the relative positioning problem which includes the ambiguity resolution problem for the CP nonlinear observation and dynamic equations. The proposed algorithm of GPS CP navigation is based on two main factors. First, even though most existing GPS CP navigation algorithms focus on obtaining the correct integer value among the integer candidates, we directly sample from the three dimensional position space and construct integers consistent with the PF. This allows the PF position estimates to be insensitive to changes of GPS satellites and cycle-slips. Second, the potential large number of samples in position space is handled with the resampling technique in the sequential particle filters. The experimental results show the performance and the advantages of the proposed approach compared to the existing methods.**

I. Introduction

Nowadays, precise relative position estimation based on GPS CP is used widely. We concentrate on the relative position estimation problem where the relative position vector (or baseline vector) is determined with given GPS code and CP measurements. In order to make use of CP in the relative position estimation problem, the integer number of wave lengths between antennas, called the integer ambiguity problem, should be resolved. Many algorithms have been introduced to solve this integer ambiguity resolution problem. These algorithms can be classified into three different methods [1]: 1) direct estimation in measurement space, 2) searching for the correct integer ambiguity among many integer hypotheses, 3) searching for the correct position among many position hypotheses.

The first method is straightforward and simple. The ambiguity is estimated directly as a bias by comparing the code measurements to the CP measurements [2]. Since the code information is not accurate enough compared to the CP measurements, direct estimation of the ambiguity from the code can cause inaccurate position estimation.

Most ambiguity resolution techniques are classified by the second method. These algorithms are based on search procedures that can distinguish a correct integer ambiguity hypothesis from all other integer hypotheses. First, the real-valued ambiguity, called the float ambiguity, is estimated, and the search space of the integer ambiguity hypotheses is constructed from integer values near the float value. The volume of the integer ambiguity searching space is usually limited by the uncertainty of measurements. Then, the most likely correct integer ambiguity hypothesis is selected. Typical methods, such as the Least Squares Ambiguity Search Technique (LSAST) [3], the Least-squares AMBiguity Decorrelation Approach (LAMBDA) [4] and recently the MHWSPT (Multiple Hypotheses Wald Sequential Probability Test) [5], have been used successfully to solve the ambiguity resolution problem. However, the hypothesis technique over the ambiguity candidates should be completed before any change in the available GPS satellites, since these algorithms have difficulty when the set of GPS satellites change. Furthermore, after the integer ambiguity is found, the system will lose accuracy when a cycle-slip can occur. Methods are required to detect cycle-slip and resolve the ambiguities again.

The Ambiguity Function Method (AFM) is a representative example of the third integer resolution method [6]. The AFM selects its hypotheses in three dimensional position space, instead of the ambiguity space. This particular aspect makes the search process insensitive to changes in GPS satellites and to cycle-slips. However, the AFM usually needs a much larger number of hypotheses than the ambiguity-search-based methods, and this disadvantages make this technique unpopular.

Note that some methods use the float ambiguity estimate directly without any effort being made to resolve the integer ambiguity problem [7]. It is shown that the integer constraint results in better precision than float ambiguity estimates when the time span of the CP observation is short [8]. The proposed algorithm is similar to the float ambiguity estimate method in that an effort to search or to fix the integer ambiguity is not made. However, the proposed algorithm can preserve the precision of the position accuracy compared to the position estimates of the fixed integer ambiguity methods since the integer property of the ambiguity is taken into account during the ambiguity resolving process.

In this paper, we propose an algorithm for the integer ambiguity resolution problem that overcomes the main disadvantages of the existing methods: 1) the dependency of the second method to changes in GPS satellites and 2) the large number of candidates required in the third method. As an alternative method for the integer ambiguity resolution

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problem, the particle filter (PF) with the samples drawn from the position space is proposed. The potential difficulty is the large number of position samples which may produce a heavy computational burden for real-time processing. To circumvent this difficulty, the "resampling" process [9] is employed which keeps only the more-likely samples, and eliminates the less-likely samples. As a result of sampling from a position space and using the PF with resampling, the proposed GPS CP positioning algorithm is shown to be insensitive to changes in GPS satellites or cycle-slips.

The paper is organized as followings. Section II reviews particle filtering. In Section III, we review the relative position determination problem using GPS code and CP information, and application of particle filters to the relative positioning problem where the particle samples are obtained in position space. The experimental results in Section IV show the performance of the proposed technique for the precise relative GPS CP position determination of satellites in Low Earth Orbit (LEO).

II. Sequential Importance Sampling (SIS) Particle Filter

To define the estimation problem, the state-space dynamics and observation equations are,

$$\mathrm{x}_k = \mathrm{f}(\mathrm{x}_{k-1}) + \mathrm{w}_{k-1}, \quad \mathrm{z}_k = \mathrm{h}(\mathrm{x}_k) + \mathrm{v}_k \tag{1}$$

where k represents time index, $\mathrm{f}(\cdot)$, $\mathrm{h}(\cdot)$ are assumed to be known nonlinear dynamic and measurement functions, and w_k and v_k are associated noises. We use $\mathrm{x_k}$ to represent both the random variable and its realization.

For nonlinear functions and non-Gaussian noise processes, an analytic calculation of the conditional density function is impractical except for simple cases [10]. The Monte Carlo method for non-linear estimation is based on the theory of recursive Bayesian filters. The goal in the Bayesian filters is to approximate the conditional posterior distribution of the states at current time k, x_k, recursively with a given measurement history up to time k, $\mathrm{z}_{1:k} = \{\mathrm{z}_1, \mathrm{z}_2, \cdots, \mathrm{z}_k\}$.

The approximation of the joint filtering density by the SIS PF at time $k-1$, is given as [11]

$$p(\mathrm{x}_{k-1}|\mathrm{z}_{1:k-1}) \approx \sum_{i=1}^{m} \bar{w}_{k-1}^i \delta(\mathrm{x}_{k-1} - \mathrm{x}_{k-1}^i) \tag{2}$$

where $\bar{w}_{k-1}^i$ is the normalized weight, and x_{k-1}^i are drawn from the known or chosen density $q(\mathrm{x}_{k-1}|\mathrm{z}_{1:k-1})$, the so-called the importance density. The normalized weight at time $k-1$ is

$$\bar{w}_{k-1}^i \propto \frac{p(\mathrm{x}_{k-1}^i|\mathrm{z}_{1:k-1})}{q(\mathrm{x}_{k-1}^i|\mathrm{z}_{1:k-1})}. \tag{3}$$

For a simple implementation [11], the importance density is chosen as $q(\mathrm{x}_k|\mathrm{x}_{k-1}^i, \mathrm{z}_{1:k}) \triangleq p(\mathrm{x}_k|\mathrm{x}_{k-1}^i)$, the sequential update equation of normalized weight can be achieved as

$$\bar{w}_k(\mathrm{x}_k^i) = \frac{p\left(\mathrm{z}_k|\mathrm{x}_k^i\right)\bar{w}_{k-1}^i}{\sum\limits_{i=1}^{m} p\left(\mathrm{z}_k|\mathrm{x}_k^i\right)\bar{w}_{k-1}^i} \tag{4}$$

Finally, the expectation of the bounded and continuous function $\mathrm{g}(\cdot)$ with the sequentially updated weights in (4) can be obtained as $\mathbf{E}_q\left[\mathrm{g}(\mathrm{x}_k)\right] = \sum\limits_{i=1}^{m} \mathrm{g}(\mathrm{x}_k^i)\bar{w}_k^i$ with the updated weights for the samples drawn from the importance density, the algorithm can be implemented in real-time.

The disadvantage of the sequential importance sampling method is that the variance of weights increases as time k increases no matter what importance density is chosen. The weight of the highly weighted particles have a tendency to become larger and larger as time k increases, while that of the low weighted particles become smaller. Therefore, the weights of most samples become ignorable and the weight of only one sample becomes dominant. This side effect, the degeneracy problem, can be handled by a resampling technique.

A. Resampling

The degeneracy problem makes it difficult to approximate the filtering density as time k increases. As a remedy to this problem, resampling techniques such as multinomial sampling, residual resampling, systematic sampling, etc., have been introduced [12].

We will consider multinomial resampling in this paper, since the multinomial distribution sampling is one of the simplest and most fundamental resampling techniques. The other versions of the resampling methods have somewhat similar performance compared to the multinomial sampling in practice. The idea of the multinomial resampling step is simply to eliminate the low-weight samples, and then, to select and copy the high-weight samples [13]. To be precise, the sampling step involves selecting $\tilde{m}$ number of new sample sets, $\{\tilde{\mathrm{x}}_k^j; j = 1, \cdots, \tilde{m}\}$ from the existing samples $\{\mathrm{x}_k^i, \bar{w}_k^i; i = 1, \cdots, m\}$. The probability of being selected is proportional to the weight $\bar{w}_k^i$. In doing so, the high weight particles are selected after resampling, and all selected particles are assigned with uniform weights (i.e. $1/\tilde{m}$). A subset of samples usually decrease. Another positive effect of the resampling step is that by eliminating the low weight particles and keeping the only high weight of the samples, i.e. by keeping $\tilde{m} \leq m$ number of samples, the number of samples can be decreased through the resampling step. Therefore, the number of samples can be adjusted for real-time implementation without an excessive computational burden.

III. Relative Positioning using Particle Filtering with Position Space Samples

A. GPS Measurements

Most GPS receivers provide two types of measurements, a pseudo-range that is related to the measured travel time, the Code measurement, and the Carrier-Phase (CP).

The code measurement contains the range between the receiver and the i'th GPS satellites and several error factors

$$\begin{aligned} p_{(1)}^{(i)} =& \rho_{(1)}^{(i)} + \delta S^{(i)} + c(t_{(1)} - T^{(i)}) + I_{(1)}^{(i)} \\ &+ Tr_{(1)}^{(i)} + Mp_{(1)}^{(i)} + n_{(1)}^{(i)} \end{aligned} \tag{5}$$

where $p_{(1)}^{(i)}$ is the measured code range, $\rho_{(1)}^{(i)}$ is the geometric range, $\delta S^{(i)}$ is the ephemeris error, c is the velocity of light, $t_{(1)}$ is the receiver clock error, $T^{(i)}$ is the GPS satellite clock error, $I_{(1)}^{(i)}$ is the ionospheric delay error, $Tr_{(1)}^{(i)}$ is the tropospheric delay error, $Mp_{(1)}^{(i)}$ is the code multipath error, $n_{(1)}^{(i)}$ is the code noise error. In term $(\cdot)_{(1)}^{(i)}$, the subscript $_{(1)}$ represents a parameter about vehicle 1 and the superscript $^{(i)}$ represents a parameter about GPS satellite i. Similarly, the CP measurements can be represented with the error components

$$\begin{aligned}\phi_{(1)}^{(i)} =& \rho_{(1)}^{(i)} - \lambda N_{(1)}^{(i)} + \delta S^{(i)} + c(t_{(1)} - T^{(i)}) \\ & - I_{(1)}^{(i)} + Tr_{(1)}^{(i)} + mp_{(1)}^{(i)} + \eta_{(1)}^{(i)}\end{aligned} \tag{6}$$

where $\phi_{(1)}^{(i)}$ is the measured CP range, λ is the CP wave length, $N_{(1)}^{(i)}$ is the integer cycle ambiguity (cycle), $mp_{(1)}^{(i)}$ is the CP multipath error, and $\eta_{(1)}^{(i)}$ is the CP noise.

The common errors in measurements are assumed to be canceled out by differencing between the measurement equations, but uncommon errors, such as noise and multipath error, can not be canceled out. The noise errors of the code and CP are assumed to be Gaussian with zero mean. We consider two cases where the Root Mean Square (RMS) errors of the code $n_{(1)}^{(i)}$ and CP noise $\eta_{(1)}^{(i)}$ are 3 m for the code, 3 cm for the CP, and 30 cm for the code, 0.3 cm for CP, respectively. The first case represents the RMS noise error for the usual GPS RTK (Real time Kinematics) receivers, and the second case represents a more accurate GPS receiver designed for specific missions such as the precise orbit determination of CHAMP (Challenging Minisatellite Payload), and GRACE (Gravity Recovery and Climate Experiment).

The multipath error has a major effect on degrading the accuracy in CP GPS position estimation and on the success of finding the correct integer ambiguity from the integer ambiguity search algorithms. The multipath error is usually modeled as the sinusoidal oscillation with zero mean for a stationary receiver. If the geometry between GPS satellites and reflector does not change quickly, the multipath error appears as a bias over the short term. In order to simulate the effect of the multipath error on the PF estimates, we introduce a bias to represent the multipath error (i.e. less than 2 cm) in CP measurements used in the experiments described in Section IV.

B. Differential Measurement

In short baseline case, many of errors such as $\delta S^{(i)}$, $c(t_{(1)} - T^{(i)})$, $I_{(1)}^{(i)}$, $Tr_{(1)}^{(i)}$, in the observation in (5) and (6) are considered as a common error for both antennas and assumed to be eliminated or reduced by Double Difference (DD) operation. The DD measurements for the code, and CP between antenna 1, 2 and GPS satellites i, j are given as

$$\begin{aligned}\nabla\Delta p_{(12)}^{(ij)} &\triangleq (p_{(2)}^{(j)} - p_{(1)}^{(j)}) - (p_{(2)}^{(i)} - p_{(1)}^{(i)}) \\ &\approx \nabla\Delta\rho_{(12)}^{(ij)} + \nabla\Delta Mp_{(12)}^{(ij)} + \nabla\Delta n_{(12)}^{(ij)}\end{aligned} \tag{7}$$

$$\nabla\Delta\phi_{(12)}^{(ij)} \approx \nabla\Delta\rho_{(12)}^{(ij)} - \nabla\Delta\lambda N_{(12)}^{(ij)} + \nabla\Delta mp_{(12)}^{(ij)} + \nabla\Delta\eta_{(12)}^{(ij)} \tag{8}$$

The measurements equation in the vector form for all visible GPS satellites i and j in common can be rearranged as

$$\begin{aligned}\nabla\Delta P &= \left[\begin{array}{cccc} \nabla\Delta p_{(12)}^{(12)} & \nabla\Delta p_{(12)}^{(23)} & \cdots & \nabla\Delta p_{(12)}^{(n_s\, n_s-1)} \end{array} \right]^T, \\ \nabla\Delta\Phi &= \left[\begin{array}{cccc} \nabla\Delta \phi_{(12)}^{(12)} & \nabla\Delta \phi_{(12)}^{(23)} & \cdots & \nabla\Delta \phi_{(12)}^{(n_s\, n_s-1)} \end{array} \right]^T\end{aligned} \tag{9}$$

where n_s is a number of common visible GPS satellites.

Equation (8) contains the DD ambiguity term, $\nabla\Delta N_{(12)}^{(ij)}$. After the DD ambiguity is known, (8) can be used as much more precise measurements than the code. However, this unknown integer ambiguity can not be computed directly in this form since the term $\nabla\Delta\rho_{(12)}^{(ij)}$ is not known either. Therefore, the hypothesis tests over the ambiguity hypotheses are usually employed.

From this point, we only use $\nabla\Delta$ symbols as a DD value and omit $(\cdot)_{(12)}^{(ij)}$ symbol, and readers can assume that measurements are DD values in the sequel.

C. Initialization

The estimate of the absolute position of receiver 1, X_1, can be obtained by differencing between the code measurements vector and the geometric range vector from a linearization point X_{1o} to all GPS satellites position

$$P_{(1)} - \varrho_{(1o)} = \mathbf{A}\delta\mathrm{X}_1 + \mathbf{e} \tag{10}$$

where

$$\varrho_{(10)} = \left[\begin{array}{cccc} \rho_{(10)}^{(1)} & \rho_{(10)}^{(2)} & \cdots & \rho_{(10)}^{(n_s)} \end{array} \right]^T, \mathbf{A} = \left[\begin{array}{cc} H_1^T & 1 \end{array} \right]$$

$$P_{(1)} = \left[p_{(1)}^{(1)}, p_{(1)}^{(2)}, \cdots, p_{(1)}^{(n_s)} \right]^T, \delta\tilde{X}_1 \triangleq \left[\begin{array}{cc} \delta\mathrm{X}_1 & c\cdot t_{(1)} \end{array} \right]^T,$$

$\delta\mathrm{X}_1$ is the difference between a linearization point X_{1o} and X_1. $\rho_{(10)}^{(i)}$ is an geometric range from X_{1o} to i^{th} GPS satellites. $H_1^{(1)(n_s)} = \frac{\partial\varrho_{1o}}{\partial\mathrm{X}}\Big|_{\mathrm{X}_{1o}}$ is the direction matrix to all GPS satellites at X_{1o}. $\mathbf{e}$ is the vector of error terms including the common errors, multipath, and noise. The estimation of $\delta\tilde{\mathrm{X}}_1$ can be obtained by the least square method as

$$\left[\begin{array}{cc} \delta\hat{\mathrm{X}}_1 & \hat{t}_{(1)} \end{array} \right]^T = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T \cdot (P_{(1)} - \varrho_{(1o)}) \tag{11}$$

For short baseline, the estimation, $\hat{\mathrm{X}}_1 = \mathrm{X}_{1o} + \delta\hat{\mathrm{X}}_1$, will be taken as the true position of the receiver 1, X_1, and the relative positioning problem comes down to estimating the relative position between $\hat{\mathrm{X}}_2$ and X_1. The initial estimation of $\hat{\mathrm{X}}_2$ is required as an initial condition for the Particle Filters

(PF). The DD code information can be used for the initial value of $\hat{X}_2$.

$$\nabla\Delta P - \nabla\Delta\varrho_{2o} = \mathbf{A}_d \delta X_2 + \nabla\Delta \mathbf{n}_d$$
$$\mathbf{A}_d = \left[H_2^{(2)(n_s)} - H_2^{(1)(n_s-1)}\right]^T$$

where δX_2 is a relative position between X_2 and X_{2o}. $H_2^{(2)(n_s)}$ is the direction matrix at the linearized point X_{2o} to the GPS satellites i, $i = 2, \cdots, n_s$ and $H_2^{(1)(n_s-1)}$ is the direction matrix at the linearized point X_{2o} to the GPS satellites j, $j = 1, \cdots, n_s - 1$. $\nabla\Delta\mathbf{n}_d$ is the double differenced code noise vector. X_{2o} is an arbitrary linearization point for X_2. $\nabla\Delta\varrho_{2o}$ is the DD vector of the geometric ranges between X_1, X_{2o} and GPS satellites. Then, the estimation of position of receiver 2 by DD Code is given by $\hat{X}_2 = X_{2o} + \delta\hat{X}_2$ where

$$\delta\hat{X}_2 = (\mathbf{A}_d^T \mathbf{A}_d)^{-1} \mathbf{A}_d^T \cdot (\nabla\Delta P - \nabla\Delta\varrho_{2o}) \qquad (12)$$

D. Sampling Particles in Position Space

We denote the sampled particles from the initial pdf of states at time $k = 0$ as $x_0^i \sim p(x_0|z_0), i = 1, \cdots, m$. The superscript i represents i^{th} sample. It is assumed that $p(x_0|z_0) = p(x_0)$. Each sample x_k^i represents a realization of X_2, the position of vehicle 2 in the ECEF coordinates. Therefore, the samples $x_k^i, i = 1, \cdots, m$ can be obtained by picking m number of independent points in three dimensional position space. The distribution of samples is assumed to be a Gaussian, or uniform where the mean and variance are estimated by the code method given in Section III-C.

Building the likelihood pdf for the states, $p(z_k|x_k^i)$ is the essence of the proposed algorithm. In order to develop the algorithm of the PF using the position space samples, consider the DD CP measurements

$$\nabla\Delta\varrho = \nabla\Delta\Phi + \lambda \cdot \nabla\Delta N + \nabla\Delta\eta \qquad (13)$$

where $\nabla\Delta\varrho$ is the vector of the DD geometric range between satellites and X_1, X_2. Equation (13) can be linearized around $\hat{X}_2$ by using estimates in (12)

$$\nabla\Delta\varrho_o + \mathrm{A_d}\delta X_2 = \nabla\Delta\Phi + \lambda \cdot \nabla\Delta N + \nabla\Delta\eta \qquad (14)$$

We use both of nonlinear measurements in (13) and linearized measurement in (14). The given position particles will be applied to the nonlinear equation. First, we make an estimate of the integer ambiguity, $\nabla\Delta N$. Consider the left null space of the direction matrix $\mathrm{A_d}$, $E = null(\mathrm{A_d^T})$ such that $E^T \cdot \mathrm{A_d} = 0$. This annihilator of $\mathrm{A_d}$ is not unique, but exists when the number of visible satellites is greater than five. Multiplying E^{T} on both sides of (14) and rearranging becomes

$$E^{\mathrm{T}} \cdot \nabla\Delta N_L \triangleq E^{\mathrm{T}}(\nabla\Delta\varrho_0 - \nabla\Delta\Phi - \nabla\Delta\eta)/\lambda \qquad (15)$$

Define N_L as an ambiguity from the linearized measurements from (15). From this point, we will omit the double differencing operation symbol $\nabla\Delta$ for simplicity. Since the position dependent term is eliminated, (15) is independent of the position samples. The ambiguity N_L is not integer valued because of η.

If we could find any integer ambiguity that when projected by the left annihilator E^{T} is close to $E^{\mathrm{T}} \cdot N_L$, then the probability of being true integer ambiguity is high. From (13), the integer ambiguity is computable if ϱ is given. In fact, each sample x^i can be substitute for X_2, and therefore, basically m samples generate m number of ϱ^i in (13).

The float ambiguity estimates for each position samples x_k^i are computed from (13) as $\tilde{N}^i \triangleq \frac{1}{\lambda}(\varrho^i - \Phi)$ where ϱ^i is the DD range between X_1 and each particles $x_k^i, i = 1, \cdots, m$. When the integer property of the ambiguity is applied, the expected estimates of the integer ambiguities corresponding to each sample can be obtained by rounding-off as

$$\bar{N}^i \triangleq \left[\tilde{N}^i\right]_{\mathrm{round-off}} \qquad (16)$$

These integer valued ambiguities are compared with the ambiguity estimate from the linearized equation in (15). By the round-off operation, the integer ambiguity estimate $\bar{N}^i$ for i^{th} sample is independent of the noise under the assumption that the summation of the DD noise η and a portion of common error assumed to be eliminated in the DD measurements is less than $\lambda/2$ ($\approx$ 19/2 cm : L_1 frequency CP). Since the common errors are assumed to be ignorable in the short baseline case, this assumption does hold as long as the summation of the noise and the multipath does not exceed $\lambda/2$ and each sample position provides an integer ambiguity, but only one of the integer ambiguities is true integer ambiguity, since the samples far from true position provide a wrong integer value. However, it is not trivial to check the magnitude of the noise and multipath in the DD measurements. We take care of the effect of the noise and multipath by simulating it in the experimental data.

If the noise in the measurements is modeled as a Gaussian, the pdf $p(z_k|x_k^i)$ can be modeled as

$$p(z_k|x_k^i) = C_1 \exp\{-\frac{1}{2}(r^i)^T {Q_E}^{-1}(r^i)\}, \qquad (17)$$

where the residual $r^i = \lambda E^{\mathrm{T}}(\bar{N}^i - N_L)$, and $Q_E = E^{\mathrm{T}} Q E$, Q is the covariance of the estimated ambiguity.

Using the structure of likelihood density in (17), the proposed PF algorithm is able to estimate the position without knowing the true integer ambiguity. In fact, the ambiguity N_L from (15) is given as the true ambiguity, and is computed based on the CP measurements of each epoch. Therefore, the particle filters in this paper keeps evaluating N_L internally at every epoch, instead of using or testing the pre-computed ambiguity hypotheses. In this way, the algorithm is able to deal with cycle-slips, and the change of ambiguities caused by the addition or loss of visible GPS satellites.

As we have remarked, the number of samples are an important factor, since a small number of samples can degrade the accuracy. Consider the samples x^i, x^j for $i \neq j$ and the corresponding float and integer ambiguities in (16), $\tilde{N}^i, \tilde{N}^j$ and $\bar{N}^i, \bar{N}^j$ respectively. If two samples are close to each other in distance, $|x^i - x^j| \ll 1$, then both samples generate the same $\bar{N}^i$. Indeed, multiple number of samples which are close to each other provide the same integer value of $\bar{N}^i$, as long as the associated $\tilde{N}^i$ satisfies the condition $\bar{N}^i - 0.5 \leqslant \left|\tilde{N}^i\right|_{round-off} < \bar{N}^i + 0.5$. Since N_L and $\bar{N}^i$ are same in the residual computation for those nearby samples, the samples within a certain bound in position space have the same value of $p(\mathrm{z}_k|\mathrm{x}_k^i)$ in (17). Therefore, when the number of samples m is small, and the samples are concentrated together, these samples may not accurately approximate $p(\mathrm{z}_k|\mathrm{x}_k)$. Thus, if the number of samples is too small, then approximation of the pdf maybe come inaccurate. A pictorial description of a one dimensional example is shown in Fig. 1. The pdf $p(\mathrm{z}_k|\mathrm{x}_k^i)$ is given as a Gaussian

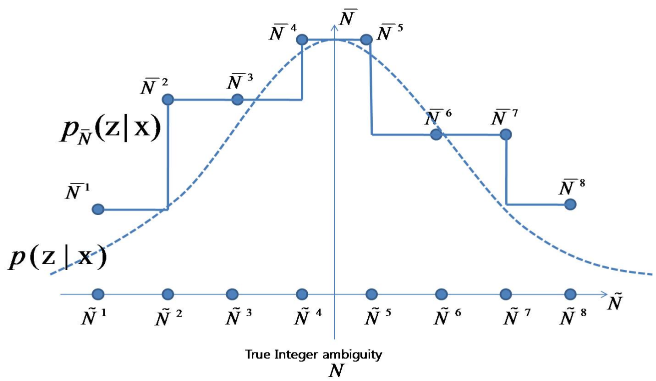

Fig. 1. $\tilde{N}^i$ and corresponding $\bar{N}^i$

distribution consistent with a Gaussian noise assumption. The density function is approximated by the discrete type of density, $p_{\bar{N}}(\mathrm{z}|\mathrm{x})$.

By substituting (17) into (4), the update of weight can be expressed as

$$\bar{w}_k^i = \frac{\bar{w}_{k-1}^i C_1 \exp\{-\frac{1}{2}(r^i)^T Q_E^{-1}(r^i)\}}{\sum_{j=1}^{m} \bar{w}_{k-1}^j C_1 \exp\{-\frac{1}{2}(r^j)^T Q_E^{-1}(r^j)\}} \tag{18}$$

Lastly, for the resampling step of the PF, samples x_{k+1}^i, drawn according to the importance density, $q(\mathrm{x}_{k+1}|\mathrm{x}_k^i, \mathrm{z}_{1:k}) = p(\mathrm{x}_{k+1}|\mathrm{x}_k^i)$, can be used for evaluating the weight of particles at time $k+1$ using (18).

Note that the objective of the PF is to approximate the pdf of the states over the samples not the states itself, and the estimation is made through the expectation over the estimated pdf. Therefore, it is natural that not all particles generates same ambiguity estimations $\tilde{N}^i$ unless the particles are concentrated in very small position space. The ambiguity estimation convergence can be monitored by an weighted-average of the integer ambiguity at epoch k. The weighted-average of the integer ambiguity can be obtained by $N_{AVR}(k) = \sum_{i=1}^{m} (\bar{N}^i \times \bar{w}_k^i)$.

IV. Experimental Results

A. Precise Orbit Determination

A simulation was implemented for evaluating the estimates of the relative position of satellites in LEO (Low Earth orbit) for a cluster of spacecrafts [14]. The basis for our experiments was a sequence of simulated GPS satellite positions and a sequence of simulated positions of two satellites trailing each other in the same LEO. The parameters of the LEO orbit is given as semimajor axis - 7054.1874 km, Eccentricity - 0.0005414, Inclination -98.13 deg, R. A. of ascending node - 81.107 deg, Argument of perigee - 270.357 deg, Mean anomaly - 179.839. An air drag perturbation and acceleration perturbation accounting the Sun and moon are included in the orbit equation. The gravitational model included in the LEO orbit uses the zonal harmonics ($J_2 = 10.82625, J_3 = -2.5326 \times 10^{-6}$) and the 3×3 non zonal harmonics ($C_{21} = S_{21} = 0, C_{22} = 1.5747 \times 10^{-6}, C_{31} = 2.1908 \times 10^{-6}, C_{32} = 0.3097 \times 10^{-6}, C_{33} = 0.1001 \times 10^{-6}, S_{22} = -0.9024 \times 10^{-6}, S_{31} = -0.2709 \times 10^{-6}, S_{32} = -0.2212 \times 10^{-6}, S_{33} = 0.1973 \times 10^{-6}$).

Both the positions of the GPS satellites and the measurements of the satellites received by GPS receiver were generated at a 2 Hz rate. The two LEO satellites have either a baseline of 1) 3 km and 2) 30 km. The orbit tracks of the GPS satellites were obtained from the International GNSS Service (IGS) products of March 1, 2000. The simulation allowed changes in the GPS satellite constellation, and introduced the cycle-slip. Under these circumstances, the stability of the converged relative position estimates was assessed. The GPS receiver for the LEO satellites was developed by JPL (Jet Propulsion Laboratory), "Black-Jack" GPS receiver. The Black-Jack receiver is designed for the high dynamics of orbit application, and shows superior performance in terms of the noise and multipath error level. For example, the Black-Jack GPS receiver is mounted on-board of the satellites in the CHAMP and GRACE projects. In this simulation, a high performance receiver, like Black-Jack, is assumed to be used for the spacecraft formation flight problem as well as the ordinary RTK GPS receiver in order to assess the estimation performance.

The generated data included the ionospheric delay error, ephemeris error, and clock error. The tropospheric delay effect on GPS measurements is ignorable in LEO where an altitude is higher than 500 km. However, a significant ionospheric delay exists at LEO altitudes. Ionospheric distortion based on the Klobchar model was imposed on the CP measurements with the total electron count (TEC) experienced varied between a minimum of 4×10^{16} and a maximum of 1.6×10^{17} [15]. Simulated trajectories of the LEO satellites were computed by integrating the orbit equations using a Cowell method. With regard to the noise, the range measurement accuracy for high performance receiver, the Black-Jack, is assumed to be 30 cm, 0.3 cm in magnitude, and for the normal geodetic GPS receiver, 3 m, and 3 cm for the code and CP, respectively. The multipath error is treated as an additional bias term whose size is 1-2 cm for both cases, and corrupts the CP measurements in the simulation.

The Particle Filters (PF) introduced in Section II, is applied to the estimation of the relative position of the spacecrafts using GPS CP information. The sampling process is implemented for obtaining the initial samples. A uniform distribution is used for the initial samples and the weights $\{\mathrm{x}^i, \bar{w}^i, i = 1, \cdots, m\}$. The reason for choosing the uniform distribution for initial sampling distribution is that if the initialization process of the PF is made by a short time of code measurements, then the initial pdf is not accurate enough to use directly for the initial conditions for the CP-based algorithm.

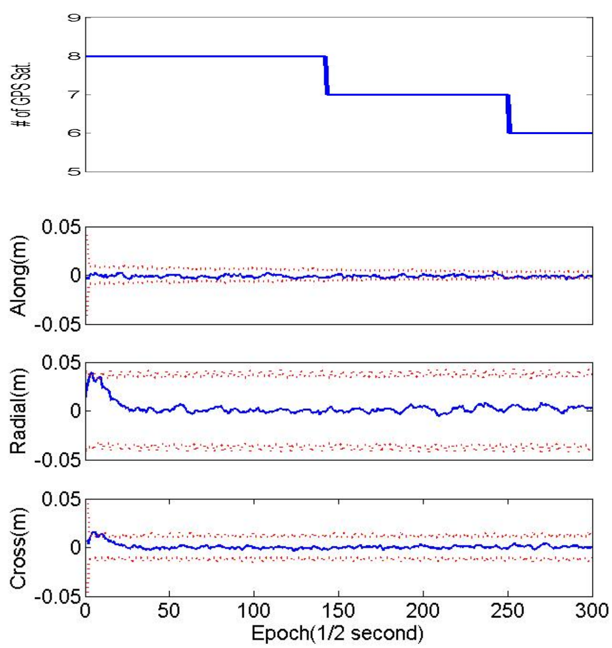

Fig. 2. # of Sat & Position Error (CP noise:3mm, Baseline:3 km)

1) Effect of GPS Satellite Change: Figure 2 shows the number of visible GPS satellites changes, and the error and standard deviation (std) of estimated relative position in three directions (i.e. Radial; unit vector from the center of orbit to the spacecraft, Along-track ; unit tangent vector to the orbit, Cross-track : unit normal vector to the orbit) for the 3 km baseline case. The measurements have accuracies of 30 cm RMS error for the code, 0.3 cm RMS error for the CP noise, i.e. the Black- Jack receiver. In this short baseline case, most of the common errors can be eliminated by the DD operation, although multipath error and noise error persist. The multipath error and noise on CP are the dominant factors in this case. The time required for the CP based GPS navigation system to converge is about five epochs (1 epoch = 1/2 seconds) excluding five epochs of initialization time by with the code estimates. Note that there is no need to search for the integer ambiguity and fix the integer ambiguity to determine position. The number of initial samples is 10K, and after first resampling step, the number of sample is decreased to 1K.

The number of GPS satellites shows that two GPS satellites went out of view at epoch 150 and 250, but no variation in the relative position is observed as expected.

2) Accuracy Comparison with Fixed Integer Ambiguity Case : The performance analysis of the proposed PF with position-based samples are compared to the ambiguity-based algorithm in two ways: 1) three dimensional relative position error 2) convergence of ambiguity estimate.

The relative position accuracy is compared to the ambiguity based case in the following way. For the position estimates of the ambiguity based algorithm, the true integer ambiguity is assumed to be fixed and known, and given in the CP measurements equation (6). Then, a nonlinear filtering technique, the Unscented Kalman Filter (UKF), is applied for the nonlinear observation equations of (13). Therefore, this is a situation after the ambiguity-based algorithm is assumed to have succeeded in searching for the true integer ambiguity. Since there is no change of the GPS satellites or the cycle-slip occurring during this experiment, then only positioning accuracy can be compared rather than the comparison of the time required for searching for the integer ambiguity or the success rate in obtaining the correct integer ambiguity. The position accuracy between the UKF estimates with the fixed-ambiguity and the PF with position-based samples are compared where the Monte Carlo simulation is implemented with 100 trials, with results given in Table I. The used data is include two cases of CP noise RMS (i.e. 3 cm, 0.3 cm), and the baseline length is 30 km. The results shows that the proposed PF is comparable to the estimates of the UKF with the fixed-ambiguity.

3) Cycle-Slip-free Carrier Phase positioning: There are many factors causing cycle-slips, such as an obstruction between antennas and the GPS satellites, or a high noisy signal due to multipath. Once a cycle-slip occurs in the integer-ambiguity searching-based algorithm, the estimation error by the cycle-slip should be detected and fixed.

The remarkable merit of sampling from the position space is that the GPS CP-based positioning algorithm is insensitive to changes in the visible GPS satellite set and to cycle-slips. For validating the impact of the cycle-slip, we simulate the occurrence of a cycle-slip. The cycle-slip data is generated by adding a random number, ranging from 100 to 1,000, to the CP measurements of all GPS satellites at the same time. In fact, even though a cycle-slip usually occurs in a particular signal of GPS satellite, we are interested in the situation where an object obstructs almost of all GPS signal at the same time. Figure 3 (top) shows the jumps in the ambiguity due to cycles-slip in every phase lock loop that is tracking the GPS satellites, and the estimated relative position effected by the cycle-slip are shown in Fig. 3 (bottom). Totally, seven GPS satellites are visible during the experiment, and cycles-slip occur in all the channels at epoch 330, such that the ambiguities of each GPS CP measurement jumps. As expected, the position estimation results support the claim that the position sample PF is insensitive to the cycle-slip.

V. Conclusion

The GPS Carrier-Phase relative position estimation using Particle Filters (PF) is addressed in this paper. In order to handle the integer ambiguity problem in GPS CP relative position estimation which is a nonlinear problem, the PF is employed. Unlike the prior integer ambiguity resolution

TABLE I

POSITION ACCURACY COMPARISON(100 MC TRIALS)

	^{b}Pos.	Mean	(cm)	^{b}Pos.	RMS	(cm)	^{c}Pos.	Mean	(cm)	^{c}Pos.	RMS	(cm)
	^{a}R	A	C	R	A	C	R	A	C	R	A	C
Fixed Ambiguity (UKF)	0.03	-0.36	-0.07	1.73	3.58	1.75	-0.14	0.08	0.01	0.20	0.47	0.20
Position based samples (PF)	-0.06	-0.33	-0.28	1.15	4.09	1.51	0.01	-0.03	-0.07	0.14	0.30	0.14

^{a} R-Radial, A-Along, C-Cross ^{b} CP noise RMS 3 cm, ^{c} CP noise RMS 0.3 cm

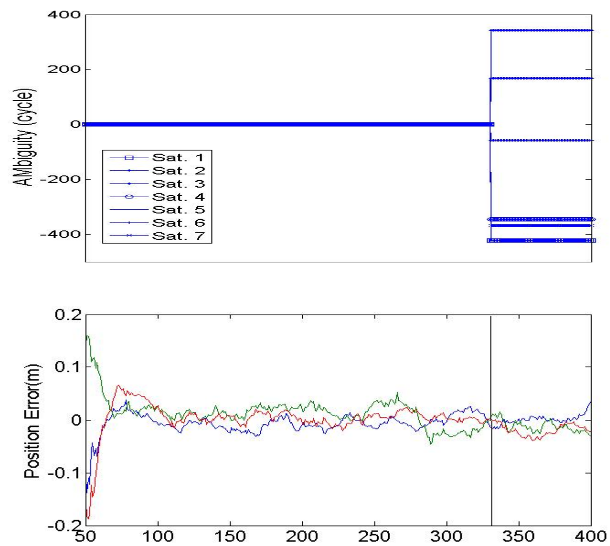

Fig. 3. Ambiguity jumps (at 330 epoch) and Position Error

algorithms that begin with hypotheses of the integer ambiguities and search for the true integer hypotheses, the proposed PF makes use of samples drawn from the position space in order to make the algorithm insensitive to changes in a GPS satellite and the occurrence of a cycle-slip. The PF algorithm employs a resampling technique to overcome the large number of samples which can be problematic for the position-hypotheses-based algorithms. The experimental results are provided by simulating formation flight for the LEO satellites. The position search method introduced in Section I usually requires a much larger number of position hypotheses, since the accuracy of the position results is basically determined by the density of the position hypotheses in the position space. This problem is circumvented by the multinomial resampling technique.

The prior integer ambiguity resolution algorithms have nontrivial management issues for frequent GPS satellites changes. Precise position estimates are not available before the ambiguity problem is resolved. Furthermore, it is particularly difficult to estimate the precise position by hypotheses methods when the system experiences a cycle-slip. It is shown in simulation that the relative position estimates obtained by the proposed approach is insensitive to changes in GPS satellites as well as a cycle-slip. The proposed algorithm does not need to take those effects (i.e. ambiguity change, cycle-slip) into account, and still the estimate remains accurate. Furthermore, the accuracy of the relative position using the position-based approach of PF is comparable to the Unscented Kalman Filter estimates of the ambiguity-based approach when the true integer ambiguity is given.

Therefore, it has been shown that the proposed position sample based particle filters has good robustness and stability properties, and will be applicable to missions where frequent changes of the available GPS satellites and cycle-slips are expected, and the rapid relative position estimation is essential.

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