2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Exponential synchronization of dynamical delayed complex networks consisting of Lur'e systems

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Abstract—**This paper deals with the exponential synchronization problem for a class of dynamical delayed complex networks with each node being a general Lur'e system. The network model considered can represent both the directed and undirected weighted networks. Based on the Lyapunov stability theory and property of the coupling matrix, the delay-dependent linear controllers are designed and the controlled networks are globally exponentially synchronized with a given convergence rate. A dynamical delayed complex network composed of identical Chua's circuits is adopted as a numerical example to demonstrate the effectiveness of the proposed results.**

I. INTRODUCTION

DYNAMICAL complex networks are becoming increasingly important in contemporary society both in science and technology. A complex network is a large set of interconnected communicating and interacting nodes where a node is a fundamental unit having specific contents and exhibiting dynamical behavior, typically. In fact, many systems in science and technology can be modeled as complex networks, and most well-known examples are: power grids, communication networks, internet, World Wide Web, metabolic systems, food webs, etc [1].

Synchronization of complex networks, one of the most important controlling activities to excite the collective behavior of complex dynamical networks, has attracted a lot of attentions of the researchers [2]-[9]. Recently, synchronization of dynamical complex networks consisting of Lur'e systems has become a topic of great interest [10]-[13]. The main reason for this is that in theory and engineering applications, a large class of systems can be expressed as Lur'e form such as E. coli cell [14], genetic oscillator [15], Goodwin model [16], repressilator [17], toggle switch [18], swarm model [19]-[21], Chua's circuit [22], [23], etc. In addition, different from general dynamical complex networks, synchronization for dynamical complex networks composed of Lur'e systems can be derived by using Lur'e system method.

In [10], the authors discuss the synchronization problem for a class of dynamical complex networks with each node being a general Lur'e system and the network is assumed to be weighed and undirected. However, most of the real-world networks, such as the World Wide Web, metabolic and citation networks, are all directed. With regard to the structural features of the real-world networks, our study is aimed at exponentially synchronizing dynamical delayed complex networks with general coupling topology and each node being a general Lur'e system via designing decentralized delay-dependent linear controllers. The network model considered is quite general. It covers the weighted and directed network model and can be linearly coupled or nonlinearly coupled.

The rest of this paper is organized as follows. In Section 2, a more general model of dynamical delayed complex networks composed of Lur'e systems is introduced. In Section 3, based on the properties of the coupling matrix and the Lur'e system, the decentralized delay-dependent linear controllers are given. A numerical example is provided to illustrate the efficiency of the given results in Section 4 and concluding remarks are given in Section 5.

II. DYNAMICAL DELAYED COMPLEX NETWORK MODEL

Consider the dynamical delayed complex network with general coupling topology that is proposed in [1]. The network is composed of N nonlinearly and diffusively coupled identical nodes. Here each of the nodes is a general Lur'e system. The state equations of the entire network are given below

$$\dot{x}_i = Ax_i + Bf(y_i) + \sum_{j=1, j\neq i}^{N} g_{ij}(h(x_j(t-\tau)) - h(x_i(t-\tau))),$$
$$y_i = Cx_i, \quad i = 1, 2, \cdots, N, \tag{1}$$

where $x_i \in R^n$, $y_i \in R^m$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{m\times n}$, f is a memoryless, possibly time-varying nonlinear function, $\tau \geq 0$ is an arbitrary but bounded constant representing the time delay and function $h(\cdot) \in R^n$ is assumed sufficiently smooth nonlinear vector field. Matrix $G = (g_{ij}) \in R^{N\times N}$ is the coupling configuration matrix representing the coupling strength and topology structure of the network; if there is a connection between node i and node j $(i \neq j)$, then $g_{ij} > 0$; and $g_{ij} = 0$ if otherwise. The diagonal elements of matrix G are defined as

$$g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}. \tag{2}$$

Manuscript received September 21, 2008. This work was supported in part by the Nation Natural Science Foundation of China under Grant 60274009.

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Denote

$$y_i = \begin{pmatrix} y_{i1} \\ \vdots \\ y_{im} \end{pmatrix} \in R^m \text{ and } f(y_i) = \begin{pmatrix} f_1(y_{i1}) \\ \vdots \\ f_m(y_{im}) \end{pmatrix} \in R^m,$$

where $f_l(y_{il}),\ i=1,\cdots,N,\ l=1,\cdots,m$ satisfy the following inequalities

$$0 \le f_l(y_{il})y_{il} \le \delta_l y_{il}^2,\ i=1,2,\cdots,N,\ l=1,2,\cdots,m\ ,$$

for all $y_{il} \in R$ and $\delta_l \ge 0$. Taking $\Delta = diag(\delta_1,\delta_2,\cdots,\delta_m)$, it can be easily seen that

$$f^T(y_i)(f(y_i) - \Delta y_i) \le 0 \tag{3}$$

for all $y_i \in R^m$.

Suppose that we want to stabilize network (1) onto a homogenous state defined by

$$x_1 = x_2 = \cdots = x_N = s,\ \dot{s} = As + Bf(Cs), \tag{4}$$

where $s \in R^n$ could be an equilibrium point, a limit cycle or a chaotic orbit.

Let $X(t;t_0;\phi) \in R^{nN}$ be a solution of delayed dynamical network (1), where $\phi = (\phi_1^T,\cdots,\phi_N^T)^T, \phi_i = \phi_i(\theta)$ are initial conditions of node i . $h: R\times\Omega \to R^n$ is continuously differentiable on $\Omega \subseteq R^n$.

Definition 1. The nonlinearity $f(y_i)$ is said to be in the sector $[0,\Delta]$ if it satisfies (3).

Definition 2. If there exist constants $\alpha > 0,\ \lambda > 0$ and a nonempty subset $\Lambda \subseteq \Omega$ with $\phi_i \in \Lambda,\ i=1,2,\cdots,N$, such that $X(t;t_0;\phi) \in \Omega\times\cdots\times\Omega$ for all $t \ge t_0$, and

$$\|X(t;t_0;\phi) - S(t;t_0;s_0)\| \le \alpha e^{-\lambda t} \sup_{-\tau\le\theta\le 0} \|\phi(\theta) - S_0\|. \tag{5}$$

where $X(t;t_0;\phi) = (x_1^T(t;t_0;\phi),\ x_2^T(t;t_0;\phi),\ \cdots,\ x_N^T(t;t_0;\phi))^T$, $S(t;t_0;s_0) = (s^T(t;t_0;s_0),\ \cdots,\ s^T(t;t_0;s_0))^T, S_0 = (s_0^T,\cdots,s_0^T)^T$, $s(t;t_0;s_0)$ is a solution of the system (4) with the initial condition $s_0 \in \Omega$, then the delayed dynamical network (1) is said to realize exponential synchronization such that λ is the exponential rate and $\Lambda\times\cdots\times\Lambda$ is called the region of synchrony of the delayed network (1).

III. MAIN RESULTS

In this section, we study the global exponential synchronization of delayed network (1) by designing linear controllers for each node. The controlled network can be described as

$$\dot{x}_i = Ax_i + Bf(y_i) + \sum_{\substack{j=1\\ j\ne i}}^N g_{ij}(h(x_j(t-\tau)) - h(x_i(t-\tau))) + u_i, \tag{6}$$

where $u_i \in R^n,\ i=1,2\cdots,N$ are the input variables of node i. Letting $e_i(t) = x_i(t) - s(t), i=1,\cdots,N$, the error dynamical system has the following form

$$\begin{aligned} \dot{e}_i &= Ae_i + B(f(Cx_i) - f(Cs)) + u_i \\ &\quad + \sum_{\substack{j=1\\ j\ne i}}^N g_{ij}(g(e_j(t-\tau)) - g(e_i(t-\tau)))\ , \end{aligned} \tag{7}$$

where $g(e_i(t-\tau)) = h(x_i(t-\tau)) - h(s(t-\tau)),\ i=1,2,\cdots,N$.

Then the global exponential synchronization problem of the dynamical network (6) is equivalent to the problem of global exponential stabilization of the error dynamical system (7).

In the sequel, we need Assumption 1 in order to obtain our main results.

Assumption 1. Suppose there exists a positive constant L, such that

$$\|h(x_i) - h(s)\| \le L\|e_i\|, \tag{8}$$

hold for $i=1,2,\cdots,N$.

Theorem 1. Suppose Assumption 1 holds and the time invariant delay $\tau \in (0,\rho]$ for some $\rho > 0$. Then the error system (7) is exponentially stable with respect to the sector $[0,\Delta]$ and the controlled network (6) is globally exponentially synchronized for any fixed time delay $\tau \in (0,\rho]$ under the set of controllers

$$u_i = u_{1i} + u_{2i},\ i=1,2,\cdots,N, \tag{9}$$

where $u_{1i} = (-A - \sqrt{\mu_{\max}(B^TB)\mu_{\max}(C^T\Delta^T\Delta C)})e_i,$

$$u_{2i} = (-\lambda + g_{ii} + \tfrac{1}{2}e^{2\lambda\rho}L^2(g_i + g_{ii}))e_i,$$

$$g_i = -\sum\nolimits_{j=1,j\ne i}^N g_{ji} < 0,\ g_{ii} < 0,\ i=1,2,\cdots,N,$$

$\mu_{\max}(M)$ denotes the maximum eigenvalue of the matrix M, L is defined in Assumption 1, and $\lambda > 0$ is the exponential rate available to be designed.

Proof. Select the following Lyapunov functional candidate

$$\begin{aligned} V(t) &= e^{-2\lambda\rho}\sum_{i=1}^N e_i^T(t)e_i(t) \\ &\quad - \sum_{i=1}^N (g_i + g_{ii})\int_{t-\tau}^t e^{2\lambda(\eta-t)}g^T(e_i(\eta))g(e_i(\eta))\,\mathrm{d}\eta, \end{aligned} \tag{10}$$

and denote

$$W = \sum_{i=1}^N (g_i + g_{ii})\int_{t-\tau}^t e^{2\lambda(\eta-t)}g^T(e_i(\eta))g(e_i(\eta))\,\mathrm{d}\eta.$$

Then the time derivative of $V(t)$ along the solution of the error system (7) is given as follows:

$$\begin{aligned} \dot{V}(t) &= 2e^{-2\lambda\rho}\sum_{i=1}^N e_i^T(t)\left\{Ae_i(t) + B\left[f(Cx_i) - f(Cs)\right] + u_i\right\} \\ &\quad + 2e^{-2\lambda\rho}\sum_{i=1}^N e_i^T(t)\sum_{\substack{j=1\\ j\ne i}}^N g_{ij}(g(e_j(t-\tau)) - g(e_i(t-\tau))) \\ &\quad - \sum_{i=1}^N (g_i + g_{ii})\|g(e_i(t))\|^2 + 2\lambda W \\ &\quad + e^{-2\lambda\tau}\sum_{i=1}^N (g_i + g_{ii})\|g(e_i(t-\tau))\|^2. \end{aligned}$$

Denote $\phi_i = f(Cx_i) - f(Cs)$, and we can deduce that the nonlinearities ϕ_i, $i=1, 2, \cdots, N$ belong to the sector $[0, \Delta]$, i.e., $\phi_i^T(\phi_i - \Delta C e_i) \le 0$. Substituting the controllers (9) into previous equation and considering (8) in Assumption 1, we have

$$\begin{aligned}\dot V(t) &\le 2e^{-2\lambda\rho}\sum_{i=1}^{N}(-\lambda + g_{ii})\|e_i(t)\|^2 + 2\lambda W \\ &+ 2e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}e_i^T(t)(g(e_j(t-\tau)) - g(e_i(t-\tau))) \\ &+ \sum_{i=1}^{N}(g_i + g_{ii})(L^2\|e_i(t)\|^2 - \|g(e_i(t))\|^2) \\ &+ e^{-2\lambda\tau}\sum_{i=1}^{N}(g_i + g_{ii})\|g(e_i(t-\tau))\|^2 .\end{aligned}$$

From Assumption 1, we have

$$\|g(e_i)\| = \|h(x_i) - h(s)\| \le L\|x_i - s\| = L\|e_i\|,$$

and this is to say

$$\sum_{i=1}^{N}(g_i + g_{ii})L^2\|e_i(t)\|^2 \le \sum_{i=1}^{N}(g_i + g_{ii})\|g(e_i(t))\|^2.$$

Hence we have

$$\begin{aligned}\dot V(t) &\le 2e^{-2\lambda\rho}\sum_{i=1}^{N}(-\lambda + g_{ii})\|e_i(t)\|^2 + 2\lambda W \\ &+ 2e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}e_i^T(t)(g(e_j(t-\tau)) - g(e_i(t-\tau))) \\ &+ e^{-2\lambda\rho}\sum_{i=1}^{N}(g_i + g_{ii})\|g(e_i(t-\tau))\|^2 \\ &= -2\lambda e^{-2\lambda\rho}\sum_{i=1}^{N}\|e_i(t)\|^2 - 2e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}\|e_i(t)\|^2 \\ &+ 2e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}e_i^T(t)g(e_j(t-\tau)) + 2\lambda W \\ &- 2e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}e_i^T(t)g(e_i(t-\tau)) \\ &- e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N}(g_{ji} + g_{ij})\|g(e_i(t-\tau))\|^2.\end{aligned}$$

It is obvious that

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ji}\|g(e_i(t-\tau))\|^2 = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}\|g(e_j(t-\tau))\|^2.$$

Therefore, it follows that

$$\begin{aligned}\dot V(t) &\le 2\lambda W - 2\lambda e^{-2\lambda\rho}\sum_{i=1}^{N}\|e_i(t)\|^2 \\ &- e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}\|e_i(t) - g(e_j(t-\tau))\|^2 \\ &- e^{-2\lambda\rho}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}\|e_i(t) + g(e_i(t-\tau))\|^2.\end{aligned}$$

It is readily seen that

$$-\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}\|e_i(t) - g(e_j(t-\tau))\|^2 - \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\ne i}}^{N} g_{ij}\|e_i(t) + g(e_i(t-\tau))\|^2$$

≤ 0.

Thus, we have

$$\begin{aligned}\dot V(t) &\le -2\lambda e^{-2\lambda\rho}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\ &+ 2\lambda\sum_{i=1}^{N}(g_i + g_{ii})\int_{t-\tau}^{t} e^{2\lambda(\eta - t)} g^T(e_i(\eta))g(e_i(\eta))\,\mathrm{d}\eta \quad (11) \\ &= -2\lambda V(t).\end{aligned}$$

By calculating integration on both sides of inequality (11), we get

$$V(t) \le e^{-2\lambda(t-t_0)}V(t_0). \quad (12)$$

Further, it is readily inferred

$$a\|e(t)\|^2 \le V(t) \le b\|e_{t0}\|^2, \quad (13)$$

where $e = (e_1^T, e_2^T, \cdots, e_N^T)^T$, $a = e^{-2\lambda\rho}$, $b = e^{-2\lambda\rho} + \rho c$, $\|e_{t0}\| = \sup_{-\tau\le\theta\le 0}\|e(t_0 + \theta)\|, c = \max\{|c_1|, \cdots, |c_N|, |c_{11}|, \cdots, |c_{NN}|\}$.

With regard to (12) and (13), finally we obtain

$$\|e(t)\| \le \sqrt{\frac{b}{a}}e^{-\lambda(t-t_0)}\|e_{t_0}\|.$$

Therefore, under the controllers (9), the error dynamical system (7) is globally exponentially stable with exponential rate λ. Consequently, the controlled network (6) is globally exponentially stabilized onto s. Thus, the proof is completed.

IV. NUMERICAL EXAMPLES

To test the effectiveness of our results, we consider a dynamical delayed network with 10 nonlinearly coupled unified Chua's oscillators, each of which can be described by the following equation [10]

$$\dot x = Ax + Bf(y), \quad y = Cx, \quad (14)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, A = \begin{bmatrix} 1.2628 & 9.1241 & 0 \\ 1 & -1 & 1 \\ 0 & -14.7059 & -0.0162 \end{bmatrix}, B = \begin{bmatrix} -9.1241 \\ 0 \\ 0 \end{bmatrix},$$

$C = [1 \quad 0 \quad 0]$, $f(y) = -0.2080(|x_1 + 1| - |x_1 - 1|)$.

The entire network is described by the following equations

$$\dot x_i = Ax_i + Bf(y_i) + \sum_{\substack{j=1\\ j\ne i}}^{10} g_{ij}(h(x_j(t-\tau)) - h(x_i(t-\tau))) + u_i \quad (15)$$

where $h(x_i(t-\tau)) = (2x_{i1}(t-\tau), x_{i2}(t-\tau), 0.1x_{i3}(t-\tau))^T$ and $\tau = 0.1$.

We simulate the above delayed network with weighted and directed coupling topology structure, where $g_{ij} = \frac{1}{k_i^{\beta}}l_{ij}$, k_i is the out-degree of node i, β is a tunable weighted parameter. In this example, let $\beta = 0.1$. Select the asymmetric coupling matrix L as follows

$$L=\begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -5 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & -8 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -6 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

According to Theorem 1, one can synchronization the network (15) by the linear controllers $u_i,\ i=1,2,\cdots,10$, where $\lambda=0.1,\ \tau=0.1,\ \rho=0.15$.

Fig.1-Fig.3 show the synchronous errors e_{i1}, e_{i2}, e_{i3} of the network (15). We can observe that all the synchronization errors do globally converge to zero.

V. Conclusions

In this paper, we focus on the exponential synchronization for a class of dynamical delayed complex network with general coupling topology and each node of the network is a general Lur'e system. The exponential synchronization problem is converted into an equivalent exponential stability problem of corresponding error system. An adequate Lyapunov function is constructed to guarantee the exponential stability of the error system. With the topology of the network and the property of the Lur'e system, decentralized delay-dependent linear controllers are designed such that the global exponential synchronization for the delayed network is solved. A numerical example of delayed network demonstrates the effectiveness of the proposed results.

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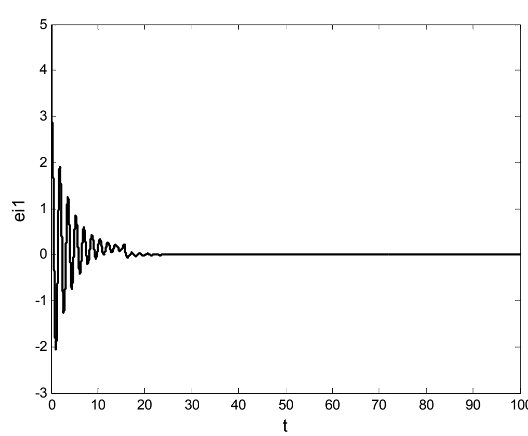

Fig. 1. Synchronization errors e_{i1} of the network

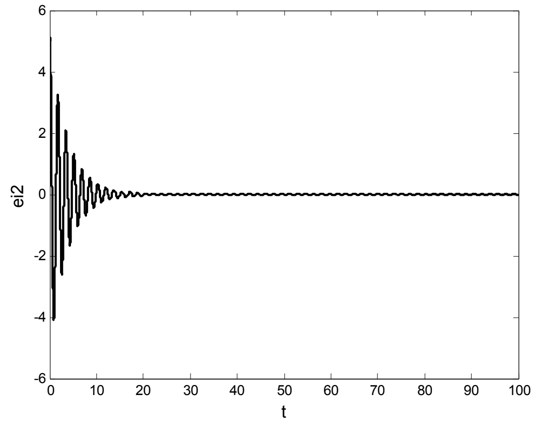

Fig. 2. Synchronization errors e_{i2} of the network

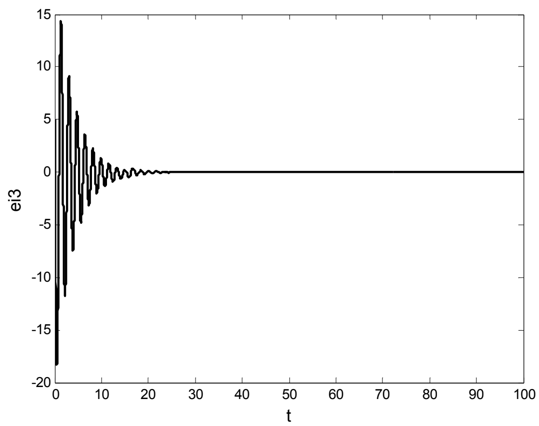

Fig. 3. Synchronization errors e_{i3} of the network