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Global Asymptotic and Finite-gain $\mathcal{L}_2$ Stabilization of Port-Controlled Hamiltonian Systems Subject to Actuator Saturation

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Abstract— **This paper considers the stabilization problem for a port-controlled Hamiltonian system subject to actuator saturation and input additive external disturbances. Conditions are identified under which a static output feedback law would achieve global asymptotic stabilization. Under some additional growth conditions on the nonlinear functions involved in the system, the same feedback law would also achieve finite gain $\mathcal{L}_2$ stabilization. In establishing these results, an estimate of the finite gain is also obtained.**

I. INTRODUCTION

As an important class of nonlinear systems, port-controlled Hamiltonian systems have attracted considerable attention in nonlinear control theory (see, for example, [1], [3], [4], [7], [9], [10], [11], [13], [14], [15], [16], [17] and the references therein). In particular, many fundamental results on both asymptotic stabilization and finite gain stabilization have been established. For example, under some mild conditions, local asymptotic stabilization can be achieved by a static output feedback (see, for example, [13]). On the other hand, a finite $\mathcal{L}_2$ gain γ is achievable if certain partial differential inequalities, parameterized in γ, are solvable for a proper storage function (see, for example, [3], [12], [14], [15], [17]).

In this paper, we consider the problem of stabilization for a port-controlled Hamiltonian system subject to actuator saturation and input additive external disturbances. In the presence of actuator saturation, local asymptotic stabilization problem reduces to one in the absence of saturation. We will thus focus only on global asymptotic stabilization. Under the zero state detectability condition and radially unboundedness property of the Hamiltonian function, we show that a static output feedback law achieves global asymptotic stabilization in the presence of actuator saturation. On the other hand, under some additional growth conditions on the nonlinear functions involved in the system, we show that the same feedback law also achieves finite gain $\mathcal{L}_2$ stabilization. Moreover, an estimate of the $\mathcal{L}_2$ gain can be established.

The results we obtained in this paper have their linear equivalences. In particular, it is known [2], [5], [8] that a linear system, either in continuous-time or discrete-time, can be made both globally asymptotically stable and finite gain $\mathcal{L}_2$ stable by a saturating linear feedback law if its open loop system is Lyapunov stable and controllable. The port-controlled Hamiltonian systems we considered in this paper, when specialized to linear systems, are indeed Lyapunov stable and their zero state detectability property is equivalent to the controllability property. The additional growth conditions we assume to establish finite gain $\mathcal{L}_2$ stabilizability for the port-controlled Hamiltonian systems are all satisfied when specialized to a linear system case.

The remainder of the paper is organized as follows. Section II briefly reviews some basic definitions and results relating to port-controlled Hamiltonian systems and formulates the problem to be solved in this paper. The main results are presented in Section III. A numerical example is worked out in Section IV to illustrate the results. Section V concludes the paper.

The notation used in this paper is standard. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $|\cdot|$ denotes the Euclidean vector norm, and $\mathcal{L}_2^m$ denotes the set of all measurable functions $x : [0,\infty) \to \mathbb{R}^m$ that satisfy

$$\int_0^\infty |x(t)|^2 dt < \infty.$$

For an $x \in \mathcal{L}_2^m$, its $\mathcal{L}_2$-norm is defined as

$$\|x\|_{\mathcal{L}_2} = (\int_0^\infty |x(t)|^2 dt)^{\frac{1}{2}}.$$

II. PRELIMINARIES AND PROBLEM FORMULATION

A port-controlled Hamiltonian system is described as [9]

$$\begin{cases} \dot{x} = (J(x) - R(x))\,\nabla H(x) + g(x)u, \\ y = g^{\mathrm{T}}(x)\nabla H(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^m$ is the output, $J(x) \in \mathbb{R}^{n\times n}$ is a skew-symmetric matrix, that is, $J(x) = -J^{\mathrm{T}}(x)$, $R(x) = R^{\mathrm{T}}(x) \geq 0$, $g(x) \in \mathbb{R}^{n\times m}$ is a gain matrix, and $\nabla H(x) \in \mathbb{R}^{n\times 1}$ is the gradient of the function $H(x) : \mathbb{R}^n \to \mathbb{R}$. We assume that all functions $J(x), R(x), g(x)$ and $H(x)$ are smooth functions of the state x, and without loss of generality, that the system possesses an equilibrium at $x = 0$.

Port-controlled Hamiltonian systems are a generalization of the classical Hamiltonian equations of motion. They model lumped-parameter physical systems with independent storage elements represented by the state variable x. In these models, $J(x)$ corresponds to the continuous interconnections

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of the power in the system, $R(x)$ represents the energy dissipation of the system, and $H(x)$ is the total stored energy.

Because of their physical significance, port-controlled Hamiltonian systems have been a subject of extensive study for many years. An important property that is required in the control of such systems is the zero-state observability (detectability).

Definition 1 ([6]): The system (1) is zero-state observable (detectable) if $u(t) = 0$ and $y(t) = 0$, $\forall t \geq 0$ imply that $x(t) = 0$ ($\lim_{t\to\infty} x(t) = 0$).

Many results have been obtained on the control of the port-controlled Hamiltonian systems. In particular, the following result on local asymptotic stabilization was established in [13].

Lemma 1 ([13]): Consider the port-controlled Hamiltonian system as described by (1). Suppose that $H(x)$ has a strict local minimum at the equilibrium $x = 0$ and the system (1) is zero-state detectable. Then, under the output feedback control law $u = -y$, the closed-loop system

$$\dot{x} = (J(x) - R(x) - g(x)g^{\mathrm{T}}(x))\nabla H(x) \tag{2}$$

is locally asymptotically stable at the equilibrium $x = 0$.

It is also easy to show that, if $H(x)$ is radially unbounded and $H(x) = 0$ if and only if $x = 0$, then the closed-loop system (2) is globally asymptotically stable at $x = 0$.

In this paper, we will identify conditions under which such a simple unity output feedback law achieves both global asymptotic stabilization and finite gain $\mathcal{L}_2$ stabilization of a port-controlled Hamiltonian system in the presence of actuator saturation and input additive disturbances.

Consider the following port-controlled Hamiltonian system subject to actuator saturation and disturbances

$$\begin{cases} \dot{x} = (J(x) - R(x))\nabla H(x) + g(x)\sigma(u + w), \\ \qquad\qquad\qquad\qquad x \in \mathbb{R}^n,\ u, w \in \mathbb{R}^m, \\ y = g^{\mathrm{T}}(x)\nabla H(x), \quad y \in \mathbb{R}^m, \end{cases} \tag{3}$$

where w is the disturbances and $\sigma(\cdot) : \mathbb{R}^m \to \mathbb{R}^m$ is the standard actuator saturation function, *i.e.*,

$$\sigma(s) = \begin{bmatrix} \sigma(s_1) \\ \sigma(s_2) \\ \vdots \\ \sigma(s_m) \end{bmatrix},$$

with

$$\sigma(s_i) = \mathrm{sgn}(s_i)\min\{1, |s_i|\}, \quad i = 1, 2, \cdots, m.$$

Here, we have slightly abused the notation by using σ to denote both the scalar valued and vector valued functions. We have also assumed, without loss of generality, the unity saturation level. Non-unity saturation level can be absorbed by the gain matrix $g(x)$ and the control function.

We will identify conditions under which the feedback law $u = -y$ will render the resulting closed-loop system of the following properties:

- In the absence of disturbances w, the equilibrium $x = 0$ is globally asymptotically stable;
- It is finite gain $\mathcal{L}_2$ stable, *i.e.*, there exists a finite constant $\gamma > 0$ such that

$$\|x\|_{\mathcal{L}_2} \leq \gamma \|w\|_{\mathcal{L}_2}, \quad \forall w \in \mathcal{L}_2^m.$$

III. MAIN RESULTS

Our main results are summarized and proved in the following theorem.

Theorem 1: Consider the port-controlled Hamiltonian system (3). Assume that the system is zero state detectable and the Hamiltonian function $H(x)$ is radially unbounded and $H(x) = 0$ if and only if $x = 0$. Then, the following results hold for the closed-loop system under the static output feedback law $u = -y$.

(i) The equilibrium $x = 0$ is globally asymptotically stable;

(ii) For system (2), which is globally asymptotically stable at the origin, assume that there are a differentiable function $V_0(x)$ and positive numbers λ, α, β and δ such that

  (a) $(\nabla V_0(x))^{\mathrm{T}} (J(x) - R(x) - g(x)g^{\mathrm{T}}(x))\nabla H(x) \leq -|x|^2$;

  (b) $-\lambda H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}} R(x)\nabla H(x) - |x|^2 + \alpha H(x) + \delta\left|g^{\mathrm{T}}(x)\nabla V_0(x)\right|^2 \leq -\beta|x|^2$;

  (c) $\lambda H^{\frac{1}{2}}(x) - |g^{\mathrm{T}}(x)\nabla V_0(x)| \geq 0$.

  Then, the closed-loop system is finite gain $\mathcal{L}_2$ stable, i.e., there exists a finite constant $\gamma > 0$ such that

$$\|x\|_{\mathcal{L}_2} \leq \gamma\|w\|_{\mathcal{L}_2}, \quad \forall w \in \mathcal{L}_2^m. \tag{4}$$

Remark 1: In the special case where $J(x) = A$, $R(x) = 0$, $g(x) = B$ and $H(x) = \frac{1}{2}x^{\mathrm{T}}x$, the port-controlled Hamiltonian system (3) takes the following linear system form

$$\begin{cases} \dot{x} = Ax + B\sigma(u + w), \quad x \in \mathbb{R}^n, u, w \in \mathbb{R}^m, \\ y = B^{\mathrm{T}}x, \quad y \in \mathbb{R}^m, \end{cases} \tag{5}$$

and the system (2) reduces to

$$\dot{x} = (A - BB^{\mathrm{T}})x. \tag{6}$$

The zero state detectability condition reduces to the detectability of the matrix pair (A, B^{T}), which, in view of the fact $A + A^{\mathrm{T}} = 0$, is equivalent to controllability of the matrix pair (A, B). Thus, the system (6) is (globally) asymptotically stable. Let

$$V_0(x) = x^{\mathrm{T}}Px, \tag{7}$$

where P is the positive definite solution to the Lyapunov equation

$$(A - BB^{\mathrm{T}})^{\mathrm{T}}P + P(A - BB^{\mathrm{T}}) = -I. \tag{8}$$

Condition (a) is implied by (8). The growth conditions (b) and (c) are given respectively by

$$\left(\frac{1}{2}\alpha - 1\right)|x|^2 + 4\delta|B^{\mathrm{T}}Px|^2 \leq -\beta|x|^2 \tag{9}$$

and

$$\frac{\sqrt{2}}{2}\lambda|x| - |B^{\mathrm{T}}x| \geq 0, \tag{10}$$

both of which can be satisfied with some small enough α, β and δ and a large enough λ. Thus, by Theorem 1, the system (5) is finite gain $\mathcal{L}_2$ stable. This result for linear systems subject to input saturation was established in [8].

Proof of Theorem 1. (i) In the absence of the disturbances w, the closed-loop system is given by

$$\begin{aligned}\dot{x} &= (J(x)-R(x))\nabla H(x) \\ &\quad +g(x)\sigma\left(-g^{\mathrm{T}}(x)\nabla H(x)\right). \end{aligned} \tag{11}$$

Let $H(x)$ be a Lyapunov function candidate. Let $g_i(x)$ be the ith column of $g(x)$. Then, noting that

$$\begin{aligned}&(\nabla H(x))^{\mathrm{T}} J(x)\nabla H(x) \\ = \;& \frac{1}{2}(\nabla H(x))^{\mathrm{T}}\left(J(x)+J^{\mathrm{T}}(x)\right)\nabla H(x) \\ = \;& 0, \end{aligned} \tag{12}$$

we can evaluate the derivative of $H(x)$ along the trajectory of the closed-loop system (11) as follows,

$$\begin{aligned}\dot{H}(x) &= (\nabla H(x))^{\mathrm{T}}\dot{x} \\ &= (\nabla H(x))^{\mathrm{T}}(J(x)-R(x))\nabla H(x) \\ &\quad +(\nabla H(x))^{\mathrm{T}} g(x)\sigma\left(-g^{\mathrm{T}}(x)\nabla H(x)\right) \\ &= \frac{1}{2}(\nabla H(x))^{\mathrm{T}}\left(J(x)+J^{\mathrm{T}}(x)\right)\nabla H(x) \\ &\quad -(\nabla H(x))^{\mathrm{T}}R(x)\nabla H(x) \\ &\quad +\sum_{i=1}^{m}(\nabla H(x))^{\mathrm{T}} g_i(x)\sigma\left(-g_i^{\mathrm{T}}(x)\nabla H(x)\right) \\ &\leq \sum_{i=1}^{m}(\nabla H(x))^{\mathrm{T}} g_i(x)\sigma\left(-g_i^{\mathrm{T}}(x)\nabla H(x)\right) \\ &= -\sum_{i=1}^{m} g_i^{\mathrm{T}}(x)\nabla H(x)\sigma\left(g_i^{\mathrm{T}}(x)\nabla H(x)\right) \\ &\leq 0, \end{aligned} \tag{13}$$

where $g_i(x)$ is the ith column of $g(x)$. Note that $\dot{H}(x)\equiv 0$ implies that

$$g_i^{\mathrm{T}}(x)\nabla H(x)\equiv 0, \quad \forall i=1,2,\cdots,m,$$

that is $y=0$, and hence $u=-y=0$. By the LaSalle invariance principle, the zero state detectability of the system and the radial unboundedness of $H(x)$ then imply that the closed-loop system is globally asymptotically stable at $x=0$.

(ii) In the presence of the disturbances w, the closed-loop system is given by

$$\begin{aligned}\dot{x} &= (J(x)-R(x))\nabla H(x) \\ &\quad +g(x)\sigma\left(-g^{\mathrm{T}}(x)\nabla H(x)+w\right). \end{aligned} \tag{14}$$

Letting $\tilde{x}=-g^{\mathrm{T}}(x)\nabla H(x)+w$, we can rewrite (14) as

$$\dot{x}=(J(x)-R(x))\nabla H(x)+g(x)\sigma(\tilde{x}). \tag{15}$$

Motivated by the choice of the Lyapunov function in [8], we consider the following Lyapunov function candidate

$$V_1(x)=\frac{2}{3}H^{\frac{3}{2}}(x). \tag{16}$$

We can evaluate the derivative of $V_1(x)$ along the trajectory of (15) as follows

$$\begin{aligned}\dot{V}_1(x) &= H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}}\dot{x} \\ &= H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}}(J(x)-R(x))\nabla H(x) \\ &\quad +H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}}g(x)\sigma(\tilde{x}) \\ &= -H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}}R(x)\nabla H(x) \\ &\quad -H^{\frac{1}{2}}(x)\tilde{x}^{\mathrm{T}}\sigma(\tilde{x})+H^{\frac{1}{2}}(x)w^{\mathrm{T}}\sigma(\tilde{x}) \\ &\leq -H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}}R(x)\nabla H(x) \\ &\quad -H^{\frac{1}{2}}(x)\tilde{x}^{\mathrm{T}}\sigma(\tilde{x})+\frac{\alpha}{\lambda}H(x)+\frac{\lambda}{4\alpha}w^{\mathrm{T}}w, \end{aligned} \tag{17}$$

where in the last step of the derivation we have used the inequality

$$ab\leq a^2+\frac{1}{4}b^2, \quad \forall a,b\in\mathbb{R}, \tag{18}$$

and the fact that $|\sigma(\tilde{x}_i)|\leq 1$, $i=1,2,\cdots,m.$

We next rewrite the equation (15) as

$$\begin{aligned}\dot{x} &= \left(J(x)-R(x)-g(x)g^{\mathrm{T}}(x)\right)\nabla H(x) \\ &\quad +g(x)\left(-\tilde{x}+\sigma(\tilde{x})+w\right), \end{aligned} \tag{19}$$

and let $V_0(x)$ be another Lyapunov function candidate. Then, the derivative of $V_0(x)$ along the trajectory of (19) can be evaluated as

$$\begin{aligned}\dot{V}_0(x) &= (\nabla V_0(x))^{\mathrm{T}}\dot{x} \\ &= (\nabla V_0(x))^{\mathrm{T}}\left(J(x)-R(x)-g(x)g^{\mathrm{T}}(x)\right)\nabla H(x) \\ &\quad +(\nabla V_0(x))^{\mathrm{T}}g(x)\left(-\tilde{x}+\sigma(\tilde{x})\right) \\ &\quad +(\nabla V_0(x))^{\mathrm{T}}g(x)w \\ &\leq -|x|^2+\left|g^{\mathrm{T}}(x)\nabla V_0(x)\right|\tilde{x}^{\mathrm{T}}\sigma(\tilde{x}) \\ &\quad +\delta\left|g^{\mathrm{T}}(x)\nabla V_0(x)\right|^2+\frac{1}{4\delta}|w|^2. \end{aligned} \tag{20}$$

In the above derivation, we have used Condition (a) of the theorem, the inequality (18) and the simple fact that

$$|-\tilde{x}+\sigma(\tilde{x})|\leq\tilde{x}^{\mathrm{T}}\sigma(\tilde{x}).$$

We now construct a Lyapunov function as follows

$$V(x)=\lambda V_1(x)+V_0(x). \tag{21}$$

In view of (17) and (20), the derivative of $V(x)$ along the trajectory of the closed-loop system is given by

$$\begin{aligned}\dot{V}(x) &\leq -\lambda H^{\frac{1}{2}}(x)(\nabla H(x))^{\mathrm{T}}R(x)\nabla H(x) \\ &\quad +\alpha H(x)-|x|^2 \\ &\quad -\left(\lambda H^{\frac{1}{2}}(x)-\left|g^{\mathrm{T}}(x)\nabla V_0(x)\right|\right)\tilde{x}^{\mathrm{T}}\sigma(\tilde{x}) \\ &\quad +\delta\left|g^{\mathrm{T}}(x)\nabla V_0(x)\right|^2 \\ &\quad +\left(\frac{\lambda^2}{4\alpha}+\frac{1}{4\delta}\right)|w|^2, \end{aligned} \tag{22}$$

from which, along with Conditions (b) and (c) of the theorem, we have

$$\dot{V}(x) \leq -\beta|x|^2+\left(\frac{\lambda^2}{4\alpha}+\frac{1}{4\delta}\right)|w|^2. \tag{23}$$

Integrating both sides of (23) from 0 to t and noting that $V(x(0)) = 0$, we obtain

$$V(x(t)) \leq -\beta \int_0^t |x(\tau)|^2 d\tau + \left(\frac{\lambda^2}{4\alpha} + \frac{1}{4\delta}\right) \int_0^t |w(\tau)|^2 d\tau. \quad (24)$$

Since $V(x(t)) \geq 0$, we have

$$\int_0^t |x(\tau)|^2 d\tau \leq \left(\frac{\lambda^2}{4\alpha\beta} + \frac{1}{4\delta\beta}\right) \int_0^t |w(\tau)|^2 d\tau, \quad (25)$$

from which it follows that

$$\|x\|_{\mathcal{L}_2} \leq \gamma \|w\|_{\mathcal{L}_2}, \quad (26)$$

with

$$\gamma = \frac{1}{2}\left(\frac{\lambda^2}{\alpha\beta} + \frac{1}{\delta\beta}\right)^{\frac{1}{2}}.$$

The proof is completed. □

IV. A Numerical Example

Consider the port-controlled Hamiltonian system (3) with

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

$$J(x) = \begin{pmatrix} 0 & x_1 + x_2 \\ -x_1 - x_2 & 0 \end{pmatrix},$$

$$R(x) = \begin{pmatrix} \sin^2 x_2 + 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$g(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and

$$H(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right) = \frac{1}{2}|x|^2.$$

Clearly, $H(x)$ is radially unbounded with $H(x) = 0$ if and only if $x = 0$. Let $V_0(x) = H(x)$. It is easy to verify that this choice of $V_0(x)$ satisfies Condition (a) of Theorem 1. Conditions (b) and (c) are given by

$$-\frac{\sqrt{2}}{2}\lambda\left(\sin^2 x_2 + 1\right) x_1^2 |x| - \left(1 - \frac{1}{2}\alpha\right)|x|^2 + \delta |x_2|^2 \leq -\beta |x|, \quad (27)$$

and

$$\frac{\sqrt{2}}{2}\lambda |x| - |x_2| \geq 0.$$

respectively. Both of these two conditions can be satisfied with

$$\lambda = \sqrt{2}, \quad \alpha = \frac{1}{2}, \quad \delta = \frac{1}{4}, \quad \beta = \frac{1}{2}.$$

Thus, according to Theorem 1, the closed-loop system under the feedback law $u = -y$ is globally asymptotically stable and finite gain $\mathcal{L}_2$ stable. An estimate of the $\mathcal{L}_2$ as obtained in the proof of the theorem is given by

$$\gamma = \frac{1}{2}\left(\frac{\lambda^2}{\alpha\beta} + \frac{1}{\delta\beta}\right)^{\frac{1}{2}} = 2.$$

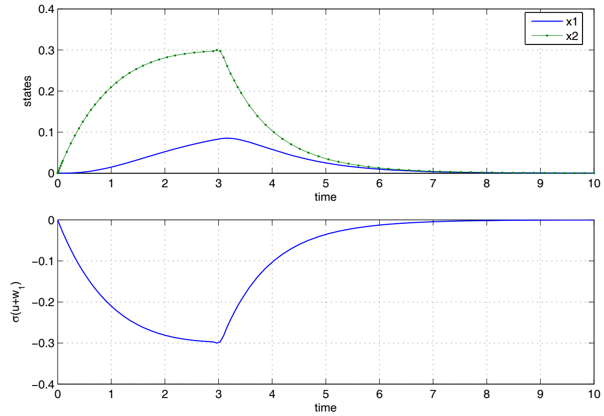

Fig. 1. State response and the actuator output: $x(0) = 0$, $w(t) = w_1(t)$.

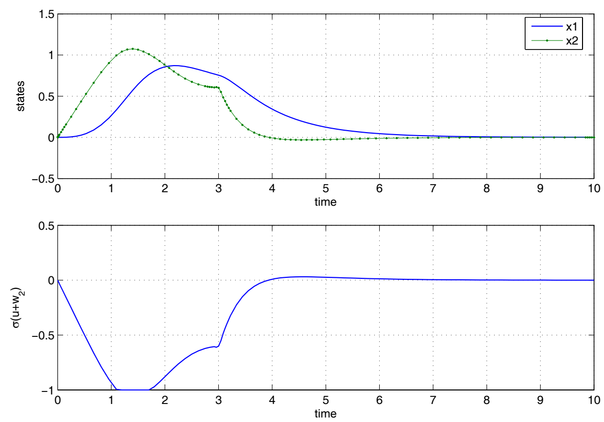

Fig. 2. State response and the actuator output: $x(0) = 0$, $w(t) = w_2(t)$.

Shown in Figs. 1-3 are some simulation results of the closed-loop system with $x(0) = 0$ and under the three different disturbances

$$w_1(t) = \frac{1}{3}\left(1(t) - 1(t-3)\right),$$
$$w_2(t) = 2\left(1(t) - 1(t-3)\right),$$
$$w_3(t) = -2\left(1(t) - 1(t-3)\right),$$

where $1(t)$ is the unit step function. As expected, all trajectories have finite energies. In fact, by numerical computation, we obtain that

$$\frac{\|x\|_{\mathcal{L}_2}}{\|w_1\|_{\mathcal{L}_2}} = 1.3748,$$

$$\frac{\|x\|_{\mathcal{L}_2}}{\|w_2\|_{\mathcal{L}_2}} = 0.9899,$$

and

$$\frac{\|x\|_{\mathcal{L}_2}}{\|w_3\|_{\mathcal{L}_2}} = 1.3202.$$

V. Conclusions

In this note, we have established conditions for global asymptotic stabilization and finite gain $\mathcal{L}_2$ stabilization for

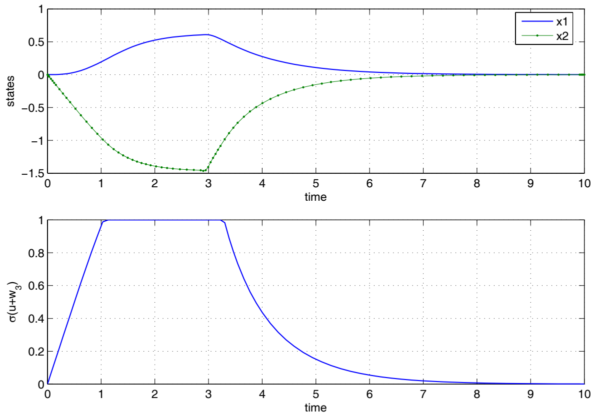

Fig. 3. State response and the actuator output: $x(0) = 0$, $w(t) = w_3(t)$.

port-controlled Hamiltonian systems subject to actuator saturation and input additive disturbances. Under these conditions, simple static output feedback laws achieve the stabilization. These conditions are automatically satisfied when the port-controlled Hamiltonian systems are specialized to their linear system counterparts. Thus, the stabilization results on the port-controlled Hamiltonian system recover the well-known corresponding results for linear systems.

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