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Least-squares based iterative parameter estimation for two-input multirate sampled-data systems

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***Abstract*— This paper studies identification problems for two-input multirate systems with colored noises (The method in the paper can be easily extended to multi-input multirate systems). The state-space models are derived for the multirate systems with two different input sampling periods and furthermore the corresponding transfer functions are obtained. To solve the difficulty of identification models with unmeasurable noises terms, the least-squares based iterative algorithm is presented by replacing the unmeasurable variables with their iterative estimates. Finally, the simulation results indicate that the proposed algorithm has good performances.**

I. INTRODUCTION

FOR traditional discrete-time sampled systems, the operation frequencies of hold and sampler are equal and synchronous at time. Systems existing two or over two operation frequencies are called multirate systems. For a two-input system, if the updating periods of the two inputs, T_1 and T_2, are not equal, then we get a multirate sampled-data system.

For decades, the studies of multirate systems have focused on not only petroleum chemical process control, but also achieved a series of research results in theory such as adaptive control [1]–[3], optimal control [4], [5], and so on. In the multirate identification literature, Li et al used multirate input-output data and system states to estimate the parameters of lifted state-space models for multirate systems [6]; Ding and Chen proposed a hierarchical identification method of lifted state-space models for general dual-rate systems [7], [8]; Li et al used the subspace approach to directly identifiy a residual model for fault detection and isolation for systems with non-uniformly sampled multirate data without any knowledge of the system [9]; Ding and Ding applied the extended least squares algorithm to identify the dual-rate systems directly from the available input-output data [10].

This paper focuses on parameter identification for two-input multirate sampled-data systems with colored noises. The fundamental idea of the proposed methods is to use the iterative techniques to deal with the identification problem of the multirate systems by adopting the iterative estimation theory: when computing the parameter estimates, the unknown variables in the information vector are replaced with their corresponding estimates at the current iteration, and these estimates of the unknown variables are again computed by the preceding parameter estimates. Based on this idea, we present a least-squares based iterative identification algorithm. The main advantage of such iterative algorithms is that they can produce more accurate parameter estimates than the recursive methods [11], see the example later. In addition, the iterative methods can be extended to other cases such as multi-input multi-output multirate systems.

This paper is organized as follows. Section II derives the state-space models and obtains the corresponding transfer functions. Section III presents a least-squares based iterative algorithm for two-input multirate systems and Section IV gives the recursive least-squares identification algorithm compared with least-squares based iterative algorithm. Section V provides an illustrative example. Finally, concluding remarks are given in Section VI.

II. PROBLEM FORMULATION

The focus of this paper is a class of multirate systems – two-input single-output multirate systems with colored noises as depicted in Figure 1, P_{c1} and P_{c2} are two continuous-time processes, the inputs $u_1(t)$ and $u_2(t)$ to P_{c1} and P_{c2} are produced separately by zero-order holds H_{T_1} and H_{T_2} with periods T_1 and T_2, processing two discrete-time signals $u_1(kT_1)$ and $u_2(kT_2)$; the noise output $e(t)$ is produced by the $v'(t)$ through the continuous-time process P_N; the output $y(t)$ by the superposition of the unmeasurable outputs $y_1(t)$ and $y_2(t)$, which is corrupted by the noise $e(t)$, are sampled by the sampler S_T with period T, yielding a discrete-time signal $y(kT)$. Without loss of generality, we assume that $T_1 = p_1 h$, $T_2 = p_2 h$, p_1 and p_2 are two coprime integers, $T := p_1 p_2 h$ is *frame period* (h is called *base period*). Since the two input updating periods are not equal to the output sampling period, the system in Figure 1 is a multirate system.

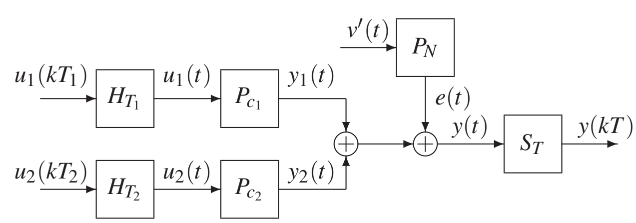

Fig. 1. The two-input single-output multirate sampled-data systems

Throughout the paper, we assume that P_{ci}, $i = 1, 2$ are liner time-invariant continuous-time processes with the following

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state-space representation:

$$P_{ci}: \quad \begin{cases} \dot{x}_i(t) = A_{ci}x_i(t) + B_{ci}u_i(t), \ i = 1,2, \\ y_i(t) = C_i x_i(t) + D_i u_i(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector of the ith subsystem, $u_i(t)$ is the control input of the ith subsystem, $y_i(t)$ is the unmeasurable output of the ith subsystem, A_{ci}, B_{ci}, C_i, D_i are matrices of appropriate sizes. For such multirate systems, the input-output data available are $\{u_1(kT + j_1T_1), u_2(kT + j_2T_2), y(kT): \ j_i = 1,2,\cdots,q_i-1, \ k = 0,1,2,\cdots\}$ ($q_i = p_1p_2/p_i$). Since the zero-hold technique is used, the input $u_i(t)$ keeps invariant over interval $[kT+(j-1)T_i, kT+jT_i)$, $i = 1,2, \ j = 1,2,\cdots,q_i$. We discretize P_{ci} with the sampling period T to get [7]

$$\begin{aligned}
& x_i((k+1)T) \\
&= \mathrm{e}^{A_{ci}T}x_i(kT) + \int_{kT}^{(k+1)T} \mathrm{e}^{A_{ci}((k+1)T-\tau)}B_{ci}\ u_i(\tau)\mathrm{d}\tau \\
&= \mathrm{e}^{A_{ci}T}x_i(kT) \\
&\quad + \sum_{j=1}^{q_i}\int_{kT+(j-1)T_i}^{kT+jT_i} \mathrm{e}^{A_{ci}(kT+q_iT_i-\tau)}B_{ci}\ u_i(\tau)\mathrm{d}\tau \\
&= \mathrm{e}^{A_{ci}T}x_i(kT) \\
&\quad + \sum_{j=1}^{q_i} \mathrm{e}^{A_{ci}(q_i-j)T_i}\int_0^{T_i}\mathrm{e}^{A_{ci}t}\mathrm{d}t\ B_{ci}\ u_i(kT+(j-1)T_i) \\
&= A_i x_i(kT) + \sum_{j=1}^{q_i} B_{ij}\ u_i(kT+(j-1)T_i), \\
& y_i(kT) = C_i x_i(kT) + D_i u_i(kT),
\end{aligned}$$

where

$$\begin{aligned}
A_i &= \mathrm{e}^{A_{ci}T} \in \mathbb{R}^{n_i\times n_i}, \ i = 1,2, \\
B_{1j} &= \mathrm{e}^{A_{c1}(p_2-j)T_1}\int_0^{T_1}\mathrm{e}^{A_{c1}t}\mathrm{d}tB_{c1} \in \mathbb{R}^{n_1}, \ j = 1,2,\cdots,p_2, \\
B_{2j} &= \mathrm{e}^{A_{c2}(p_1-j)T_2}\int_0^{T_2}\mathrm{e}^{A_{c2}t}\mathrm{d}tB_{c2} \in \mathbb{R}^{n_2}, \ j = 1,2,\cdots,p_1.
\end{aligned}$$

The output equation is

$$y(kT) = y_1(kT) + y_2(kT) + e(kT).$$

Let z^{-1} be a unit backward shift operator, i.e., z is a unit forward shift operator, $z^{-1}u_i(kT+T_i) = u_i(kT+T_i-T)$, $zx(kT) = x(kT+T)$, we have

$$\begin{aligned}
y(kT) &= C_1(zI-A_1)^{-1}\sum_{j=1}^{p_2} B_{1j}u_1(kT+(j-1)T_1) \\
&\quad + C_2(zI-A_2)^{-1}\sum_{j=1}^{p_1} B_{2j}u_2(kT+(j-1)T_2) \\
&\quad + D_1u_1(kT) + D_2u_2(kT) + e(kT) \\
&=: \frac{1}{\alpha_1(z)}\sum_{j=1}^{p_2}\beta'_{1j}(z)u_1(kT+(j-1)T_1) \\
&\quad + \frac{1}{\alpha_2(z)}\sum_{j=1}^{p_1}\beta'_{2j}(z)u_2(kT+(j-1)T_2) + e(kT),
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
\alpha_i(z) &= z^{-n_i}\det\ [zI-A_i] \\
&=: 1 + \alpha_{i1}z^{-1} + \alpha_{i2}z^{-2} + \cdots + \alpha_{in_i}z^{-n_i}, \ i = 1,2, \\
\beta'_{i1}(z) &= z^{-n_i}C_i\mathrm{adj}\ [zI-A_i]B_{i1} + D_i\alpha_i(z) \\
&=: \beta'_{i1}(0) + \beta'_{i1}(1)z^{-1} + \beta'_{i1}(2)z^{-2} + \cdots + \beta'_{i1}(n_i)z^{-n_i}, \\
\beta'_{1j}(z) &= z^{-n_1}C_1\mathrm{adj}\ [zI-A_1]B_{1j} \\
&=: \beta'_{1j}(1)z^{-1} + \beta'_{1j}(2)z^{-2} + \cdots + \beta'_{1j}(n_1)z^{-n_1}, \\
&\quad\ j = 2,3,\cdots,p_2, \\
\beta'_{2j}(z) &= z^{-n_2}C_2\mathrm{adj}\ [zI-A_2]B_{2j} \\
&=: \beta'_{2j}(1)z^{-1} + \beta'_{2j}(2)z^{-2} + \cdots + \beta'_{2j}(n_2)z^{-n_2}, \\
&\quad\ j = 2,3,\cdots,p_1.
\end{aligned}$$

Similarly, we discretize P_N with the period T to get

$$e(kT) = \frac{\eta(z)}{\gamma(z)}v(kT), \tag{3}$$

where

$$\begin{aligned}
\gamma(z) &= 1 + \gamma_1z^{-1} + \gamma_2z^{-2} + \cdots + \gamma_{n_c}z^{-n_c}, \\
\eta(z) &= 1 + \eta_1z^{-1} + \eta_2z^{-2} + \cdots + \eta_{n_e}z^{-n_e}.
\end{aligned}$$

Thus, substituting (3) into (2) gives

$$\begin{aligned}
y(kT) &= \frac{1}{\alpha_1(z)}\sum_{j=1}^{p_2}\beta'_{1j}(z)u_1(kT+(j-1)T_1) \\
&\quad + \frac{1}{\alpha_2(z)}\sum_{j=1}^{p_1}\beta'_{2j}(z)u_2(kT+(j-1)T_2) + \frac{\eta(z)}{\gamma(z)}v(kT).
\end{aligned} \tag{4}$$

Multiplying both sides of (4) by $\alpha_1(z)\alpha_2(z)$, and let $\alpha(z) := \alpha_1(z)\alpha_2(z)$, $\beta_{1j}(z) := \alpha_2(z)\beta'_{1j}(z)$, $\beta_{2j}(z) := \alpha_1(z)\beta'_{2j}(z)$, $\zeta(z) := \alpha_1(z)\alpha_2(z)\eta(z)$, then the above equation can be rewritten as

$$\begin{aligned}
\alpha(z)y(kT) &= \sum_{j=1}^{p_2}\beta_{1j}(z)u_1(kT+(j-1)T_1) \\
&\quad + \sum_{j=1}^{p_1}\beta_{2j}(z)u_2(kT+(j-1)T_2) + \frac{\zeta(z)}{\gamma(z)}v(kT),
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
\alpha(z) &= 1 + \alpha_1(z)z^{-1} + \alpha_2z^{-2} + \cdots + \alpha_nz^{-n}, \\
\beta_{i1}(z) &=: \beta_{i1}(0) + \beta_{i1}(1)z^{-1} + \beta_{i1}(2)z^{-2} + \cdots + \beta_{i1}(n)z^{-n}, \\
&\quad\ i = 1,2, \\
\beta_{1j}(z) &=: \beta_{1j}(1)z^{-1} + \beta_{1j}(2)z^{-2} + \cdots + \beta_{1j}(n)z^{-n}, \\
&\quad\ j = 2,3,\cdots,p_2, \\
\beta_{2j}(z) &=: \beta_{2j}(1)z^{-1} + \beta_{2j}(2)z^{-2} + \cdots + \beta_{2j}(n)z^{-n}, \\
&\quad\ j = 2,3,\cdots,p_1, \\
\gamma(z) &=: 1 + \gamma_1z^{-1} + \gamma_2z^{-2} + \cdots + \gamma_{n_c}z^{-n_c}, \\
\zeta(z) &= 1 + \zeta_1z^{-1} + \zeta_2z^{-2} + \cdots + \zeta_{n_d}z^{-n_d}.
\end{aligned}$$

The objective of the paper is to present the least-squares based iterative identification algorithm to identify the parameters α_j, $\beta_{ij}(r)$, γ_j and ζ_j.

III. The Least-squares based iterative algorithms

Define the information vector $\psi(kT)$ and the parameter vector ϑ as

$$\begin{aligned}
\psi(kT) &:= \begin{bmatrix} \varphi_{\rm s}(kT) \\ \varphi_{\rm n}(kT) \end{bmatrix} \in \mathbb{R}^{n_0},\ n_0 := m+n_c+n_d, \\
&\quad m = (p_1+p_2+1)n+2, \\
\varphi_{\rm s}(kT) &:= [-y(kT-T), -y(kT-2T), \cdots, \\
&\quad -y(kT-nT), \phi_1^{\rm T}(kT), \phi_2^{\rm T}(kT)]^{\rm T} \in \mathbb{R}^m, \\
\phi_i(kT) &:= [u_i(kT), u_i(kT-T), u_i(kT-2T), \cdots, \\
&\quad u_i(kT-nT), u_i(kT-T+T_i), \\
&\quad u_i(kT-2T+T_i), \cdots, u_i(kT-nT+T_i) \cdots, \\
&\quad u_i(kT-T+(q_i-1)T_i), \\
&\quad u_i(kT-2T+(q_i-1)T_i), \cdots, \\
&\quad u_i(kT-nT+(q_i-1)T_i)]^{\rm T} \in \mathbb{R}^{q_in+1}, \\
\varphi_{\rm n}(kT) &:= [-e(kT-T), -e(kT-2T), \cdots, \\
&\quad -e(kT-n_cT),\ v(kT-T), v(kT-2T), \cdots, \\
&\quad v(kT-n_dT)]^{\rm T} \in \mathbb{R}^{n_c+n_d}, \\
\vartheta &:= \begin{bmatrix} \theta_{\rm s} \\ \theta_{\rm n} \end{bmatrix} \in \mathbb{R}^{n_0}, \\
\theta_{\rm s} &:= [\alpha_1, \alpha_2, \cdots, \alpha_n, \beta_{11}(0), \beta_{11}(1), \beta_{11}(2), \cdots, \\
&\quad \beta_{11}(n), \beta_{12}(1), \beta_{12}(2), \cdots, \beta_{12}(n), \cdots, \\
&\quad \beta_{1p_2}(1), \beta_{1p_2}(2), \cdots,\ \beta_{1p_2}(n), \beta_{21}(0), \beta_{21}(1), \\
&\quad \beta_{21}(2), \cdots, \beta_{21}(n), \beta_{22}(1), \beta_{22}(2), \cdots, \beta_{22}(n), \\
&\quad \cdots, \beta_{2p_1}(1), \beta_{2p_1}(2), \cdots, \beta_{2p_1}(n)]^{\rm T} \in \mathbb{R}^m, \\
\theta_{\rm n} &:= [\gamma_1, \gamma_2, \cdots, \gamma_{n_c}, \zeta_1, \zeta_2, \cdots, \zeta_{n_d}]^{\rm T} \in \mathbb{R}^{n_c+n_d}.
\end{aligned}$$

By the above definition, we have

$$e(kT) = \varphi_{\rm n}^{\rm T}(kT)\theta_{\rm n} + v(kT), \tag{6}$$
$$y(kT) = \varphi_{\rm s}^{\rm T}(kT)\theta_{\rm s} + e(kT). \tag{7}$$

Then (5) can be written as

$$y(kT) = \psi^{\rm T}(kT)\vartheta + v(kT). \tag{8}$$

The information vector $\psi(kT)$ in (8) contains unmeasurable variables $e(kT-jT)$ and $v(kT-jT)$. To solve the difficulty, these unmeasurable variables are replaced with their iterative estimates by the iterative method. For convenience, let $t = kT$, $\psi(t) := \psi(kT)$. Thus, (8) can be written as

$$y(t) = \psi^{\rm T}(t)\vartheta + v(t). \tag{9}$$

Consider the newest q and define the stacked output vector $Y(t)$, stacked information vector $\Psi(t)$ and white noise vector $V(t)$ as

$$Y(t) := \begin{bmatrix} y(t) \\ y(t-T) \\ \vdots \\ y(t-qT+T) \end{bmatrix} \in \mathbb{R}^q, \tag{10}$$

$$\Psi(t) := \begin{bmatrix} \psi^{\rm T}(t) \\ \psi^{\rm T}(t-T) \\ \vdots \\ \psi^{\rm T}(t-qT+T) \end{bmatrix} \in \mathbb{R}^{q\times n_0}, \tag{11}$$

$$V(t) := \begin{bmatrix} v(t) \\ v(t-T) \\ \vdots \\ v(t-qT+T) \end{bmatrix} \in \mathbb{R}^q. \tag{12}$$

If we take $q = L$, $t = L$ (L is the data length), then $Y(t)$ and $\Psi(t)$ contain all the measured data. From (9)-(12), we have

$$Y(t) = \Psi(t)\vartheta + V(t). \tag{13}$$

Notice that $V(t)$ is a white noise vector with zero mean and define a quadratic criterion function [12],

$$J(\vartheta) = [Y(t) - \Psi(t)\vartheta]^{\rm T}[Y(t) - \Psi(t)\vartheta]. \tag{14}$$

Provided that $\psi(t)$ is persistently exciting, $\Psi^{\rm T}(t)\Psi(t)$ is an invertible matrix. Minimizing $J(\vartheta)$ in (14) gives the least-squares estimate of ϑ:

$$\hat{\vartheta}(t) = [\Psi^{\rm T}(t)\Psi(t)]^{-1}\Psi^{\rm T}(t)Y(t). \tag{15}$$

Since $\psi(t-jT)$ in $\Psi(t)$ contains unmeasurable variables $e(t-jT)$ and $v(t-jT)$ (see the definitions of $\varphi_{\rm n}(t)$ and $\Phi(t)$), so the estimate of ϑ is impossible to compute by (15). Here, a new iterative identification algorithm based on the hierarchical identification principle is developed to solve the difficulty. The details are as follows. Let $l = 1, 2, 3, \cdots$ be an iteration variable, and $\hat{\vartheta}_l(t) := \begin{bmatrix} \hat{\theta}_{{\rm s},l}(t) \\ \hat{\theta}_{{\rm n},l}(t) \end{bmatrix}$ be the estimate of $\vartheta := \begin{bmatrix} \theta_{\rm s} \\ \theta_{\rm n} \end{bmatrix} \in \mathbb{R}^{n_0}$, the unknown variables $e(t-jT)$ and $v(t-jT)$ in the information vector are replaced with their estimates $\hat{e}_l(t-jT)$ and $\hat{v}_l(t-jT)$, then the estimate of $\varphi_{\rm n}(t)$ is

$$\begin{aligned}
\hat{\varphi}_{{\rm n},l}(t) &:= [-\hat{e}_l(kT-T), -\hat{e}_l(kT-2T), \cdots, \\
&\quad -\hat{e}_l(kT-n_cT), \hat{v}_l(tT-T), \hat{v}_l(t-2T), \cdots, \\
&\quad \hat{v}_l(t-n_dT)]^{\rm T} \in \mathbb{R}^{n_c+n_d}.
\end{aligned} \tag{16}$$

Replacing $\varphi(t)$ in $\psi(t)$ with $\hat{\varphi}_{{\rm n},l}(t)$, then we have the estimate of $\psi(t)$ as

$$\hat{\psi}_l(t) = \begin{bmatrix} \varphi_{\rm s}(t) \\ \hat{\varphi}_{{\rm n},l}(t) \end{bmatrix} \in \mathbb{R}^{n_0}. \tag{17}$$

From (7), we have

$$e(t-jT) = y(t-jT) - \varphi_{\rm s}^{\rm T}(t)\theta_{\rm s}.$$

If θ_s in the above equation is replaced with $\hat{\theta}_{s,l-1}(t)$, then the estimate of $e(t-jT)$ can be computed by

$$\hat{e}_l(t-jT) = y(t-jT) - \varphi_s^{\mathrm{T}}(t-jT)\hat{\theta}_{s,l-1}(t). \tag{18}$$

From (9), we have

$$v(t-jT) = y(t-jT) - \psi^{\mathrm{T}}(t-jT)\vartheta.$$

Replacing $\psi(t)$ and ϑ in the above equation with their estimates $\hat{\psi}_{l-1}(t)$ and $\hat{\vartheta}_{l-1}$, respectively, we can compute the estimate $\hat{v}_l(t-jT)$ by

$$\hat{v}_l(t-jT) = y(t-jT) - \hat{\psi}_{l-1}^{\mathrm{T}}(t-jT)\hat{\vartheta}_{l-1}(t). \tag{19}$$

Define

$$\hat{\Psi}_l(t) = [\hat{\psi}_l(t), \hat{\psi}_l(t-T), \cdots, \hat{\psi}_l(t-qT+T)]^{\mathrm{T}} \in \mathbb{R}^{q\times n_0}. \tag{20}$$

The least-squares estimate of the parameter vector ϑ is obtained by replacing $\Psi(t)$ with $\Psi_l(t)$ in (15),

$$\hat{\vartheta}_l(t) = [\hat{\Psi}_l^{\mathrm{T}}(t)\hat{\Psi}_l(t)]^{-1}\hat{\Psi}_l^{\mathrm{T}}(t)Y(t),\ l = 1,2,3,\cdots. \tag{21}$$

Equations (16)-(21) form the least-squares based iterative (LSI) algorithm for estimating ϑ:

$$\hat{\vartheta}_l(t) = [\hat{\Psi}_l^{\mathrm{T}}(t)\hat{\Psi}_l(t)]^{-1}\hat{\Psi}_l^{\mathrm{T}}(t)Y(t), \tag{22}$$

$$Y(t) = [y(t), y(t-T), \cdots, y(t-qT+T)]^{\mathrm{T}}, \tag{23}$$

$$\hat{\Psi}_l(t) = [\hat{\psi}_l(t), \hat{\psi}_l(t-T), \cdots, \hat{\psi}_l(t-qT+T)]^{\mathrm{T}}, \tag{24}$$

$$\hat{\psi}_l(t) = \begin{bmatrix} \varphi_s(t) \\ \hat{\varphi}_{n,l}(t) \end{bmatrix}, \tag{25}$$

$$\begin{aligned}\varphi_s(t) = [&-y(t-T), -y(t-2T), \cdots, -y(t-nT),\\ &\phi_1^{\mathrm{T}}(t), \phi_2^{\mathrm{T}}(t)]^{\mathrm{T}},\end{aligned} \tag{26}$$

$$\begin{aligned}\phi_i(kT) = [&u_i(kT), u_i(kT-T), u_i(kT-2T), \cdots,\\ &u_i(kT-nT), u_i(kT-T+T_i),\\ &u_i(kT-2T+T_i), \cdots, u_i(kT-nT+T_i),\\ &\cdots, u_i(kT-T+(q_i-1)T_i),\\ &u_i(kT-2T+(q_i-1)T_i), \cdots,\\ &u_i(kT-nT+(q_i-1)T_i)]^{\mathrm{T}},\end{aligned} \tag{27}$$

$$\begin{aligned}\varphi_{n,l}(t) = [&-\hat{e}_l(t-T), -\hat{e}_l(t-2T), \cdots, -\hat{e}_l(t-n_cT),\\ &\hat{v}_l(t-T), \hat{v}_l(t-2T), \cdots, \hat{v}_l(t-n_dT)]^{\mathrm{T}},\end{aligned} \tag{28}$$

$$\begin{aligned}\hat{e}_l(t-jT) &= y(t-jT) - \varphi_s^{\mathrm{T}}(t-jT)\hat{\theta}_{s,l-1}(t),\\ &\quad j = 1,2,\cdots,n_c,\end{aligned} \tag{29}$$

$$\begin{aligned}\hat{v}_l(t-jT) &= y(t-jT) - \hat{\psi}_{l-1}^{\mathrm{T}}(t-jT)\hat{\vartheta}_{l-1}(t),\\ &\quad j = 1,2,\cdots,n_d.\end{aligned} \tag{30}$$

The LSI algorithm adopts the idea of updating the estimate ϑ using a fixed data batch with the data length L at each iteration, and thus has higher parameter estimation accuracy than the recursive least-squares (RLS) algorithm.

To initialize the LSI algorithm, we take $\hat{\vartheta}_0(t) = \mathbf{1}_{n_0}$ with $\mathbf{1}_{n_0}$ being an n-dimensional column vector whose elements are all 1, $\hat{e}_0(t-j)$ and $\hat{v}_0(t-j)$ are random numbers.

To summarize, we list the steps involved in the LSI algorithm to compute $\hat{\vartheta}_l(t)$ as l increases:

1) Choose the data window length $q \gg n_0$, collect the input and output data $\{u_1(kT+j_1T_1), u_2(kT+j_2T_2), y(t)$: $j_i = 1,2,\cdots,q_i-1$, $k = 1,2,\cdots,q-1\}$, and given the estimation accuracy ε and let $t = kT = qT$ and $\hat{\vartheta}_0(t) = \mathbf{1}_n$.
2) Collect the input and output data to form output vector $Y(t)$ by (23), form $\phi_1(t)$ and $\phi_2(t)$ by (27) and $\varphi_s(t)$ by (26).
3) To initialize, let $l = 1$, $\hat{e}_0(t-jT)$ and $\hat{v}_0(t-jT)$ equal random numbers, form $\hat{\varphi}_{n,0}(t)$ by (28) and $\hat{\psi}_0(t)$ by (25).
4) Compute $\hat{e}_l(t)$ and $\hat{v}_l(t)$ by (29) and (30).
5) Form $\hat{\psi}_{n,l}(t)$ by (28), $\hat{\psi}_l(t)$ by (25) and $\hat{\Psi}_l(t)$ by (24).
6) Update the estimate $\hat{\vartheta}_l(t)$ by (22).
7) Compare $\hat{\vartheta}_l(t)$ with $\hat{\vartheta}_{l-1}(t)$: if
$$\|\hat{\vartheta}_l(t) - \hat{\vartheta}_{l-1}(t)\| < \varepsilon,$$
then terminate the procedure and obtain the iterative times $l_0 = l$ and estimate $\hat{\vartheta}_{l_0}(t) = \hat{\vartheta}_{l_0}(kT)$, and increment k by 1, i.e., $t := (k+1)T$, set $\hat{\vartheta}_0(t) = \hat{\vartheta}_{l_0}(kT)$ and go to step 2; otherwise, increment l by 1 and go to step 4.

IV. The Recursive Least Squares Algorithms

For comparison, we give the recursive least squares algorithm to identify parameter vector ϑ to identify (5),

$$\begin{aligned}
\hat{\vartheta}(t) &= \hat{\vartheta}(t-T) + L(t)[y(t) - \hat{\psi}^{\mathrm{T}}(t)\hat{\vartheta}(t-T)],\\
L(t) &= \frac{P(t-T)\hat{\psi}(t)}{1+\hat{\psi}^{\mathrm{T}}(t)P(t-T)\hat{\psi}(t)},\\
P(t) &= [I - L(t)\hat{\psi}^{\mathrm{T}}(t)]P(t-T),\\
\hat{\psi}(t) &= [\varphi_s^{\mathrm{T}}(t), -\hat{e}(t-T), -\hat{e}(t-2T), \cdots, -\hat{e}(t-n_cT),\\
&\qquad \hat{v}(tT-T), \hat{v}(t-2T), \cdots, \hat{v}(t-n_dT)]^{\mathrm{T}},\\
\hat{e}(t) &= y(t) - \varphi_s^{\mathrm{T}}(t)\hat{\theta}_s(t),\ \hat{v}(t) = y(t) - \hat{\psi}^{\mathrm{T}}(t)\hat{\vartheta}(t).
\end{aligned}$$

To initialize the above algorithm, we take $P(0) = p_0I$ with p_0 normally a large positive number, e.g., $p_0 = 10^6$, and $\hat{\vartheta}(0)$ some small real vector, e.g., $\hat{\vartheta}(0) = \mathbf{1}_{n_0}/p_0$ with $\mathbf{1}_{n_0}$ being an n-dimensional column vector whose elements are all 1.

V. Example

Example For the system depicted in Figure 1, take the process model to be

$$P_{c1}(s) = \frac{1}{50s+1}, \quad P_{c2}(s) = \frac{1}{60s+1},$$

and $h = 1$s, $p_1 = 2$, $p_2 = 3$, hence $T_1 = 2$s, $T_2 = 3$s and $T = 6$s, the corresponding discrete-time state space models are

$$\begin{cases} x_1(kT+T) = 0.88692x_1(kT) \\ \qquad\qquad +[1.8098, 1.8837, 1.9605]\begin{bmatrix} u_1(kT) \\ u_1(kT+2) \\ u_1(kT+4)\end{bmatrix} \\ y_1(kT) \quad = 0.02x_1(kT), \end{cases}$$

and

$$\begin{cases} x_2(kT+T) = 0.90484x_2(kT) \\ \qquad\qquad +[2.7835, 2.9262]\begin{bmatrix} u_2(kT) \\ u_2(kT+3)\end{bmatrix} \\ y_2(kT) \quad = 0.016667x_2(kT), \end{cases}$$

then the input and output representation is

$$\begin{aligned}&(1-1.91202z^{-1}+0.91393z^{-2})y(kT)\\&\quad=(0.036196z^{-1}-0.034431z^{-2})u_1(kT)\\&\qquad+(0.037673z^{-1}-0.035836z^{-2})u_1(kT+2)\\&\qquad+(0.039211z^{-1}-0.037299z^{-2})u_1(kT+4)\\&\qquad+(0.046392z^{-1}-0.044573z^{-2})u_2(kT)\\&\qquad+(0.048771z^{-1}-0.046859z^{-2})u_2(kT+3).\end{aligned}$$

Introducing a noise model $e(kT)=(1-0.7z^{-1})v(kT)$ in the above equation, we have

$$\begin{aligned}&(1-1.91202z^{-1}+0.91393z^{-2})y(kT)\\&\quad=(0.036196z^{-1}-0.034431z^{-2})u_1(kT)\\&\qquad+(0.037673z^{-1}-0.035836z^{-2})u_1(kT+2)\\&\qquad+(0.039211z^{-1}-0.037299z^{-2})u_1(kT+4)\\&\qquad+(0.046392z^{-1}-0.044573z^{-2})u_2(kT)\\&\qquad+(0.048771z^{-1}-0.046859z^{-2})u_2(kT+3)\\&\qquad+(1-0.7z^{-1})v(kT).\end{aligned}$$

Here, $\{u_i(kT+j_iT_i)\}$ is taken as a persistent excitation signal sequence with zero mean and unit variance, and $\{v(kT)\}$ as a white noise sequence with zero mean and variance $\sigma^2=0.80^2$ and $\sigma^2=2.0^2$. Applying the LSI algorithm and RLS algorithm to estimate the parameters of the above transfer function model, the parameter estimation errors $\delta:=\|\hat{\vartheta}_l(kT)-\vartheta\|/\|\vartheta\|$ (the LSI algorithm with $l=6$ iterations) or $\delta:=\|\hat{\vartheta}(kT)-\vartheta\|/\|\vartheta\|$ (the RLS algorithm) versus $t=kT$ are shown in Figures 2-3.

From Figures 2-3, we can arrive at the following conclusions: For different noise variances, the parameter estimation errors δ of both the LSI algorithm and RLS algorithm are becoming gradually smaller as $t=kT$ increases; the parameter estimation errors of the LSI algorithm are smaller than those of the RLS algorithm for the given noise variances. In other words, under the same data length, the parameter estimates given by the LSI algorithm have higher accuracy than those by RLS algorithm.

VI. Conclusions

This paper presents a least-squares based iterative algorithm for two-input multirate systems with colored noises. The method can be extended to other cases, for example, general multi-input multi-output multirate systems with each of the input and output channels having different sampling periods, and with colored noises.

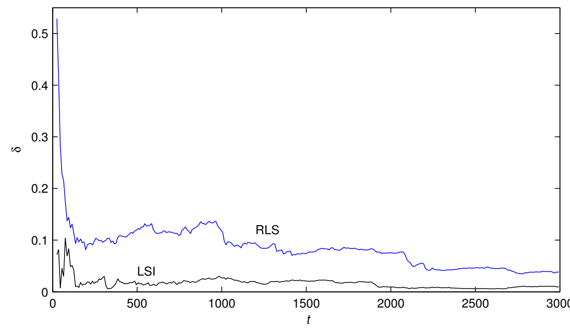

Fig. 2. The parameter estimation errors δ versus t ($\sigma^2=0.80^2$)

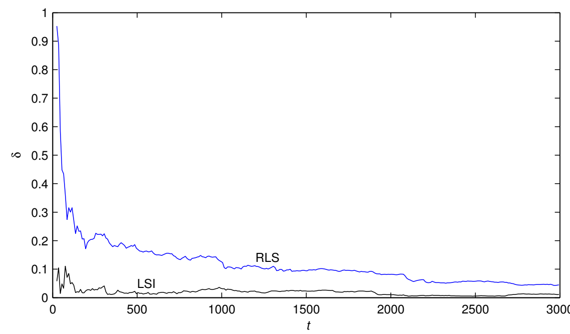

Fig. 3. The parameter estimation errors δ versus t ($\sigma^2=2.00^2$)

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