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Robust estimation of Base Sheet Ash during a wet end break, scanner warm up and initialization

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***Abstract*— During a Wet End break, the loss of paper feed through the paper machine causes the two main scanners to go offline and the remaining parts of the process are operated in open-loop. This causes the stock composition in the Headbox to deviate substantially from the nominal specifications, causing paper quality (after start up) and paper machine runability issues. In this work, the Base Sheet Ash measurement of the scanner is estimated using a Least Absolute Value (LAV) model which can then be used for control of the chalk valve during the breaks to keep the Headbox Ash within specified limits. The model is developed around key Wet End variables which have been identified using PCA and sensitivity analysis. Modeling is performed on plantwide data obtained from actual running of a paper machine in the UK.**

I. Introduction to paper making

Papermaking is a very complex process involving several mechanical and chemical stages with high levels of interaction between them. Although a comprehensive treatment of this subject is beyond the scope of this paper, a brief overview of the main areas concerning this paper are given below.

A. The Paper Machine

A Paper Mill is generally divided into three main sections, Stock Preparation, the Wet End and the Dry End. The Wet End and the Dry End are joined and in combination are usually referred to as the Paper Machine. An average sized paper machine is around 200 meters long and 10 meters wide. Paper can be produced at a rate of over 20 tonnes per hour and the speed of the paper through the machine is around 50 Km/h.

The Stock Preparation section start by Pulping; the process of separating the individual fibres in the Virgin pulp using water. After this the fibres are further modified to help them bond. This mechanical process is known as Refinement. Different pulp types are usually added in varying proportions to form a recipe or Furnish for forming a particular grade of paper. Chemicals, water and fillers are then added to form the Thick Stock, a 5% consistency slurry which is subsequently thinned down during several stages of cleaning and dilution to form the Thin Stock, a less than 1% consistency mix. Chemicals which aid the formation of the paper, improved filler/fibre bonding, improved flocculation and improved retention of fillers and fibres are then mixed into the Think Stock.

The Thin Stock then enters the Wet End of the paper machine through the Headbox where it is sprayed onto the paper forming mesh called the Wire. The purpose of the Wire section is to retain the solids while the water is drained away. The next stage is the press section which comprises of several stages of press rolls where the paper is flattened, consolidated and further dried. Following the press section are the first stage of dryer sections where the paper is dried using steam heated rollers. At the end of the Wet End, the product is a paper web commonly referred to as the Base Sheet.

The Base Sheet then enters the Dry End of the machine. The main functions of the Dry End are first to add the coating to the Base Sheet, and then a final drying section. This is then succeeded by the calendering stage, where irregularities in the sheet formation are corrected and its surface smoothness is improved. At the end of the Dry End, the paper web is rolled onto a large reel referred to as the Jumbo Reel and the this final paper is referred to as the Reel End paper.

B. Filler content

Fillers (also referred to as Ash or Chalk) play an important role in paper making. Fillers are minerals (clay) which are added to the Thick Stock and they serve two important functions. First they fill the gap which is formed between wood fibres and secondly, the larger the quantity of filler used in the Thick Stock, the less amount of fibre needs to be used. This helps to make paper making economical since fillers are substantially cheaper than wood. They also aid in improving papers printing properties.

Although adding more filler leads to economical benefits, it has detrimental effects on the paper making process and the paper itself. A high proportion of filler leads to paper discoloration, brittleness and poor opacity. However, the worst effect of a high filler content is on the paper machine runability. Wood fibres are long and thin strands which form a mesh type of structure on the Wire. The interlocking between the fibres helps to build Tensile Strength in the paper web. Filler particles on the other hand are particle shaped and do not bond well to either themselves or wood fibres. A high amount of filler will lead to reduced paper Tensile Strength in both the Machine Direction (MD) and Cross Direction (CD). In the MD case this will lead to an increased occurrence of Web Breaks, either at the Wet End or the Dry End. A web or paper break occurs when the web of paper in the machine breaks (tears) and depending on where the break occurs it may take several hours for a new feed to be fed into the machine. Tensile Strength is important because the paper is pulled through the machine by a relatively large force and at very high speeds.

C. Paper Breaks

What distinguishes paper making from the majority of other manufacturing and process production lines is the phenomenon of paper breaks. A typical paper machine is about 200 meter long from the Headbox to the Reel End, but the path of the paper web through the machine (i.e. rolling around cylinders, etc.) is typically 600 meters. This 600 meter long web of paper can tear anywhere along its length for a variety of reasons, be it poor paper formation, low tensile strength, incorrect cylinder loads, incorrect chemical additions, etc. Once the web is torn, the machine needs to be stopped and a new web fed into the machine. The machine down time is hugely costly for the manufacturer therefore it is desirable to not have such breaks in the first place, and if they happen, to recover as quickly as possible.

Despite improvements in the performance of chemicals and also the control systems, paper breaks are still relatively common. A well run paper machine will experience two or three breaks a day. The average recovery time from a break is one to two hours, which depends on where in the machine the paper breaks. Along the length of the machine, there are several points where the sheet of paper can be diverted away from the machine. If the break is in the Wet End, the stock can be diverted to the Couch Pit and if the break occurs in the Dry End the sheet can be diverted to

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the Broke Hydrapulper from several points. These material will then be recycled by being mixed back into the virgin fibre mix. In severe break situations the machine will be stopped completely, in which case it will take several hours for it to start up again.

For example suppose that a paper break has occurred at the Soft Nib Calendar which is situated at the end of the the Dry end of the paper machine, just before the Reel End. In this case, the machine speed is reduced, but the Stock preparation section and the Wet End will continue to operate (i.e. Stock comes on the wire). The paper web will then go through the normal stages of press, drying and film press sections, however right before the Soft Nib Calendars, the sheet is diverted away from the machine and Broked. Borke refers to when the paper which cannot be sold, is broken down with water and the resulting slurry is used again in the paper furnish. Once the machine men are ready to resume paper making, they will then divert the paper sheet back to its normal path and the machine speed is gradually increased to its setpoint.

This approach allows the sections of the machine which are located before the paper break to keep operating. Although it appears that material and energy is wasted by keeping the machine running during the breaks, keeping the machine ticking over allows the machine to make paper faster than from a cold start which is more economical over any given period. However, keeping the machine running during a break poses difficult control challenges, one of which is the subject of this work and is described in the next section.

II. Control of the Paper Machine

A typical paper machine has thousands of sensors which measure various process values to help control the machine. However, there are two points of measurement which are vital for the control of the paper machine. These are the two Sheet Scanners; the Base Sheet Scanner and the Reel End Scanner. These are large scanners which sweep across the paper web in a zig-zag fashion and measure several paper properties such as Sheet Weight, Sheet Ash (filler), floc size, floc intensity, caliper, etc. These measurements are then fed through to the plants DCS systems to adjust various chemical and product additions.

The Base Sheet and Reel End scanners are respectively situated at the beginning and end of the Dry End of the paper machine. Of the two scanners, it is the measurements obtained from the Base Sheet Scanner which is largely used for control of the paper machine. The Reel End scanner is primarily used to control the paper coating process. When a paper break occurs in the Dry End of the paper machine, paper will still pass through the Base Sheet Scanner, but not the Reel End scanner. This does not pose too much of a challenge since the control system still has measurement from the Base Sheet to maintain the system operation.

A serious difficulty arises when the paper break is in the Wet End. In this situation paper does not pass through either of the scanners and there are no paper measurements using which the machine can be controlled. Common practice in paper making is to operate the machine in open loop during such breaks. During open loop control of the Wet End, all control variables which require Base Sheet or Reel End scanner measurement are frozen to their last value a priori occurrence of the break. In some cases, such as Ash control, there are potential for serious machine disturbances.

A. *Chalk Valve control*

The Chalk Valve controls the amount of filler which is added to the Thick Stock at the Machine Chest. The position of this valve is controlled in according to the measurement of the Base Sheet Ash and the current grade and grammage of the paper being made. As mentioned, when a Wet End break occurs, no information is available from the Base Sheet Scanner and the chalk valve position is frozen to its last control value. This may lead to several scenarios. If the break duration is short and the machine was in steady operation before the break, then the stock mix will generally not vary too much.

However, if for some reason the controller was making corrective action just prior the break occurred, then the valve position is fixed far away from its nominal value and the longer the break lasts, the higher the chances of either too much or too little filler being mixed into the Tick Stock. The latter case is a less serious issue and other than affecting paper properties such as caliper and smoothness, does not generally affect the machine runability. However, the former case leads to a reduction in paper tensile strength and problems in starting the machine after the break. This manifests itself into a train of breaks, where a primary break occurs causing the thick stock ratios to deviate far from specifications, which then leads to several secondary breaks because the stock is of poor quality and the machine has trouble starting up again.

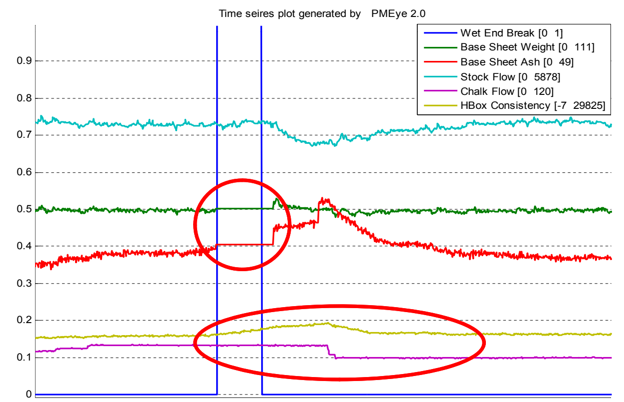

Fig. 1. The Wet End Break is shown by the thick blue line. When the break occurs, the Base Sheet Ash reading freezes. The DCS system is blind to the rising value of the Headbox Ash which is reflected in the bump in the Headbox consistency value after the break.

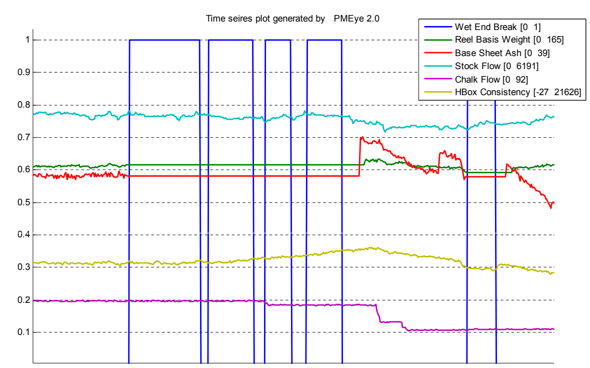

Fig. 2. This figures shows the undesirable situation of a train of breaks. If the Wet End is greatly disturbed during the first (primary) break, it usually leads to problems starting up again and a series of breaks follow. Here it can be seen that during the train of breaks, the Base Sheet Ash reading is not available which means that the DCS system cannot correct Chalk flow and the Headbox Consistency keeps rising

Figures 1 and 2 show two examples of such breaks at the paper machine. Data are from a run of 90gsm paper. In addition to the main problem stated above, the observant reader would have also noticed two additional issues. First, that the scanner does not come back online immediately after the break has ended. This is because the scanners take 5 minutes to warm up. In both figures, there is also a 'jump' in the Ash reading which occurs around 30 minutes after scanner's first live reading. This is due to the fact that the scanners are 'initialized' (recalibrated) every 30 minutes. Part of this process is to calculated an 'air gap' temperature correction factor which is used by the scanner to adjust its reading. When the scanner has been operating for a while the temperature of the air in the gap between the sheet and the scanner is relatively steady. However, when paper is first fed through the scanner, the temperature of the air will rise rapidly from room temperature to around 70 degrees (recall that the Base Sheet which goes through this scanner has just left the steam drying section). Consequently at the first recalibration point after the break, there is a large jump (correction) in the Base Sheet Ash reading

There are therefore three periods during which the Base Sheet Scanner reading is either invalid or unreliable. During the break itself when there is no paper through the scanner, during the first 5 minutes following the break when there is paper through the scanner, but there is no scanner reading, and during the following 30 minutes during which the air gap temperature correction factor is incorrect leading to an incorrect Ash measurement. Our methodology in this work is to estimate the correct value of the Base Sheet Ash during these three periods. This estimate can then be used as a soft-sensor instead of the real Sheet Ash measurement so that during the break the Headbox Ash levels remain within specification. This is a novel approach not employed previously in the paper industry.

III. Modeling challenges and methods

A. Modeling challenges

Modeling in the paper industry is particulary difficult due to several features which are unique to this industry. Some of the most pertinent challenges are described below;

Multiple products A typical paper machine produces a large variety of products. Products are classified according to their Basis Weight, Grade of finish, and coating. Together they may lead to over a hundred combinations of product grammage/grade/finish. In this situation it becomes increasingly difficult to want to develop either a unified model which caters for all these operating points, or to want to have a dedicated model for each of the combinations. Within an industrial settings, there are simply not the resources or manpower to want to manage such a complex modeling scheme.

Complex processes Paper making involves several complex mechanical and chemical processes which are heavily interacting. This is especially the case at the Wet End of the machine where important factors such as filler and ash retention are sharply affected by both the performance of the chemical reactions taking place, and also the physical characteristics of the Wire and the set up of the forming foils and vacufoils. Many of these phenomena cannot be easily medelled. Under these circumstances it is very difficult to develop analytical models which can accurately predict paper properties.

Prodcut Trials Hundreds of chemicals are involved in the paper making process. The suppliers of these chemicals are always keen to replace their product portfolio with newer and more profitable products. However, for a paper maker to replace a chemical product in their process requires several trial runs to ensure that the new chemical does not lead to undesirable side effects and that the promised performance improvements are seen en-situ. As a result a typical paper machine will go through many trial runs per week and the paper made during these trial runs is expected to meet the standard specifications and sell. It may even be that two or more product trial runs coincide. Such trials usually push the machine into new operation regions and it is not possible to develop a new model every time a new trial is planned.

Machine changes Similar to the product trials, the paper machine itself is also subject to constant variations. Components and sensors across the machine from the Stock Preparation to the Dry End of the machine are constantly upgraded for improved performance. Changes can almost be on a daily basis. This may be a major change such a new steam system for the dryers, or a major change such a new plate for the refiners or new agitator blades.

Disturbances There are two kinds of disturbances in a paper machine. Cross Direction disturbances and Machine Direction disturbances. Cross Direction disturbances are generally fewer and if present usually are a result of some mechanical defect (such as uneven rollers, fibre build up on rollers, incorrect jet commander settings etc.). Machine direction issues are however more complicated to diagnose and can have more severe effects on the paper machine. Many of the Machine Direction disturbances are caused by incorrect operational procedures or poor maintenance regimes. For example whenever a filter in a chemical addition line is not primed correctly prior being taken online, it will lead to a temporary stoppage of chemical addition. For some chemicals such as Percol and Bentonite which have a strong and instant effect on the retention of stock on the wire, this will show as sudden large spikes in the value of Base Sheet Ash on the scanners. Other sources of disturbances are poor agitation in the mixing tanks which will lead to the sudden release of additives into the blend pipe. Regardless of the type of disturbance, the common feature between them is that they are not gaussian and are often very large in magnitude.

B. Modeling technique

It is clear that an analytical approach will not be useful given the above challenges. Simple data driven modeling techniques such as Least Squares (LS) modeling are a good candidate because their ease of computation would allow for a model which is updated continuously online. The advantage of such an approach is that the model will follow the machine into the new operating space. Although these type of models will not be able to give accurate predictions for a long period, this is actually not required in this case. The objective is simply to be able to estimate the Base Sheet Ash during the breaks where stock still comes on the Wire. This usually means estimating for a maximum of around 2 hour. If the break is expected to last for more than one hour, the stock flow is stopped and the Wire runs dry.

The de facto modeling methods of choice in these situations is the least squares regression . The dominance and popularity of the least squares regression can be ascribed, at least partially, to the fact that the theory is simple, well developed and documented. The least squares regression is optimal and results in the maximum likelihood estimators of the unknown parameters of the model if the errors are independent and follow a normal distribution with mean zero and a common variance [1]. The least squares regression is very far from optimal in many non-Gaussian situations, especially when the errors follow distributions with longer tails [2]. For the regression problems, Huber [3] stated that "just a single grossly outlying observation may spoil the least squares estimate, and, moreover, outliers are much harder to spot in the regression than in the simple location case". The outliers occurring with extreme values of the regressor variables can be especially disruptive.

Least Absolute Value (LAV) regression overcomes these drawbacks and provides an attractive alternative [4]. It is less sensitive than least squares regression to extreme errors, has implicit mechanics to reject bad data and does not require a normal distribution of data which is very unrealistic in practical situations [5]. To see why the LAV approach offers this implicit bad data rejection property, let x_i be an arbitrary number and define m_2 by the value of m which minimizes the sum of the squared differences (i.e. the least squares solution) between m and x,

$$m_2 := \arg\min_m \sum_{i=1}^{N} (m - x_i)^2 \tag{1}$$

It is straightforward to find the minimum by setting the partial derivative of the sum with respect to m equal to zero. We obtain,

$$0 = \sum_{i=1}^{N} 2(m_2 - x_i) \quad (2)$$

or,

$$m_2 = \frac{1}{N} \sum_{i=1}^{N} x_i \quad (3)$$

Notice that in the LS case, the solution is the mean of the data observation values. In this case if any of the x_i are very large (say due to a measurement error) it will directly effect the solution. To see the same effect in the LAV case, define now m_1 to be the solution obtained by minimizing the least absolute value as,

$$m_1 := \arg\min_m \sum_{i=1}^{N} \mid m - x_i \mid \quad (4)$$

To find the minimum, the partial derivative with respect to m is set equal to zero,

$$0 = \sum_{i=1}^{N} sgn(m_1 - x_i) \quad (5)$$

where,

$$sgn(x) = \begin{cases} +1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases} \quad (6)$$

The solution indicates that m_1 should be chosen so that m_1 exceeds x_i for $N/2$ terms; m_1 is less than x_i for $N/2$ terms; and if there is an x_i left in the middle, m_1 equals that x_1. This defines m_1 as the median (for N even, the solution is an interval). Relating the mean and the median to the LS and LAV problems helps to easily see why the LAV approach is much more inherently robust. For example suppose that there are three observations from an experiment $\{1, 1.1, 0.9\}$. the LAV solution in this case is 1 and the LS solution is also 1. However, if due to equipment error, or a measurement anomaly, the observations became $\{1, 100, 0.9\}$, then the LAV solution is *still* 1, but the LS solution will change to ≈ 10. This is because in the LAV case, as long as the *bad* data remains in the same side of the median, it has zero effect on the actual value of the median, but in the LS case, any change will be directly reflected in the LS solution.

C. Solving the LAV problem

Let the linear regression model with n observations be identified as shows below,

$$y_i = \sum_{i=1}^{m} x_{ij}\beta_j + \epsilon_i, \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m \quad (7)$$

Let $(x_{i1}, x_{i2}, \ldots, x_{im}, y_i)$ be the ith observations and let $b_0, b_1, \ldots, b_m$ estimated by minimizing the overall absolute values of the differences between the values of $\hat{y}$ and y be the estimates of $\beta_0, \beta_1, \ldots, \beta_m$. So the LAV method is given as follows,

$$\min \left(\sum_{i=1}^{n} \mid y_i - \hat{y}_i \mid \right) \quad (8)$$

The LAV minimization problem actually predates the least squares solution [6] [7]. It's use however was not popular for many years because unlike least squares the solution is difficult and not straightforward. It was not until the implementation of the linear programming algorithms on the digital computer that LAV estimates could be obtained for problems of reasonable size [8]. Charnes et al [9] appear to be the first to point out that (8) could be rewritten as a linear programming problem. Subsequently several other LP versions of the LAV problem were developed by [10][11], [12], [13] The best model known for the linear regression model is the primary linear regression model developed by [10] and [14] In this model, the aim is to minimize the overall absolute difference between observations and estimation values, in other words, minimizing the overall error terms. In this respect, goal programming is used to develop the linear programming model which will be used to minimize the overall total positive and negative deviations. That is achieved by [15],

$$\min \left(\sum_{i=1}^{n} (d_i^+ + d_i^-) \right) \quad (9)$$

subject to,

$$y_i - (b_0 + \sum_{i=1}^{m} x_{ij} b_j + d_i^+ - d_i^-) = 0, \quad (10)$$
$$i = 1, 2, \ldots, n \quad (11)$$
$$j = 1, 2, \ldots, m \quad (12)$$

This is the model that will be used in this work for estimation of the Base Sheet Ash

IV. Base Sheet Estimation

As mentioned the two sheet scanners are not able to give reliable information during the breaks or for the 45 minutes following a break. However, other measurement devices in the Wet end continue to log data. In particular case of the paper machine considered there are around 200 variables in the Wet End process line (i.e. simply variables related to flow of material, and not drive, pump etc measurements) that continue to have a value during the break as long as Stock still flows onto the Wire. Note that if no stock flows on the Wire, then basically the flow of the material through the machine has stopped completely and all data are invalid.

Although it might seem that using as much of the data as possible will lead to an improved model, this is not the case for several reasons. First, the delay associated with each of these variables is different and too many variables means a tedious synchronization process. Secondly, each of these variables will have many disturbances and data anomalies associated with them. Using too may of these will lead to a large quantity of bad data and data irregularities which are often not possible to manually detect and filter. Moreover, and most importantly machine operators will not be able to effectively use a complex model which requires 100s of variables to estimate the Base Sheet Ash. Rather, what is very useful is a simple model which relates some key process variables to the Base Sheet Ash. This helps the machine operators to focus their attention on parts of the process which will be likely to cause a large disturbances.

Some of these wet end variables were ruled out because they were clearly not related to the Ash circulation in the wet end. The results of the variable selection process are omitted here. However, following a tedious process of sensitivity, PCA, Frequency Power spectrum analysis and physical system considerations, 4 variables were selected for the modeling purposes. Namely; Headbox Total Consistency, HeadBox Ash, Whitewater Total Consistency and Whitewater Ash.

The significance of these four variables is explained with the aid of Figure 3 which shows the schematic of the Wire section at the Blackburn Mill. The headbox is a key component of the

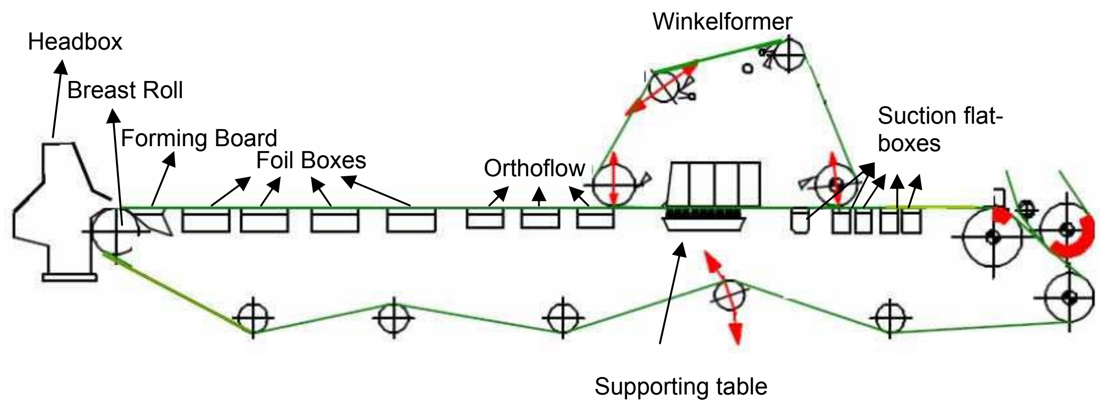

Fig. 3. Showing the Wire section of the paper machine. The Wire itself is shown in green. The 'width' of the Wire at this particular machine is 4.5 meters.

Wet End and it is where Stock is sprayed into the Wire. Several measurement are taken at the Headbox such as pressures, gasses, etc, but two key measurements are the total amount of solids (Total Consistency) and filler (Ash) content. These two measurements are made using spectroscopy techniques and whilst the Total consistency measurement is generally accurate, the filler reading is not very accurate since it is not measured directly. Instead, a particle size threshold is defined, smaller than which all solid particles are considered to be minerals. However, the refining process creates very small fibre fragments which will incorrectly be counted as minerals. When stock is sprayed into the wire, it has a solid contents of less than 1% and by the time it leaves the wire section the consistency is increased to around 20%. Whitewater is the water which is drained on the wire and is collected to be used as 'sweetener' back in the pulping process. The reason for the name of this water is that as the water drains it is cloudy due to the fine mineral particles which are not stopped on the wire mesh and drain away. Any filler particle which has not flocculated or bounded with filler particles will be drained away because the Wire mesh size is not sufficiently small to retain these fine particles.

A. Decision making flow chart

If the machine is operating normally, data from the four variables is stored into a modeling buffer. The buffer only stores up to the last 200 minutes of data. Experiential trials found that for longer periods, the model was able to give better longer term predictions, but not very accurate short term predictions. When a Wet End Break occurs, the data in the model buffer is used to generate a new Base Sheet Ash model and control is locked onto the model value. Note that the sampling time for the Wet End DCS system is 5 seconds, but a new model is generated on average in less than 1 second (the value differs due to a different size of buffer data at break time). This means if the paper breaks at the current sample time, there is sufficient time to generate a new model and compute the replacement Ash estimate, all by the next sample time.

The model will continue to be used to generate the Base Sheet Ash for the duration of the break for as long as Stock flow is not zero. Once the break has ended, control remains locked onto the model until the second scanner initialization at which point it reverts back to the scanner reading. The simplicity of this approach is that the model will update itself as the machine is moved from one paper grade to a different one. This eliminates the need for complex scheduling systems and more importantly it means that if new products are introduced into the product portfolio, no action will be required as far as the estimation system goes. A flow chart for the process is given in Figure 4.

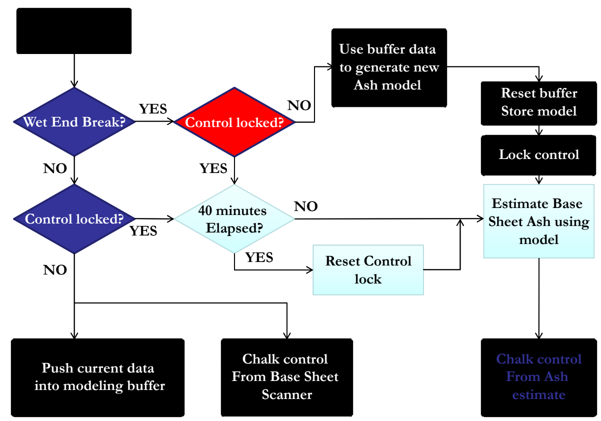

Fig. 4. flowchart for modeling procedure

B. Results

The methodology described above was programmed and implemented on the paper machine data. Numerous different studies of the model have been made relating to different operating points and product grades and to assess the stability and robustness of the model. The Ash estimates values were validated with manual lab readings. Due to lack of space we suffice to only include some examples below. In all the examples the prediction time span is highlighted. Note that in all the figures, when the model is not estimating, it is following the actual scanner output and so the Base Ash Estimate and actual Base Sheet Ash scanner trends are superimposed.

V. Conclusions

This work is part of a larger project whose ultimate aim is to reduce, minimize and possibly eliminate the disturbances which effect the Wet End of a paper machine when a paper break occurs. When the Wet End in general, and the Furnish composition specifically, are largely disturbed as a results of a paper break and prolonged open loop control, this will lead to at best bad paper being made after the break (where the paper is not saleable) or at worst it will lead to runability issues where the machine has trouble starting back up because the paper keeps tearing due to a dropped tensile strength caused by the increases in the Headbox Ash. The ultimate aim of this project is to make the Wet end 'blind' to the occurrence of a break. In other words, through application of such techniques, to estimate and replace all the information which is not available during the break such that the DCS can maintain the

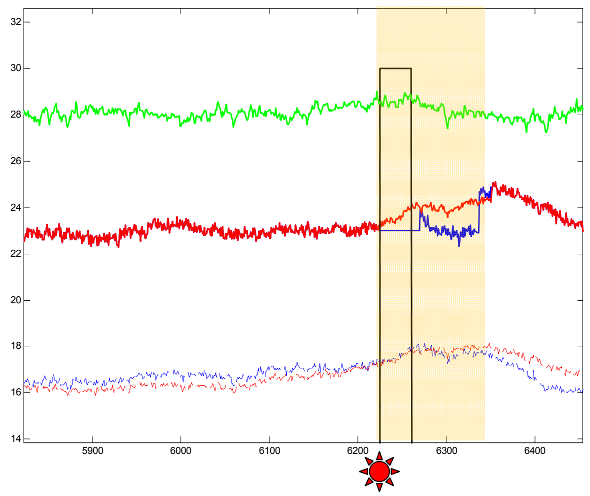

Fig. 5. Figure legend:- Black:Wet End break, Green:Stock Flow, Solid-Blue: Base Sheet Scanner Ash, Dashed-Blue: Headbox Ash, Dashed-Red: Headbox consistency, Solid-Red: Base Sheet Ash model estimate, Red-Sun: New model generated, Shaded region: Prediction interval

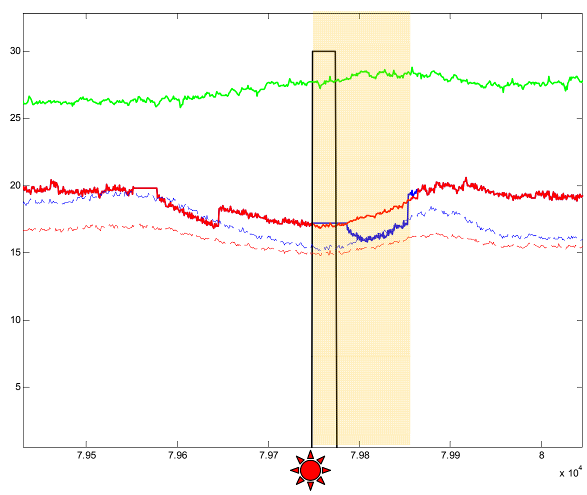

Fig. 6. Figure legend:- Black:Wet End break, Green:Stock Flow, Solid-Blue: Base Sheet Scanner Ash, Dashed-Blue: Headbox Ash, Dashed-Red: Headbox consistency, Solid-Red: Base Sheet Ash model estimate, Red-Sun: New model generated, Shaded region: Prediction interval

Wet End at its desirable state as if it was making, and ready to make good paper once the break issue has been resolved.

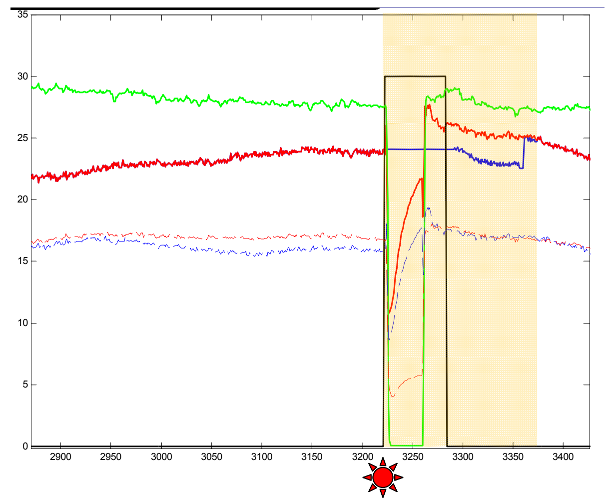

Fig. 7. Figure legend:- Black:Wet End break, Green:Stock Flow, Solid-Blue: Base Sheet Scanner Ash, Dashed-Blue: Headbox Ash, Dashed-Red: Headbox consistency, Solid-Red: Base Sheet Ash model estimate, Red-Sun: New model generated, Shaded region: Prediction interval

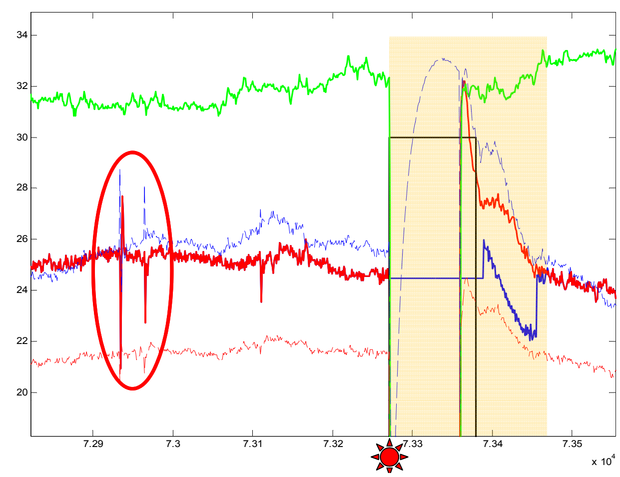

Fig. 8. Notice the large disturbances before the break, despite which the prediction is still very accurate. Figure legend:- Black:Wet End break, Green:Stock Flow, Solid-Blue: Base Sheet Scanner Ash, Dashed-Blue: Headbox Ash, Dashed-Red: Headbox consistency, Solid-Red: Base Sheet Ash model estimate, Red-Sun: New model generated, Shaded region: Prediction interval

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