2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Combined Adaptive Fuzzy Control for Uncertain MIMO Nonlinear Systems

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Abstract—**A combined adaptive fuzzy control method of a class of uncertain MIMO nonlinear systems is studied in this paper. In this method, the proposed controllers consist of two parts: the direct and indirect adaptive control terms. Compared with existing methods for controlling MIMO systems, this novel method can trade off fuzzy descriptions for control rules at the same time to achieve better adaptation properties and improve control effect. In addition, most methods need to assume that the minimum approximation error is required to satisfy the square-integrable condition. The method proposed in this paper doesn't need this assumption, and the effect of minimum approximation error could be removed by the adaptive compensation term. Based on Lyapunov stability theory, it can be ensured that all signals of closed-loop system are bounded, and the tracking errors converge to a small neighborhood around zero. Simulation results indicate the validity of the proposed method.**

I. Introduction

During the last two decades, the controller design for nonlinear systems has drawn a lot of attention in the control community, and it has achieved great success based on geometrical technology, and special feedback linearization method [1,2]. However, these methods can be only used for nonlinear systems whose dynamic characteristics are exactly known. To relax these restrictions, some adaptive design approaches are proposed in [3, 4, 5]. In these schemes, an accurate model of the plant is assumed to be available, and known nonlinear functions with respect to unknown parameter linearly appear. However, in many practical situations, these assumptions are not sufficient because it is difficult to describe an accurately dynamic model of a system with known function.

Recently, fuzzy logic control has been widely used in complex and ill-defined systems. Based on the universal approximation theorem, many stable adaptive fuzzy control schemes have been developed to incorporate the expert knowledge systematically. The stability study in such schemes is performed by using the Lyapunov synthesis method. According to the definition in [6], there are two distinct approaches that have been formulated in the design of the fuzzy adaptive control system: direct and indirect schemes. In the direct method, a fuzzy system is used to describe the control action and the parameters of the fuzzy system are adjusted directly to meet the required control objective [7,8-11,12,13]. In the indirect adaptive approach, fuzzy logic systems are used to estimate the plant dynamics and a controller can be obtained based on these estimates [7,8-11, 14,15]. A combined adaptive fuzzy control method has already been proposed in [16] for a class of SISO uncertain nonlinear systems. However, from the viewpoint of the engineering application, most of the plants are MIMO in nature. At present stages, few works on the combined adaptive fuzzy control method are extended to stabilize MIMO nonlinear systems. This paper will give a combined adaptive fuzzy control method for uncertain MIMO systems.

For a class of uncertain nonlinear MIMO systems, this paper proposes a combined adaptive fuzzy control method. In the novel method, the combined adaptive fuzzy controller consists of two parts: the tracking controller and adaptive compensation controller. The tracking controller is the weighted average of direct and indirect adaptive fuzzy controllers. Knowledge of both controlled plant and control action are fully exploited in the controller. The proposed method can trade off fuzzy descriptions for control rules at the same time to achieve better adaptation properties and improve control effect. The adaptive compensation controller is used to compensate the approximation error in fuzzy logic system, and then the square-integrable condition for minimum approximate error should be removed. Base on Lyapunov analysis, all the signals of closed-loop system are proved to be bounded and tracking errors converge to a small neighborhood around zero. The validity of the proposed method is verified by simulation results.

II. Problem Description and Propaedeutics

Consider the following MIMO nonlinear dynamical system

$$\begin{aligned} y_1^{(r_1)} &= f_1(x) + \sum_{j=1}^{p} g_{1j}(x)u_j \\ &\vdots \qquad\qquad \vdots \\ y_p^{(r_p)} &= f_p(x) + \sum_{j=1}^{p} g_{pj}(x)u_j \end{aligned} \tag{1}$$

where $x = [y_1, \dot{y}_1, \cdots, y_1^{(r_1-1)}, \cdots, y_p, \dot{y}_p, \cdots, y_p^{(r_p-1)}]^T$ is state

This work was supported in part by National Natural Science Foundation of China (60674056, 60874056) and the Science Foundation of Educational Department of Liaoning Province (2008312).

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978-1-4244-4524-0/09/$25.00 ©2009 AACC

vector, $u=[u_1,\cdots,u_p]^T$ is control input vector, $y=[y_1,\cdots,y_p]^T$ is output vector, and $f_i(x)$, $g_{ij}(x)$ where $i,j=1,2,\cdots,p$ are unknown nonlinear smooth functions.

Let

$y^{(r)}=[y_1^{(r_1)},\cdots,y_p^{(r_p)}]^T$, $F(x)=[f_1(x),\cdots,f_p(x)]^T$

and

$$G(x)=\begin{bmatrix} g_{11}(x) & \cdots & g_{1p}(x) \\ \vdots & & \vdots \\ g_{p1}(x) & \cdots & g_{pp}(x) \end{bmatrix}.$$

Then, system (1) can be modified as

$$y^{(r)}=F(x)+G(x)u \tag{2}$$

The control objective is to design a combined adaptive fuzzy controller so that the system output y can track the ideal output $y_d(t)=[y_{d_1}(t),\cdots,y_{d_p}(t)]^T$ as accurate as possible, where $y_{d_i}(t)$, $i=1,2,\cdots,p$ and its derivative is known and bounded.

Let

$e_1=y_{d_1}-y_1,\cdots,e_p=y_{dp}-y_p$,

$\underline{e}=[e_1,\dot{e}_1,\cdots,e_1^{(r_1-1)},\cdots,e_p,\dot{e}_p,\cdots,e_p^{(r_p-1)}]^T$

the error dynamic equation can be calculated as

$$\underline{\dot{e}}=A\underline{e}+B[-F(x)-G(x)u+y_d^{(r)}] \tag{3}$$

where $A=diag[A_{01},\cdots,A_{0P}]$ and $B=diag[B_1,\cdots,B_p]$,

$$A_{0i}=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\ B_i=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \text{ and } i=1,2,\cdots p.$$

To design a stable adaptive fuzzy controller, the following assumption is made for the system.

Assumption 1: the matrix $G(x)$ is positive definite, and $\exists\sigma_0,\sigma_1>0,\sigma_0,\sigma_1\in R$, so $\sigma_0 I_p\le G(x)\le\sigma_1 I_p$, where σ_0 and σ_1 can be known or unknown and I_p is a unit matrix.

III. A Combined Adaptive Fuzzy Controller Design

According to the approximation property of the fuzzy logic systems, $\hat{F}(x/\theta_f)$, can be used to approximate the unknown function $F(x)$. Suppose $\hat{f}(x/\theta_f)$ is the i th component in fuzzy logic system $\hat{F}(x/\theta_f)$, the following system is obtained by making use of product inference engine, singleton fuzzier and center average defuzzier:

$$\hat{f}_i(x/\theta_f)=\xi_{fi}^T(x)\theta_{fi},\ i=1,2,\cdots,p \tag{4}$$

where $\theta_{fi}=[\theta_{fi1},\cdots,\theta_{fim_{fi}}]^T\in R^{m_{fi}}$ is parameter vector and $\xi_{fi}(x)=[\xi_{fi1}(x),\cdots,\xi_{fim_{fi}}(x)]^T\in R^{m_{fi}}$ is fuzzy basis function vector, and

$$\xi_{fil}(x)=\frac{\prod_{j=1}^{r}\mu_{F_j^l}(x_j)}{\sum_{l=1}^{m_{fi}}(\prod_{j=1}^{r}\mu_{F_j^l}(x_j))},\ l=1,\cdots,m_{fi} \tag{5}$$

where membership function $\mu_{F_j^l}(x_j)$, $1\le j\le n$ is given beforehand, and fuzzy system $\hat{F}(x/\theta_f)$ can be written as

$$\hat{F}(x/\theta_f)=\xi_f(x)\theta_f \tag{6}$$

where $\theta_f=[\theta_{f1},\cdots,\theta_p]^T\in R^{m_f}$, $m_f=\sum_{i=1}^{p}m_{fi}$, $\xi_f(x)=diag[\xi_{f1}^T(x),\cdots,\xi_{fp}^T(x)]$ is fuzzy basis matrix.

Next, we design a function $\hat{G}(x/\theta_g)$ to approximate unknown item $G(x)$, and select $\hat{g}_{ij}(x/\theta_{gij})=\xi_{gij}^T(x)\theta_{gij}$ to approximate function $g_{ij}(x)$, where $1\le i,j\le p$, $\xi_{gij}(x)\in R^{m_{gij}},\theta_{gij}\in R^{m_{gij}}$ and $m_{gij}>0$. Suppose $G_i(x)$ is the i th column vector of $G(x)$, and it is approximated as

$$\hat{G}_i(x/\theta_{gi})=\xi_{gi}(x)\theta_{gi},\ i=1,2,\cdots,\ p \tag{7}$$

where $\theta_{gi}=[\theta_{gi1},\cdots,\theta_{gip}]^T\in R^{m_{gi}}$, $m_{gi}=\sum_{j=1}^{p}m_{gij}$, $\xi_{gi}(x)=diag[\xi_{gi1}^T(x),\cdots,\xi_{gip}^T(x)]$. Then, $G(x)$ could be approximated as

$$\hat{G}(x/\theta_g)=\xi_g(x)\theta_g \tag{8}$$

where $\theta_g=diag[\theta_{g1},\cdots,\theta_{gp}]$ and $\xi_g(x)=[\xi_{g1}(x),\cdots,\xi_{gp}(x)]$.

Function $u_d(x/\theta_d)$ is designed to approximate the optimal controller u^*. Suppose that

$$u_d(x/\theta_d)=\xi_d(x)\theta_d \tag{9}$$

where $\theta_d=[\theta_{d1},\cdots,\theta_{dp}]^T$, $\xi_d(x)=diag[\xi_{d1}^T(x),\cdots,\xi_{dp}^T(x)]$.

The following combined-type adaptive controller is designed under system (1):

$$u=u_T+u_C \tag{10}$$

where u_T is the tracking controller and u_C is the adaptive compensation controller. u_T is designed as

$$u_T=\alpha u_i+(1-\alpha)u_d \tag{11}$$

where

$$u_i=\hat{G}^{-1}(x/\theta_g)(-\hat{F}^{-1}(x/\theta_f)+y_d^{(r)}+K\underline{e}) \tag{12}$$

In expression (11), weighted factor $\alpha\in[0,1]$ is defined. If fuzzy control rules are more important and more reliable than fuzzy description information, there should be a small α, and vice versa. Especially, there will be the direct adaptive fuzzy controller if $\alpha=0$ and the indirect if $\alpha=1$. In expression (12), parameter $K=diag[K_1,\cdots,K_p]$, where $K_i=[k_{ir_i},\cdots,k_{i1}]$ and $i=1,2,\cdots,p$, is selected to make all roots of polynomial $s^{r_i}+k_{i1}s^{(r_i-1)}+\cdots+k_{ir_i}$ remain in left-side open complex plane. The ideal controller is defined as

$$u^*=G^{-1}(x)(-F(x)+y_d^{(r)}+K\underline{e}) \tag{13}$$

If $F(x)$ and $G(x)$ are known, $\underline{e}$ could converge to zero in u^*. However, $F(x)$ and $G(x)$ are unknown as a matter of fact, so $\hat{F}$ and $\hat{G}$ are used as substitutes.

Combining (1), (10), (11), (12) and (13), the system error equation can be obtained under a series derivation and simplification such that

$$\dot{\underline{e}} = (A - BK)\underline{e} + B[\alpha(\hat{F} - F) + \alpha(\hat{G} - G)u_i + (1-\alpha)G(u^* - u_d) - Gu_C] \tag{14}$$

where $A - BK = diag\left[A_1, \cdots, A_P\right]$, and

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_{ir_i} & -k_{ir_{i-1}} & -k_{ir_{i-2}} & \cdots & -k_{i1} \end{bmatrix}, \; i = 1, 2, \cdots, p.$$

Let $\Lambda = A - BK$, the Lyapunov function $V = \frac{1}{2}\underline{e}^T P \underline{e}$ is considered, where $P = diag\left[P_1, \cdots, P_P\right]$ and P_i is a $r_i \times r_i$ positive definite matrix, and satisfies Lyapunov function $\Lambda^T P + P\Lambda = -Q$, where $Q = diag\left[Q_1, \cdots, Q_P\right]$, Q_i is a $r_i \times r_i$ positive definite matrix like P_i.

The adaptive compensation controller is designed as

$$u_C = k_C \operatorname{sgn}(\underline{e}^T PB) \tag{15}$$

where k_C is a nonnegative constant.

IV. Design of Adaptive Law

The constraint set of parameter vector θ_f, θ_{gi} and θ_d are defined as $\Omega_f = \left\{\theta_f : \left\|\theta_f\right\| \le M_f\right\}$, $\Omega_{gi} = \left\{\theta_{gi} : \left\|\theta_{gi}\right\| \le M_{gi}\right\}$ and $\Omega_d = \left\{\theta_d : \left\|\theta_d\right\| \le M_d\right\}$ respectively, and the compact set of state vector x is defined as $U_x = \left\{x : \left\|x\right\| \le M_x\right\}$, where M_f, M_{gi}, M_d and M_x are constants defined.

Refer to literature [9], the optimum parameters in this paper are defined as

$$\theta_f^* = \arg\min_{\theta_f}\left[\sup_{x\in U_x}\left|\hat{F}(x/\theta_f) - F(x)\right|\right]$$

$$\theta_g^* = \arg\min_{\theta_g}\left[\sup_{x\in U_x}\left|\hat{G}(x/\theta_g) - G(x)\right|\right]$$

$$\theta_d^* = \arg\min_{\theta_d}\left[\sup_{x\in U_x}\left|u_d(x/\theta_d) - u^*\right|\right],$$

and the minimum approximation errors are defined as

$$\omega_i = (\hat{F}(x/\theta_f^*) - F(x)) + (\hat{G}(x/\theta_g^*) - G(x))u_i$$

$$\omega_d = u_d(x/\theta_d^*) - u^*.$$

To design a stable adaptive controller, the following assumption is made.

Assumption 2: The minimum approximation errors are bounded, i.e., there exist constants $\omega_1, \omega_2 > 0$ so that $\left\|\omega_i\right\| \le \omega_1$, $\left\|\omega_d\right\| \le \omega_2$.

Let $\tilde{\theta}_f = \theta_f - \theta_f^*$, $\tilde{\theta}_g = \theta_g - \theta_g^*$ and $\tilde{\theta}_d = \theta_d^* - \theta_d$, according to fuzzy logic systems (6), (8) and (9), the error equation (14) can be written as

$$\dot{\underline{e}} = \Lambda\underline{e} + B[\alpha\xi_f(x)\tilde{\theta}_f + \alpha\xi_g(x)\tilde{\theta}_g u_i + (1-\alpha)G\xi_d(x)\tilde{\theta}_d + \alpha\omega_i - (1-\alpha)G\omega_d - Gu_C] \tag{16}$$

The task of adaptive law is to determine a regulation mechanism for θ_f, θ_g and θ_d such that the tracking error $\underline{e}$ and parameter errors $\tilde{\theta}_f$, $\tilde{\theta}_g$ and $\tilde{\theta}_d$ have minimum values. To achieve this goal, the following candidate Lyapunov function is considered:

$$V = \frac{1}{2}\underline{e}^T P\underline{e} + \frac{1}{2\gamma_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2\gamma_2}tr(\tilde{\theta}_g^T\tilde{\theta}_g) + \frac{1}{2\gamma_3}\tilde{\theta}_d^T\tilde{\theta}_d \tag{17}$$

where γ_1, γ_2 and γ_3 are positive designed parameters. Considering equation (15), we can get that

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} + (\frac{1}{\gamma_1}\dot{\theta}_f^T + \alpha\underline{e}^T PB\xi_f)\tilde{\theta}_f + \frac{1}{\gamma_2}tr(\tilde{\theta}_g\dot{\theta}_g) + \alpha\underline{e}^T PB\xi_g(x)\tilde{\theta}_g u_i + [(1-\alpha)\underline{e}^T PBG\xi_d(x) - \frac{1}{\gamma_3}\dot{\theta}_d^T]\tilde{\theta}_d + \underline{e}^T PB[\alpha\omega_i - (1-\alpha)G\omega_d - Gu_C] \tag{18}$$

where

$$\frac{1}{\gamma_2}tr(\tilde{\theta}_g\dot{\theta}_g) + \alpha\underline{e}^T PB\xi_g(x)\tilde{\theta}_g u_i = \sum_{i=1}^{p}(\alpha u_{ii}\underline{e}^T PB\xi_{gi}(x) + \frac{1}{\gamma_2}\dot{\theta}_{gi}^T)\tilde{\theta}_{gi} \tag{19}$$

where u_{ii} is the i th component of controller u_i.

The adaptive laws of θ_f, θ_g and θ_d are designed as

$$\dot{\theta}_f = -\alpha\gamma_1\xi_f^T(x)B^T P\underline{e} \tag{20}$$

$$\dot{\theta}_{gi} = -\alpha\gamma_2 u_{ii}\xi_{gi}^T(x)B^T P\underline{e} \tag{21}$$

$$\dot{\theta}_d = (1-\alpha)\gamma_3\sigma_1\xi_d^T(x)B^T P\underline{e} \tag{22}$$

V. Analysis of Stability and Performance

The systems designed above can ensure the global stability of closed-loop system, and the results about stability are given as follows.

Theorem I: Considering the nonlinear dynamic system of equation (1), if conditions of assumption 1 and 2 hold, combining the control laws of equation (10), (11), (12) and (15) and the adaptive laws of equation (20), (21) and (22), the following properties can be ensured:

1) All signals in the closed-loop system are bounded.
2) The tracking error and its derivative converge to zero.

Proof: According to (18) and (20), we get $\frac{1}{\gamma_1}\dot{\theta}_f^T + \alpha\underline{e}^T PB\xi_f = 0$, and then combined with (21) and

(19), we get $\sum_{i=1}^{p}(\alpha u_{ii}\underline{e}^T PB\xi_{gi}(x)+\frac{1}{\gamma_2}\dot{\theta}_{gi}^T)\tilde{\theta}_{gi}=0$, and then (18) can be modified as

$$\dot{V}=-\frac{1}{2}\underline{e}^T Q\underline{e}+[(1-\alpha)\underline{e}^T PBG\xi_d(x)-\frac{1}{\gamma_3}\dot{\theta}_d^{\,T}]\tilde{\theta}_d + \underline{e}^T PB[\alpha\omega_i-(1-\alpha)G\omega_d-Gu_C] \quad (23)$$

From $G(x)\le\sigma_1 I_p$ in assumption 1 and expression (22), expression (23) can be modified as

$$\dot{V}\le-\frac{1}{2}\underline{e}^T Q\underline{e}+\underline{e}^T PB[\alpha\omega_i-(1-\alpha)G\omega_d]-\underline{e}^T PBGu_C \quad (24)$$

From assumption 1, we can get that

$$\dot{V}\le-\frac{1}{2}\underline{e}^T Q\underline{e}+\left\|\underline{e}^T PB\right\| \times\left[\alpha\left\|\omega_i\right\|+\sigma_0(1-\alpha)\left\|\omega_d\right\|-\sigma_0 k_C\left\|\mathrm{sgn}(\underline{e}^T PB)\right\|\right] \quad (25)$$

From assumption 2, we get that

$$\dot{V}\le-\frac{1}{2}\underline{e}^T Q\underline{e}+\left\|\underline{e}^T PB\right\| \times\left[\alpha\omega_1+\sigma_0(1-\alpha)\omega_2-\sigma_0 k_C\left\|\mathrm{sgn}(\underline{e}^T PB)\right\|\right] \quad (26)$$

where $\left\|\mathrm{sgn}(\underline{e}^T PB)\right\|=\sqrt{r_1+r_2+\cdots+r_p}$.

Let $n=\sqrt{r_1+r_2+\cdots+r_p}$ and $k_C\ge\frac{\alpha\omega_1+\sigma_0(1-\alpha)\omega_2}{\sigma_0 n}$, then the following condition can be ensured:

$$\dot{V}\le-\frac{1}{2}\underline{e}^T Q\underline{e} \quad (27)$$

where $\underline{e}\in L_\infty$ can be ensured. Furthermore, according to the hypotheses of systems and the control law expressions (10), (11), (12) and (15), control terms u_T and u_C are both bounded. Therefore, the right side of expression (14) is bounded, viz. $\dot{\underline{e}}\in L_\infty$. From expression (27), we also get that

$$\dot{V}\le-\frac{1}{2}\underline{e}^T Q\underline{e}\le-\frac{\lambda_{Q\min}}{2}\left\|\underline{e}\right\|^2 \quad (28)$$

where $\lambda_{Q\min}$ is the minimum eigenvalue of Q. Doing integral to both sides of expression (28) such that

$$\int_0^\infty\left\|\underline{e}(t)\right\|^2 dt\le\frac{2}{\lambda_{Q\min}}(V(0)-V(\infty)) \quad (29)$$

From the above expression, we get that $\underline{e}\in L_2$. On the basis of the above-mentioned results and according to the Barbalat lemma ([5]: if $\underline{e}\in L_2\cap L_\infty$ and $\dot{\underline{e}}\in L_\infty$, then $\lim_{t\to\infty}\underline{e}(t)=0$), we can get that $\lim_{t\to\infty}\underline{e}(t)=0$.

VI. Simulation Example

Considering a damping system of mass-spring, the motion equations of mechanical system are as follows:

$$\begin{cases} M_1\ddot{y}_1=u_1-f_{K1}(x)-f_{B1}(x)+f_{K2}(x) \\ \qquad +f_{B2}(x)-f_{C1}(x)+f_{C2}(x) \\ M_2\ddot{y}_2=u_2-f_{K2}(x)-f_{B1}(x)-f_{C2}(x) \end{cases} \quad (30)$$

where

$$f_{K1}(x)=K_{10}y_1+\Delta K_1 y_1^3$$
$$f_{K2}(x)=K_{20}(y_2-y_1)+\Delta K_2(y_2-y_1)^3$$

are defined as elastic forces, and

$$f_{B1}(x)=B_{10}\dot{y}_1+\Delta B_1\dot{y}_1^2$$
$$f_{B2}(x)=B_{20}(\dot{y}_2-\dot{y}_1)+\Delta B_2(\dot{y}_2-\dot{y}_1)^2$$

are defined as friction forces, where $x=[y_1,\dot{y}_1,y_2,\dot{y}_2]^T$.

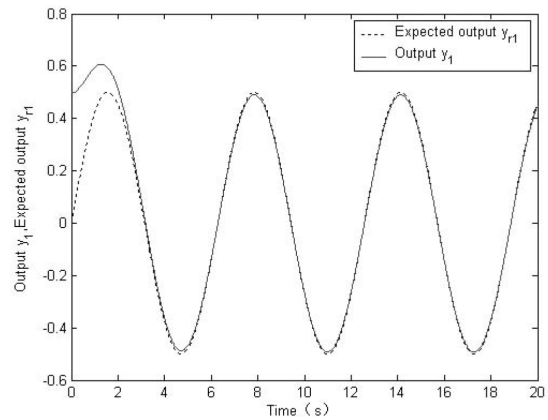

Fig. 1. The curves of system output y_1 and expected output y_{r1}

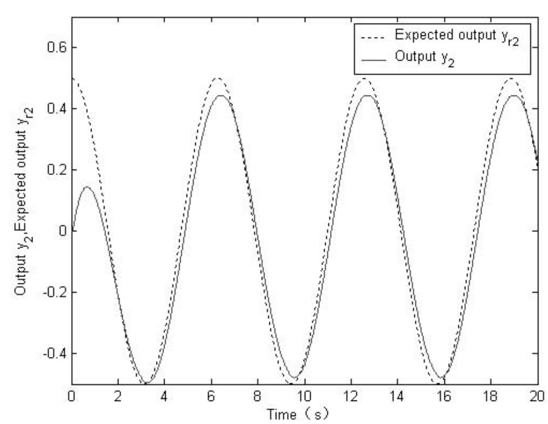

Fig. 2. The curves of system output y_2 and expected output y_{r2}

Approximation parameters are given as follows: $M_1=0.2$, $M_2=0.2$, $K_{10}=1$, $K_{20}=2$, $B_{10}=2$, $B_{20}=2.2$。 Disturbance parameters are given as follows: $\Delta M_1=0.05\sin(y_1)$, $\Delta M_2=0.05\sin(y_1-y_2)$, $\Delta K_1=0.1$, $\Delta K_2=0.12$, $\Delta B_1=0.2$ and $\Delta B_2=0.15$. Suppose the Coulomb friction forces are $f_{C1}=0.02\,\mathrm{sgn}(\dot{y}_1)$ and $f_{C2}=0.02\,\mathrm{sgn}(\dot{y}_2-\dot{y}_1)$, and there are a little force $\Delta u_{12}=0.1u_2$ to M_1 and a little force $\Delta u_{21}=0.15u_1$ to M_2, and the ideal output trajectories are $y_{r1}(t)=0.5\sin(t)$ and $y_{r2}(t)=0.5\cos(t)$, the goal of control is to make the system output tracking the ideal output.

Five fuzzy sets are defined to each $x_i(i=1,2,3,4)$ and the labeling is F_i^1 (minus infinity), F_i^2 (approximate to -0.5),

F_i^3 (approximate to 0), F_i^4 (approximate to 0.5) and F_i^5 (positive infinity) respectively, and the membership functions are

$$\mu_{F_i^1}(x_i) = 1/(1+\exp(5(x_i+1))) \quad ,$$

$$\mu_{F_i^2}(x_i) = \exp(-2(x_i+0.5)^2)$$

$$\mu_{F_i^3}(x_i) = \exp(-2x_i^2)\ ,\ \mu_{F_i^4}(x_i) = \exp(-2(x_i-0.5)^2)$$

$$\mu_{F_i^5}(x_i) = 1/(1+\exp(-5(x_i-1)))$$

Some fuzzy rules are set as follows:

$R^{(l)}$: If x_1 is F_1^m and x_3 is F_3^m, then y is F^l, where $m=1,\cdots,5,\ l=1,\cdots,5$;

$R^{(l)}$: If x_2 is F_2^m and x_4 is F_4^m, then y is F^l, where $m=1,\cdots,5,\ l=6,\cdots,10$.

Then the conditional equation can be obtained as

$$\xi_{f1} = \left[\xi_{f11}\ \xi_{f12}\cdots\xi_{f110}\right]^T \in R^{10}$$

where

$$\xi_{f11} = \mu_{F_1^1}(x_1)\mu_{F_3^1}(x_3)\big/D_1 \quad , \quad \cdots \quad ,$$

$$\xi_{f15} = \mu_{F_1^5}(x_1)\mu_{F_3^5}(x_3)\big/D_1$$

$$\xi_{f16} = \mu_{F_2^1}(x_2)\mu_{F_4^1}(x_4)\big/D_2 \quad \cdots \quad ,$$

$$\xi_{f110} = \mu_{F_2^5}(x_2)\mu_{F_4^5}(x_4)\big/D_2$$

$$D_1 = \sum_{m=1}^{5}\mu_{F_1^m}(x_1)\mu_{F_3^m}(x_3)\ ,\ D_2 = \sum_{m=1}^{5}\mu_{F_2^m}(x_2)\mu_{F_4^m}(x_4)\ .$$

Let $\xi_{f2} = \xi_{f1}$, and then $\hat{F}(x/\theta_f) = \xi_f(x)\theta_f$, where $\xi_f = diag[\xi_{f1}^T, \xi_{f2}^T]$, $\theta_f = [\theta_{f1}, \theta_{f2}]^T$, $\theta_{f1} = [\theta_{f11}, \cdots, \theta_{f110}]^T$, $\theta_{f2} = [\theta_{f21}, \cdots, \theta_{f210}]^T$.

Notice that it has to accord with the known physical properties (viz. elastic forces always rest with y_1 and y_2, and friction forces always rest with $\dot{y}_1$ and $\dot{y}_2$.), the fuzzy approximator is simplified and segmented to be used as follows: the performance of elastic force is known by $\sum_{l=1}^{5}\xi_{fil}\theta_{fil}\ \ (i=1,2)$, and the performance of friction force can be known by $\sum_{l=6}^{10}\xi_{fil}\theta_{fil}$.

To approximate the function $g_{ij}(\mathbf{x})$, three fuzzy sets $G_i^r\ (r=1,2,3)$ are defined to x_1 and x_3 respectively, and the corresponding membership functions are

$$\mu_{G_i^1}(x_i) = 1\big/\left(1+\exp\left(5(x_i+0.5)\right)\right),\ \mu_{F_i^2}(x_i) = \exp\left(-2x_i^2\right)$$

$$\mu_{G_i^3}(x_i) = 1\big/\left(1+\exp\left(-5(x_i-0.5)\right)\right).$$

Some other fuzzy rules are set as

$R^{(l)}$: if x_1 is G_1^n and x_3 is G_3^n, then y is G^l, where $n=1,2,3$ and $l=1,2,3$;

Then we get that $\xi_{g1} = \left[\xi_{g11}, \xi_{g12}, \xi_{g13}\right]^T \in R^3$, and here we select $\xi_{g2} = \xi_{g1}$.

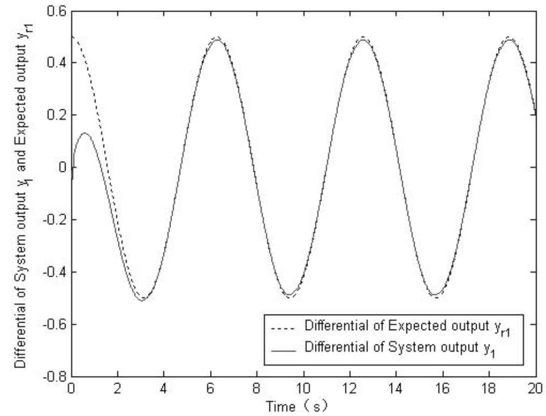

Fig. 3. The curves of $\dot{y}_1$ and $\dot{y}_{r1}$

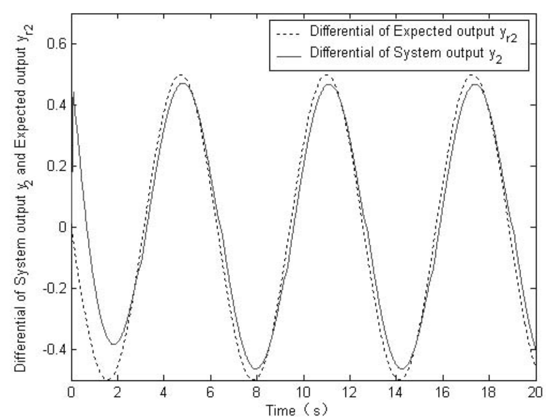

Fig. 4. The curves of $\dot{y}_2$ and $\dot{y}_{r2}$

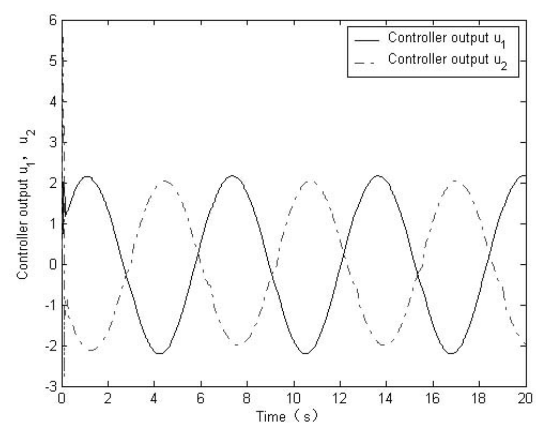

Fig. 5. The output curves of controller

To approximate the function $u_{di}(\mathbf{x})$, three other fuzzy sets $U_{di}^s\ (s=1,2,3)$ are defined to x_1 and x_3 respectively and $U_{di}^s = G_i^r$ is selected.

Set the feedback gain vector as $K_1 = K_2 = [1,2]$, and the initial value is given as

$$\theta_{f1}(0) = \theta_{f2}(0) = \left[0.2,0.2,0.5,0.3,0.1,0.4,0.5,0.6,0.7,0.8\right]^T$$

$$\theta_{g1}(0) = \theta_{g2}(0) = \left[2,3,4\right]^T .$$

The positive definite matrix is set as

$Q_1 = Q_2 = diag\left(10,10\right)$, $B_1 = B_2 = \left[0,1\right]^T$,

$\gamma_1 = 2,\ \gamma_2 = 2,\ \gamma_3 = 1$, $\alpha = 0.4,\ k_C = 1$.

Let $y_1(0) = 0.5$ and $y_2(0) = \dot{y}_1(0) = \dot{y}_2(0) = 0$ as the initial state of system. The simulation results are as fig.1-fig.5.

VII. CONCLUSION

In this paper, a combined adaptive fuzzy control approach of a class of uncertain MIMO nonlinear systems is studied. In this novel method, both the knowledge of controlled objects and control rules are sufficiently used. In addition, most methods need to assume that the minimum approximation error is required to satisfy the square-integrable condition. The method proposed in this paper doesn't need this assumption, and the effect of minimum approximation error could be removed by the adaptive compensation term. Based on Lyapunov stability theory, it can be ensured that all signals of closed-loop system are bounded, and the tracking errors converge to a small neighborhood around zero. Simulation results indicate the validity of the proposed method.

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