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Delay-Dependent State-Derivative Feedback with an α-Stability Constraint for Time Delay Systems

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Abstract—**This paper concerns the problem of state-derivative feedback with exponential convergence rate for a linear system with a constant time-delay. This is promising for better applicability in practical systems where the state-derivative signals are easier to obtain than the state ones. Based on a system state transformation and appropriate Lyapunov functions, a sufficient condition for the design of the state-derivative feedback controller is derived in terms of delay-dependent linear matrix inequalities (LMIs). Two practical application examples are included to illustrate the effectiveness of the proposed method.**

I. Introduction

THE phenomena of time-delay are often encountered in many practical systems, such as aircraft systems, neural network, chemical processes, communication systems, nuclear reactor, electrical networks, etc. Whenever the time-delay is significant as compared with the system time-constants, some erratic behavior can occur leading to the generation of oscillation, performance degradation and instability. In view of this, considerable attention from many researchers has been devoted to the issue of time-delay systems. Especially, several results on stability analysis and control synthesis for such systems have been reported in [1]-[3]. However, all the approaches considered in these papers are the state feedback method. So if the state signals are more difficult to obtain than the state-derivative signals in some case, the results with taking account of the state feedback are not practical for the system. In practice, there exist some problems where the state-derivative signals are more available than the state ones [4]. Consequently, state-derivative feedback has been an attractive topic for both theoretical and practical reasons. As to theory analysis[5]-[8], the state-derivative feedback has been utilized to achieve eigenstructure assignment in singular systems, *H-infinite* control of linear state-delay descriptor systems, robust control of descriptor linear systems, and robust pole assignment in descriptor second-order dynamical systems. In [6], sufficient conditions for delay-dependent/delay-independent stability and L_2-gain analysis of linear descriptor systems are obtained in terms of linear matrix inequalities (LMIs). In [8], for descriptor second-order dynamical systems with proportional plus derivative state feedback, the eigenvalue assignment with minimum sensitivity is considered based on a complete parametric eigenstructure assignment approach. For the practical application[9]-[12], typical examples are the active and passive vibration control of landing gear components, robust vibration control for car wheel suspension systems, control of bridge cable vibration, and High-Performance Induction Motor Speed Control. In [12], a novel nonlinear speed/position control strategy for the induction motor is presented utilizing exact feedback linearization with state and state derivative feedback.

The convergence rate of states of a control system is another important issue concerned with. Usually, asymptotic stability is insufficient for a control system to ensure a satisfactory dynamical performance. In such a case, exponential stability (α-stability) with a given decay rate is very useful. To the best of authors' knowledge, there had been no result reported for exponential state-derivative feedback for linear delayed systems in the past.

Different approaches, as for instance, the Riccati equation approach, Smith-like predictors, and the linear matrix inequality (LMI) formulation, have been used to deal with the problems of stability analysis and control design for linear systems with delayed states. It is known that, over the other ones, the LMI approach generally has significant advantages of simplifying the design procedure and handling modeling uncertainty, state constraints and multi-objective performance specifications without losing the convexity. Hence, the underlying problem can be numerically solved by LMI efficiently. Depending on whether the criterion itself contains the sizes of time delays, the criteria for time-delay systems can be classified into two categories, namely delay-independent criteria and delay-dependent criteria. As the name implies, delay-independent results guarantee stability for arbitrarily large delays. Delay-dependent results take into account the maximum delay that can be tolerated by the system and, thus, are more useful in applications.

So, motivated by the facts above-mentioned, the problem of delay-dependent state-derivative feedback with exponential convergence rate for linear systems with delayed state is investigated by means of a system state transformation and appropriate Lyapunov functions. The case of a single constant time-delay is considered. The focal point of this paper is on developing a sufficient condition to stabilize linear delayed systems with exponential state-derivative feedback in terms of LMIs which depend on the size of the

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time-delay. Finally, the feasibility and effectiveness of the proposed method are verified on the grounds of computer simulation results.

II. Problem Statement and Main Results

Consider linear time-delay systems described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\tau) + Bu(t) \\ x(t) &= \phi(t),\ t \in [-\tau, 0] \end{aligned}, \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $\tau > 0$ is the constant time-delay of the system, the initial vector $\phi(t)$ is a continuously differentiable function from $[-\tau, 0]$ to R^n, A, A_d, and B are known real constant matrices of appropriate dimensions.

Consider the system (1) with the state-derivative feedback controller

$$u(t) = -K\dot{x}(t) \quad (2)$$

where $K \in R^{m\times n}$ is a constant gain matrix to be designed. The closed-loop system is then given by

$$\dot{x}(t) = \overline{A}x(t) + \overline{A}_d x(t-\tau), \quad (3)$$

where $\overline{A} = (I+BK)^{-1}A$, $\overline{A}_d = (I+BK)^{-1}A_d$.

The design problem to be addressed in this paper is to find a state-derivative feedback gain matrix $K \in R^{m\times n}$, such that the following requirements satisfy:

(i) $(I+BK)$ has a full rank,

(ii) the closed-loop system (3) is exponentially stable with a given decay rate $\alpha > 0$, i.e.

$$\lim_{t\to\infty} e^{\alpha t} \| x(t) \| = 0 \quad (4)$$

holds for all trajectories $x(t)$, $t \ge 0$.

The following lemmas will be used to prove our result.

Lemma 1 [13]**.** Let D, E, F be real matrices of appropriate dimensions with $\| F \| \le 1$. Then, for any scalar $\varepsilon > 0$, we have

$$DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} \le \varepsilon^{-1}DD^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E$$

Lemma 2 [14]**.** For any matrix $M \in R^{p\times p}$, $M + M^{\mathrm{T}} < 0$ if and only if $M < 0$ holds.

The following theorem presents a delay-dependent exponential state-derivative feedback stabilizability criterion based on the LMI method.

Theorem 1. The closed-loop system (3) is exponentially stable with decay rate $\alpha > 0$ if for a given scalar $0 < \varepsilon < 1$, there exist a symmetric positive definite matrix Q and a matrix Y, such that

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & 0 & 0 \\ * & * & \Xi_{33} & 0 \\ * & * & * & \Xi_{44} \end{bmatrix} < 0, \quad (5)$$

where * represents the symmetric form in the matrix and

$$\begin{aligned} \Xi_{11} = {} & AQ + QA^{\mathrm{T}} + AY^{\mathrm{T}}B^{\mathrm{T}} + BYA^{\mathrm{T}} + \tau f(\alpha,\varepsilon,\tau)A_d A_d^{\mathrm{T}} \\ & + e^{\alpha\tau}(A_d Q + QA_d^{\mathrm{T}} + A_d Y^{\mathrm{T}}B^{\mathrm{T}} + BYA_d^{\mathrm{T}}) \end{aligned},$$

$\Xi_{12} = \tau e^{0.5\alpha\tau}(QA^{\mathrm{T}} + BYA^{\mathrm{T}} + \alpha I)$, $\Xi_{22} = -\tau\varepsilon I$,

$\Xi_{13} = \tau e^{\alpha\tau}(QA_d^{\mathrm{T}} + BYA_d^{\mathrm{T}})$, $\Xi_{33} = -\tau(1-\varepsilon)I$, $\Xi_{14} = Q + BY$,

$\Xi_{44} = -Q/(2\alpha)$, $f(\alpha,\varepsilon,\tau) = \varepsilon e^{\alpha\tau} + (1-\varepsilon)e^{2\alpha\tau}$.

The state feedback gain is given by $K = YQ^{-1}$.

Proof: Utilizing the following transformation

$$z(t) = e^{\alpha t}x(t), \quad (6)$$

where $\alpha > 0$ is decay rate, system (3) is transformed into

$$\dot{z}(t) = (\overline{A} + \alpha I)z(t) + e^{\alpha\tau}\overline{A}_d z(t-\tau). \quad (7)$$

Let $z(t)$ be a trajectory of system (7). Hence, we have that for $t \ge \tau$

$$\begin{aligned} z(t-\tau) &= z(t) - \int_{-\tau}^{0} \dot{z}(t+s)ds \\ &= z(t) - \int_{-\tau}^{0} [(\overline{A}+\alpha I)z(t+s) + e^{\alpha\tau}\overline{A}_d z(t-\tau+s)]ds. \end{aligned} \quad (8)$$

Substituting (8) into (7), we know that $z(t)$ satisfies

$$\begin{aligned} \dot{z}(t) = {} & (\overline{A} + \alpha I + e^{\alpha\tau}\overline{A}_d)z(t) - e^{\alpha\tau}\overline{A}_d \\ & \times \int_{-\tau}^{0} [(\overline{A}+\alpha I)z(t+s) + e^{\alpha\tau}\overline{A}_d z(t-\tau+s)]ds. \end{aligned}$$

In view of the above, consider the following time-delay system

$$\begin{aligned} \dot{\xi}(t) = {} & (\overline{A} + \alpha I + e^{\alpha\tau}\overline{A}_d)\xi(t) - e^{\alpha\tau}\overline{A}_d \\ & \times \int_{-\tau}^{0} [(\overline{A}+\alpha I)\xi(t+s) + e^{\alpha\tau}\overline{A}_d \xi(t-\tau+s)]ds, \\ \xi(s) = {} & \psi(s), \qquad \forall s \in [-2\tau, 0], \end{aligned} \quad (9)$$

where $\psi(\cdot)$ is the initial condition. Observe that (9) requires initial data on $[-2\tau, 0]$.

Define the Lyapunov functional candidate

$$V(\xi, t) = \xi^{\mathrm{T}}(t)P\xi(t) + W(\xi, t), \quad (10)$$

where P is a symmetric positive definite matrix and

$$\begin{aligned} W(\xi,t) = {} & \varepsilon^{-1}e^{\alpha\tau}\int_{-\tau}^{0}\int_{t+s}^{t} \| (\overline{A}+\alpha I)\xi(\theta) \|^2 d\theta ds \\ & + (1-\varepsilon)^{-1}e^{2\alpha\tau}\int_{-\tau}^{0}\int_{t+s-\tau}^{t} \| \overline{A}_d\xi(\theta) \|^2 d\theta ds, \end{aligned} \quad (11)$$

where $0 < \varepsilon < 1$ is a scalar to be chosen. Then, the time derivative of $V(\xi, t)$ is given by

$$\begin{aligned} & \dot{V}(\xi, t) \\ & = \xi^{\mathrm{T}}(t)[P(\overline{A} + \alpha I + e^{\alpha\tau}\overline{A}_d) + (\overline{A} + \alpha I + e^{\alpha\tau}\overline{A}_d)^{\mathrm{T}}P]\xi(t) \\ & \quad - 2\xi^{\mathrm{T}}(t)Pe^{\alpha\tau}\overline{A}_d\int_{-\tau}^{0}(\overline{A}+\alpha I)\xi(t+s)ds - 2\xi^{\mathrm{T}}(t)Pe^{2\alpha\tau}\overline{A}_d \\ & \quad \times \int_{-\tau}^{0}\overline{A}_d\xi(t-\tau+s)ds + \varepsilon^{-1}\tau e^{\alpha\tau}\xi^{\mathrm{T}}(t)(\overline{A}+\alpha I)^{\mathrm{T}} \\ & \quad \times (\overline{A}+\alpha I)\xi(t) + (1-\varepsilon)^{-1}\tau e^{2\alpha\tau}\xi^{\mathrm{T}}(t)\overline{A}_d^{\mathrm{T}}\overline{A}_d\xi(t) \\ & \quad - \varepsilon^{-1}e^{\alpha\tau}\int_{-\tau}^{0}\xi^{\mathrm{T}}(t+s)(\overline{A}+\alpha I)^{\mathrm{T}}(\overline{A}+\alpha I)\xi(t+s)ds \\ & \quad - (1-\varepsilon)^{-1}e^{2\alpha\tau}\int_{-\tau}^{0}\xi^{\mathrm{T}}(t-\tau+s)\overline{A}_d^{\mathrm{T}}\overline{A}_d\xi(t-\tau+s)ds. \end{aligned} \quad (12)$$

Note that by lemma 1,

$$\begin{aligned}
&-2\xi^{\mathrm{T}}(t)Pe^{\alpha\tau}\bar{A}_{\mathrm{d}}\int_{-\tau}^{0}(\bar{A}+\alpha I)\xi(t+s)ds \\
&= e^{\alpha\tau}\int_{-\tau}^{0}[-2\xi^{\mathrm{T}}(t)P\bar{A}_{\mathrm{d}}(\bar{A}+\alpha I)\xi(t+s)]ds \\
&\le e^{\alpha\tau}\int_{-\tau}^{0}[\varepsilon\xi^{\mathrm{T}}(t)P\bar{A}_{\mathrm{d}}\bar{A}_{\mathrm{d}}^{\mathrm{T}}P\xi(t) \\
&\quad +\varepsilon^{-1}\xi^{\mathrm{T}}(t+s)(\bar{A}+\alpha I)^{\mathrm{T}}(\bar{A}+\alpha I)\xi(t+s)]ds \\
&= \varepsilon\tau e^{\alpha\tau}\xi^{\mathrm{T}}(t)P\bar{A}_{\mathrm{d}}\bar{A}_{\mathrm{d}}^{\mathrm{T}}P\xi(t) \\
&\quad +\varepsilon^{-1}e^{\alpha\tau}\int_{-\tau}^{0}\xi^{\mathrm{T}}(t+s)(\bar{A}+\alpha I)^{\mathrm{T}}(\bar{A}+\alpha I)\xi(t+s)]ds, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
&-2\xi^{\mathrm{T}}(t)e^{2\alpha\tau}P\bar{A}_{\mathrm{d}}\int_{-\tau}^{0}\bar{A}_{\mathrm{d}}\xi(t-\tau+s)ds \\
&= e^{2\alpha\tau}\int_{-\tau}^{0}[-2\xi^{\mathrm{T}}(t)P\bar{A}_{\mathrm{d}}\bar{A}_{\mathrm{d}}\xi(t-\tau+s)]ds \\
&\le e^{2\alpha\tau}\int_{-\tau}^{0}[(1-\varepsilon)\xi^{\mathrm{T}}(t)P\bar{A}_{\mathrm{d}}\bar{A}_{\mathrm{d}}^{\mathrm{T}}P\xi(t) \\
&\quad +(1-\varepsilon)^{-1}\xi^{\mathrm{T}}(t-\tau+s)\bar{A}_{\mathrm{d}}^{\mathrm{T}}\bar{A}_{\mathrm{d}}\xi(t-\tau+s)]ds \\
&= \tau e^{2\alpha\tau}(1-\varepsilon)\xi^{\mathrm{T}}(t)P\bar{A}_{\mathrm{d}}\bar{A}_{\mathrm{d}}^{\mathrm{T}}P\xi(t) \\
&\quad +e^{2\alpha\tau}(1-\varepsilon)^{-1}\int_{-\tau}^{0}\xi^{\mathrm{T}}(t-\tau+s)\bar{A}_{\mathrm{d}}^{\mathrm{T}}\bar{A}_{\mathrm{d}}\xi(t-\tau+s)ds,
\end{aligned}$$

for any scalar $0<\varepsilon<1$.

Hence, using (13) and (14) in (12), we obtain

$$\dot{V}(\xi,t)\le\xi^{\mathrm{T}}(t)\Sigma\xi(t), \qquad (15)$$

where

$$\begin{aligned}
\Sigma=&P(\bar{A}+\alpha I+e^{\alpha\tau}\bar{A}_{\mathrm{d}})+(\bar{A}+\alpha I+e^{\alpha\tau}\bar{A}_{\mathrm{d}})^{\mathrm{T}}P+\tau f(\alpha,\varepsilon,\tau)P\bar{A}_{\mathrm{d}} \\
&\times\bar{A}_{\mathrm{d}}^{\mathrm{T}}P+\varepsilon^{-1}\tau e^{\alpha\tau}(\bar{A}+\alpha I)^{\mathrm{T}}(\bar{A}+\alpha I)+(1-\varepsilon)^{-1}\tau e^{2\alpha\tau}\bar{A}_{\mathrm{d}}^{\mathrm{T}}\bar{A}_{\mathrm{d}}.
\end{aligned}$$

By $P^{-1}=Q$, $Y=KQ$ and Schur complement, Ξ in (5) is equivalent to

$$\Pi=\begin{bmatrix}\Pi_{11} & \Pi_{12} & \Pi_{13}\\ * & -\tau\varepsilon I & 0\\ * & * & -\tau(1-\varepsilon)I\end{bmatrix}<0, \qquad (16)$$

where

$$\begin{aligned}
\Pi_{11}=&AP^{-1}(I+BK)^{\mathrm{T}}+2\alpha(I+BK)P^{-1}(I+BK)^{\mathrm{T}} \\
&+e^{\alpha\tau}A_dP^{-1}(I+BK)^{\mathrm{T}}+(I+BK)P^{-1}A^{\mathrm{T}} \\
&+e^{\alpha\tau}(I+BK)P^{-1}A_d^{\mathrm{T}}+\tau f(\alpha,\varepsilon,\tau)A_dA_d^{\mathrm{T}}
\end{aligned}$$

$$\Pi_{12}=\tau e^{0.5\alpha\tau}(I+BK)P^{-1}A^{\mathrm{T}}+\alpha\tau e^{0.5\alpha\tau},$$

$$\Pi_{13}=\tau e^{\alpha\tau}(I+BK)P^{-1}A_d^{\mathrm{T}}.$$

By left and right multiplying both sides of (16) by the diagonal matrix $\mathrm{diag}\{P(I+BK)^{-1},(I+BK)^{\mathrm{T}},(I+BK)^{\mathrm{T}}\}$ and the diagonal matrix $\mathrm{diag}\{(I+BK)^{-\mathrm{T}}P,(I+BK)^{-\mathrm{T}},(I+BK)^{-\mathrm{T}}\}$, respectively, taking account of $\bar{A}=(I+BK)^{-1}A$ and $\bar{A}_{\mathrm{d}}=(I+BK)^{-1}A_{\mathrm{d}}$, and Schur complement, we obtain

$$\Sigma<0. \qquad (17)$$

From (15) and (17), we get $\dot{V}(\xi,t)<0$. Then, according to the Lyapunov theory, system (3) is exponentially stable.

It's obvious that the last three parts of Σ are positive definite. So, from (17), we have

$$P(\bar{A}+\alpha I+e^{\alpha\tau}\bar{A}_{\mathrm{d}})+(\bar{A}+\alpha I+e^{\alpha\tau}\bar{A}_{\mathrm{d}})^{\mathrm{T}}P<0. \qquad (18)$$

From lemma 2 and (18), it follows that

$$P(\bar{A}+\alpha I+e^{\alpha\tau}\bar{A}_{\mathrm{d}})<0, \qquad (19)$$

which also indicates that

$$P(\bar{A}+e^{\alpha\tau}\bar{A}_{\mathrm{d}})=P(I+BK)^{-1}(A+e^{\alpha\tau}A_{\mathrm{d}})<0. \qquad (20)$$

Hence, $P(I+BK)^{-1}(A+e^{\alpha\tau}A_{\mathrm{d}})$ has a full rank and so, $(I+BK)$ has a full rank, as required in (i). This proof is completed.

Remark 1. Note that from (20), it follows that the matrix $(A+e^{\alpha\tau}A_{\mathrm{d}})$ must have a full rank and thus, all its eigenvalues are not equal to zero.

III. Simulation Results

In this section, computer simulations are carried out to show the effectiveness of the proposed method.

Example1. Consider a practical example of VTOL control for a helicopter [15] which is given in the form of (1) with

$$A=\begin{bmatrix}-0.0366 & 0.0271 & 0.0188 & -0.4555\\ 0.0482 & -1.0100 & 0.0024 & -4.0208\\ 0.1002 & 0.3681 & -0.7070 & 1.4200\\ 0 & 0 & 1.0000 & 0\end{bmatrix},$$

$$B=\begin{bmatrix}0.4422 & 0.1761\\ 3.5446 & -7.5922\\ -5.5200 & 4.4900\\ 0 & 0\end{bmatrix},\ A_d=0.3A.$$

The state vector $x(t)=\begin{bmatrix}x_1(t) & x_2(t) & x_3(t) & x_4(t)\end{bmatrix}^{\mathrm{T}}$ contains the horizontal velocity, the vertical velocity, the pitch rate and the pitch angle, respectively. For the state-derivative feedback, $\dot{x}_1(t)$, $\dot{x}_2(t)$ are measured by accelerator sensors, $\dot{x}_3(t)$ is available by angle accelerator sensors, and $\dot{x}_4(t)$ is obtained by angle rate sensors. Therefore, the proposed method can be used to solve the problem.

Setting $\varepsilon=0.1$, $\alpha=0.1$, and solving LMI (5) from Theorem 1, the upper bound $\bar{\tau}$ of τ is found to be 0.3692 and a feasible solution is given in the Appendix. Note that, as discussed before, the obtained feasible solution ensures that $(I+BK)$ has a full rank. The closed-loop dynamic response by virtue of the methods proposed in this paper is depicted in Fig. 1 with $\tau=0.1785$ and $x(0)=\begin{bmatrix}1 & 1 & 0.5 & 0.5\end{bmatrix}^{\mathrm{T}}$.

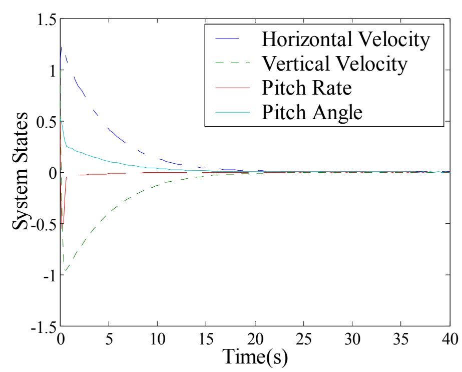

Fig. 1. The state trajectories of VTOL helcopter.

From Fig. 1, it is obvious that flight states of VTOL helicopter converge to equilibrium by the designed controller. So it is easy to get the conclusion that the controller, which takes advantage of state-derivative signals, can overcome the adverse effect caused by flight delay.

To provide relatively complete information, we calculate the upper bound $\overline{\tau}$ for different decay rate α, listed in Table I. As seen, for different ε values, the upper bound $\overline{\tau}$ becomes small as the α increases.

TABLE I
UPPER BOUNDS $\overline{\tau}$ WITH GIVEN ε FOR DIFFERENT α

α	0.1	0.2	0.3	0.4
τ ($\varepsilon=0.1$)	0. 3692	0.1872	0.0644	0.0296
τ ($\varepsilon=0.2$)	0.4792	0.3587	0.1265	0.0584
τ ($\varepsilon=0.3$)	0.5606	0.5122	0.1857	0.0863
τ ($\varepsilon=0.9$)	0.6776	0.6603	0.3612	0.1785

Example2. A satellite system consisting of two rigid bodies (main module and sensor module) connected by an elastic link that is modeled as a spring with viscous damping $f \in [0.0038, 0.04]$ and torque constant $k \in [0.09, 0.4]$ is considered [16]. The system matrices of the satellite system are given as follows

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix}, B=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$A_d=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.01 & 0 & 0 & 0 \\ 0 & -0.001 & 0 & -0.001 \end{bmatrix}.$$

For the case when $k=0.4$ and $f=0.04$, according to Theorem 1with $\varepsilon=0.1$ and $\alpha=0.1$, it is found that the system is exponentially stable for the upper bound $\overline{\tau}=0.0239$. A feasible solution is given in the Appendix, which ensures $\det(I+BK)\neq 0$ for requirement (i). Time responses with $\tau=0.02$ are depicted in Fig. 2 which illustrate that the state-derivative feedback controller (2) stabilizes the satellite system well.

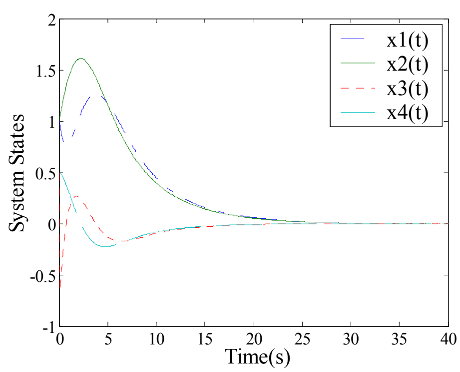

Fig. 2. The state trajectories of satellite system.

For different decay rate α, the upper bound $\overline{\tau}$ of τ is shown in Table II.

TABLE II
UPPER BOUNDS $\overline{\tau}$ WITH GIVEN ε FOR DIFFERENT α

α	0.1	0.2	0.3	0.4
τ ($\varepsilon=0.5$)	0.1200	0.0242	0.0085	0.0037
τ ($\varepsilon=0.6$)	0.1439	0.0290	0.0102	0.0044
τ ($\varepsilon=0.7$)	0.1676	0.0339	0.0119	0.0052
τ ($\varepsilon=0.9$)	0.2135	0.0435	0.0153	0.0067

IV. CONCLUSION

The problem of state-derivative feedback with exponential convergence rate for linear systems with delayed state has been addressed, which is more suitable for practical systems where the state-derivative signals are easier to obtain than the state ones. By introducing a transformation of the system states and appropriate Lyapunov functions, a delay-dependent stabilizability criterion is derived in the framework of LMIs, which determines the upper bound guaranteeing the exponential stabilizability for the considered systems. Simulation results demonstrate that the proposed method can stabilize practical systems and secures the allowed delay bounds for different exponential decay rate.

APPENDIX

A feasible solution for example 1 is given as follows

$$Q=1.0\text{e+}005*\begin{bmatrix} 0.2752 & -0.8583 & -1.1591 & 0.0000 \\ -0.8583 & 7.5205 & 0.0168 & -0.0000 \\ -1.1591 & 0.0168 & 7.5552 & -0.0000 \\ 0.0000 & -0.0000 & -0.0000 & 0.0000 \end{bmatrix},$$

$$Y=1.0\text{e+}005*\begin{bmatrix} -0.4868 & 1.3040 & 2.2096 & -0.0000 \\ -0.3403 & 1.5993 & 1.0338 & -0.0000 \end{bmatrix},$$

$$K=\begin{bmatrix} 0.2736 & 0.2039 & 0.3340 & 0.0156 \\ 0.1843 & 0.2333 & 0.1646 & -0.0130 \end{bmatrix}.$$

A feasible solution for example 2 is obtained in the following

$$Q=1.0\text{e+}004*\begin{bmatrix} 0.0013 & 0.0009 & -0.0004 & -0.0002 \\ 0.0009 & 0.0011 & 0.0000 & -0.0001 \\ -0.0004 & 0.0000 & 7.0332 & -0.0000 \\ -0.0002 & -0.0001 & -0.0000 & 0.0001 \end{bmatrix},$$

$$Y=1.0\text{e+}004*\begin{bmatrix} 0.0007 & 0.0002 & -7.0331 & -0.0001 \end{bmatrix},$$

$$K=\begin{bmatrix} 0.0298 & 0.0400 & -1.0000 & -0.9311 \end{bmatrix}.$$

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