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A Decentralized $\mathcal{H}_\infty$ Routing Control Strategy for Mobile Networked Multi-Agents

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Abstract— **This paper presents a novel Markovian jump model for developing routing algorithms in mobile networks that encounter variable number of nodes as well as changing number of destinations. The changes in the number of active destination nodes are represented by singular switching systems where a unified $\mathcal{H}_\infty$ decentralized routing control is proposed and implemented. The resulting optimization problem and the corresponding associated physical constraints are then expressed as Linear Matrix Inequality (LMI) conditions.**

I. Introduction

In recent years, autonomous mobile wireless networks which has diverse applications from space missions to intelligence, surveillance, and reconnaissance (ISR) missions have stimulated attractive research topics on self-organizing networks such as *ad hoc* wireless networks. Routing problem which is one of the main challenges in mobile networks, in general deals with minimization of certain objective functions such as shortest path, link congestion, end-to-end delay, and packet loss [1], [2]. In [3], the dynamic routing problem was defined as a team optimization problem and an approximate solution based on neural networks was obtained. In [4], the authors have introduced robust centralized as well as decentralized routing control strategies for networks with a fixed topology based on the minimization of the worst-case queueing length, which is related to the queueing delays.

In mobile networks, the neighboring sets of nodes may change due to the mobility and variations in the network topology, left over energy resources, and fading or increasing number of nodes. Therefore, the dynamics of the network characterizing the traffic flow will become time varying. In this paper, the approach introduced in [4] is generalized to mobile networks. To achieve this goal, the mobile network routing model is described as a Markovian jump linear system (MJLS). A decentralized $\mathcal{H}_\infty$ control is then introduced to stabilize the network and provide a routing solution for mobile networks.

Recently, considerable attention has been devoted to MJLSs with time-delays [5], [6] (and references therein). In [6], a sufficient condition for exponential estimates of a class of MJLSs systems with time-varying state delays was introduced. In [7], a stabilizing control for MJLSs with input delays was presented. However, no switching gain was designed for state feedback namely, a fixed gain K was found for all the switching modes of the system resulting in conservative conditions that potentially reduce the possibility of obtaining a feasible solution. To the best of our knowledge, no decentralized $\mathcal{H}_\infty$ control has been introduced in the literature for MJLSs with time-varying delays.

The queueing dynamics considered in [4] was derived based on the fluid flow conservation principle. Each state of the subsystem (node) represents a queue corresponding to a given destination node. Another issue that is addressed in this paper is routing problem with changing number of destinations nodes. In other words, for some destination nodes no external traffic has to be routed in certain periods. However, due to system dynamics and time-delays some messages may still be present in queues that should be routed to these destinations as quickly as possible. Given that in the considered dynamical model defined in [4] the states are queueing length at each node that corresponds to a destination node, the number of states depends on the active destination nodes. On the other hands, simply eliminating the inactive destination states leads to loss of integrity and stability of the overall system. It also ignores the leftover messages that are kept in the eliminated queues. To cope with these problems, we propose to model such network behavior as a singular MJLS.

A delay dependent stabilization for singularly perturbed MJLSs with a fixed singular matrix was presented in [8]. The $\mathcal{H}_\infty$ control scheme for a singular system with time delays was developed for MJLSs in [9]. In [10], some robust control strategies are introduced for stochastic singular systems with random abrupt changes. However, these systems are not affected by delay.

The Markovian jump model with time-varying delays that is introduced in this paper will enable one to develop an appropriate queueing dynamics for the network routing traffic problem to simultaneously address random mobility of the nodes as well as varying the number of the destination nodes.

II. Problem Formulation

According to the flow conservation law, a network traffic dynamics for a given node can be expressed as [4]:

$$\dot{x}_i = B_i u_i(t) + B_{wi} w_i(t) + \sum_{j \in \wp(i)} B_{dij} u_j(t - \tau_{ji}(t)) \quad (1)$$

where each node is considered as representing a subsystem that includes all the queues present in the node corresponding to different destinations. Therefore, x_i denotes the queue

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lengths in node i for different destinations, $u_i(t)$ denotes the flows sent from node i, $\tau_{ji}(t)$ is an *unknown but bounded time-varying* total delay in transmitting, propagating, and processing messages at node i from node j, $w_i(t)$ is the external input flow entering node i, and $B_i \in \Re^{n\times l}$ and $B_{dij} \in \Re^{n\times l}$ represent network connectivity matrices. In fact, each element of $B_i(B_{dij})$ is equal to $-1(1)$ if its corresponding flow is outgoing (incoming) flow to node i and is zero otherwise, $B_{wi} = I_{n\times n}$, and $\wp(i)$ is the neighboring set of node i.

The $\mathcal{H}_\infty$ robust optimal control design strategy is a suitable framework for dealing with system uncertainties and unknown time-delays. Therefore, at each node (subsystem), the routing problem can be stated as that of finding an $\mathcal{H}_\infty$ state feedback control, $u_i = K_i x_i$, such that it simultaneously guarantees the stability of the network traffic in the presence of time-varying delays and minimizes a global objective function which is considered as the *worst-case queueing length* due to external inputs. In other words, by selecting the regulated output as $z_i(t) = C_i x_i(t)$, where C_i is a weight matrix that is full rank, the routing problem can be cast into the following optimization problem:

$$\begin{aligned} \min\gamma \quad s.t. \quad J(w) &< 0 \qquad (2) \\ J(w) &= \int_0^\infty (z^T z - \gamma^2 w^T w)dt, \quad \gamma > 0 \end{aligned}$$

where $z(t) = vec\{z_i(t)\}, \quad w(t) = vec\{w_i(t)\}$.

When the topology of a network changes due to either node mobility, loss of node power, or addition of new nodes, the neighboring sets $\wp(i)$, and consequently the connectivity matrices B_i and B_{dij} in (1) will change. On the other hand, the nature of node mobility is generally not deterministic and involves random transitions and switches. Furthermore, only the existing neighboring sets $\wp(i)$ and connectivity matrices B_i and B_{dij} do affect the selection of new neighboring sets in the next transition step. In other words, changes in a neighboring set are independent from previous neighboring sets. Therefore, a Markovian jump process is a viable framework for describing and modeling the mobility behavior. Consequently, the dynamics of system (1) is now modified to the following MJLS representation for mobile networks

$$\begin{aligned} \dot{x}_i(t) &= B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &+ \sum_{j\in\wp_{r(t)}(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \qquad (3) \end{aligned}$$

where $r(t)$ is a function that represents the rule for changing the neighboring sets. Let us consider $r(t)$ as a continues-time Markov process taking values in a finite space $\mathcal{S} = \{1, ..., M\}$ which describes the switching between different modes, and whose evolution is governed by the following probability transitions:

$$\mathbb{P}[r(t+h) = k | r(t) = l] = \left\{ \begin{array}{cc} \pi_{kl}h + o(h) & k \neq l \\ 1 + \pi_{kk}h + o(h) & k = l \end{array} \right\}$$

where $\pi_{kl} > 0$ is the transition rate from mode k to mode l, $\pi_{kk} = -\sum_{l=1, l\neq k}^{M} \pi_{kl}$, and $o(h)$ is a function satisfying $\lim_{h\to 0} \frac{o(h)}{h} = 0$. To simplify the notation, we denote B_{ir} to represent $B_i(r(t))$ when $r(t) = r$. This notation is also applied to other matrices. Let us now define the concept of $\mathcal{H}_\infty$ control of stochastic systems.

Definition 1 [7]: Let $\gamma > 0$ be a positive constant. System (3) is said to be stochastically stable with γ-disturbance attenuation if there exits a constant $M(\phi, r_0)$ with $M(0, r_0) = 0$ such that

$$\|z\|_{\mathbb{E}_2} =\leq [\gamma^2 \|w(t)\|_2 + M(\phi, r_0)]^{1/2} \qquad (4)$$

A typical set of constraints expressing the physical properties of the network traffic are listed below

$$\begin{aligned} u_i(t) &\geq 0 \qquad (5) \\ x_i(t) &\geq 0 \qquad (6) \\ G_{k_i} u_i(t) &\leq c_{ki}(r(t)) \quad k_i = 1, ..., li, \quad i = 1, ..., n \qquad (7) \\ Q_{dji} x_i(t) &\leq x_{max_{dji}} \qquad (8) \end{aligned}$$

where $x_{max_{di}} = q^d_{max_i}$ and li is number of links in subsystem i. The first two constraints (i.e., (5) and (6)) are the so-called non-negativity constraints and are introduced for obvious reasons. The capacity constraint (7) states that the total flow in each link cannot exceed its capacity $c_{ki}(r(t))$ at each mode. The last condition, i.e., (8) indicates that to avoid packet loss the length of the queue should always remain smaller than a maximum value that is specified for the buffer $x_{max_{dji}}$. Therefore, G_{k_i} should be defined such that by multiplying G_{k_i} with u_i one yields the total flows that should go through the link k_i, and Q_{dji} should be defined such that $Q_{dji} x_i$ leads to the queueing length of the buffer dji, for $d = 1, ..., \bar{d}, \ i, j = 1, ..., n$. We now state our assumption on the characteristics of the delay function.

Assumption 1: The delays $\tau_{ji}(t)$ are unknown differentiable functions that for all $t \geq 0$ satisfy

$$0 \leq \max\{\tau_{ji}(t)\} \leq h_{ji}, \ \max\{|\dot{\tau}_{ji}(t)|\} \leq d_{ji} < 1$$

Even though, the above delays are not known *a priori* and are time-varying, the utilization of efficient processes and processors make them not to vary quickly when compared to the main source of the delay which is the queueing delay. Therefore, assuming that $|\dot{\tau}_{ji}(t)|$ is less than 1 s is quite realistic for real application. For simplicity, it is also assumed that the delay between any two nodes in both directions are the same, i.e. $\tau_{ji} = \tau_{ij}$. For more details refer to [11].

The following lemma is used in our subsequent results whose proof can be found in [8].

Lemma 1: [8] For any matrices $U, V \in \mathcal{R}^{n\times n}$ with $V > 0$, one has $UV^{-1}U^T \geq U + U^T - V$.

It is worth noting that the neighbor set of each node may vary. Therefore, the interconnections of each subsystem in the MJLS model (3) is mode-dependent. This implies that the interconnected terms vary at each switching mode. In the next section a decentralized $\mathcal{H}_\infty$ control for the MJLS with mode-dependent interconnected terms is proposed to provide a routing solution that guarantees internal stability of the traffic network and that minimizes the worst case network queueing length. Appropriate LMIs are also provided to satisfy the associated physical constraints.

III. A Markovian Jump $\mathcal{H}_\infty$ Control Strategy for Routing Problems in Mobile Networks

Theorem 1: Consider a mobile traffic network whose dynamics is governed by (3) for which $w_i \in L_2[0,\infty)$. The state feedback routing controllers $u_i = K_{ir}x_i$ with the gain of $K_{ir} = M_{ir}Y_{ir}^{-1}$ guarantee that the closed-loop system is stochastically stable and $J(w) < 0$, provided that there exist matrices M_{ir}, and symmetric positive definite matrices $Y_{ir},\ \bar{R}_{ir},\ \bar{Q}_i$ for $i = 1,...,n,\ r = 1,...,M$ such that the following LMI conditions are satisfied

$$W_{ir1} = \begin{bmatrix} \theta_{ir1} & \theta_{ir2} & B_{wik} & Y_{ik}^TC_{ik}^T & \theta_{ir3} & \theta_{ir4} & \theta_{ir5} \\ * & \theta_{ir6} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{ir7} & 0 & 0 \\ * & * & * & * & * & \theta_{ir8} & 0 \\ * & * & * & * & * & * & \theta_{ir9} \end{bmatrix} < 0$$

$$W_{ir2} = \begin{bmatrix} \theta_{ir10} & \tilde{\pi}_k \\ * & \tilde{R}_{ir} \end{bmatrix} \geq 0 \qquad (9)$$

where $\theta_{ir1} = M_{ik}^TB_{ik}^T + B_{ik}M_{ik} + \pi_{kk}Y_{ik}$,$\theta_{ir2} = \tilde{B}_{dik}\tilde{R}_{ik}$, $\theta_{ir3} = (\bar{\pi}_kY_{ik})^T$, $\theta_{ir4} = m_{ik}M_{ik}^T$, $\theta_{ir5} = h_{ji}M_{ik}^T$ $\theta_{ir6} = -(1-d_{ji})\tilde{R}_{ik}$, $\theta_{ir8} = -m_{ik}\bar{R}_{ik}$ $\theta_{ir7} = -diag\{Y_{i1},...,Y_{i(k-1)},Y_{i(k+1)},...,Y_{iM}\}$, $\theta_{ir9} = -h_{ji}\bar{Q}_i$, $\theta_{ir10} = 2(1-m_{ik}\pi_{kk})I - \bar{Q}_i + m_{ik}\pi_{kk}\bar{R}_{ik}$ $\bar{\pi}_k = [\sqrt{\pi_{k1}}\ \ ...\ \ \sqrt{\pi_{k(k-1)}}\ \ \sqrt{\pi_{k(k+1)}}\ \ ...\ \ \sqrt{\pi_{kM}}]^T$ $\tilde{\pi}_k = [\sqrt{m_{i1}\pi_{k1}} ... \sqrt{m_{i(k-1)}\pi_{k(k-1)}}\ \sqrt{m_{i(k+1)}\pi_{i(k+1)}} ... \sqrt{m_{iM}\pi_{kM}}]^T$, m_{ik} = number of subsystems where the subsystem i belongs to their $\wp_k(.)$ in mode k, $\tilde{R}_{ik} = diag_{j\in\wp_k(i)}\{\bar{R}_{jk}\}$, $\tilde{M}_{jk} = diag j \in \wp_k(i)\{M_{jk}\}$, and $\tilde{B}_{dik} = vec\{B_{dijk}\}$, for $j \in \wp_k(i)$.

Proof: The proof is provided in Appendix I.

A. LMI Conditions for the Physical Constraints

In this section, the physical constraints (5)-(8) are represented as LMI feasibility conditions.

1) Capacity Constraint: The capacity constraint for each subsystem is defined in (7). Consider the following ellipsoid for a selected $\varrho_i > 0$, i.e., $\Sigma_i = \{x_i(t)|x_i^T(t)Y_{ir}^{-1}x_i(t) \leq \varrho_{ir},\ Y_{ir} = Y_{ir}^T > 0\}$. By applying invariant set method, and performing some manipulations the capacity constraints for a mobile network can be expressed by the following LMI conditions for $i = 1,...,n, r = 1,...,M, k_i = 1,...,li$

$$W_{c1ir} \triangleq \gamma \leq \max_{i,r}\{(\varrho_{ir} - L_2)/L_1\} \qquad (10)$$

$$W_{c2irk_i} \triangleq \begin{bmatrix} Y_{ir} & M_{ir}^TG_{kir}^T \\ G_{kir}M_{ir} & c_{kir}^2/\varrho_{ir} \end{bmatrix} \geq 0 \qquad (11)$$

where $L_1 = \int_0^\infty w_i^T(t)w_i(t)dt$ is an upper bound on the energy of the external input $w_i(t)$, and $L_2 = V(x_0, r_0)$. For further details refer to [12].

2) Upper Bound on the Buffer Size: Following along the similar lines as those used for the capacity constraint and considering the ellipsoid considered for the capacity constraint, equation (8) can be satisfied by the following LMI conditions

$$W_{c3ir} \triangleq \begin{bmatrix} Y_{ir} & Y_{ir}^TQ_{di}^T \\ Q_{di}Y_{ir} & x_{max_{di}}^2/\varrho_{ir} \end{bmatrix} \geq 0 \qquad (12)$$

3) Non-negative Orthant Stability: The non-negativity constraint (6) can be expressed in terms of the non-negative orthant stability condition that is mentioned in [13]. Noting the state feedback controller is $u_i = K_{ir}x_i$, by selecting the positive definite matrix Y_{ir} to be a diagonal matrix and by setting $K_{ir} = M_{ir}Y_{ir}^{-1}$ for subsystem i the (essential) non-negativity of $(B_{dijr}K_{ir})\ B_{ir}K_{ir}$, which ensures the non-negativity constraint (6), can be expressed as follows

$$W_{c4ir} \triangleq (B_{ir}M_{ir})_{sm} \geq 0, \quad s \neq m, i = 1,...,n, \qquad (13)$$

$$W_{c5ir} \triangleq (B_{dijr}M_{jr})_{sm} \geq 0,\ m,s = 1,...,\bar{d}, j \in \wp_r(i) \qquad (14)$$

Once the non-negativity condition $x_i \geq 0$ is satisfied, the second non-negativity condition $u_i \geq 0$, as given by (5) can be easily satisfied if we specify $K_{ijr} > 0$. Therefore, by noting that Y_{ir} is a diagonal positive definite matrix, (5) is satisfied if the following LMI condition holds

$$W_{c6ir} \triangleq M_{ir(sm)} \geq 0,\ s,m = 1,...,\bar{d} \qquad (15)$$

Remark 1: It should be noted that since the elements of B_{ir} are either -1 or 0, satisfying condition (15) results in a square matrix $B_{ir}M_{ir}$ with negative or zero elements. On the other hand, satisfying W_{c4ir} leads to a diagonal negative definite matrix $B_{ir}M_{ir}$. This is also validated by the fact that the queues at each node are decoupled from each other. Therefore, $B_{ir}K_{ir}$ should always be diagonal. Moreover, since the elements of B_{dijr} are either 1 or 0, satisfying condition (15) results in a square matrix $B_{dijr}M_{ir}$ with positive or zero elements. Therefore, W_{c5ir} is trivially satisfied.

Consequently, the above results can be summarized by the following theorem.

Theorem 2: An $\mathcal{H}_\infty$ routing control scheme for a traffic network that is governed by the queueing model (3) is obtained by solving the following optimization problem:

$$\min_{M_{ir}, Y_{ir},\ \bar{R}_{ir},\ \bar{Q}_i} \gamma \qquad (16)$$

subject to the selection of positive definite matrices Y_{ir}, $\bar{R}_{ir}$, $\bar{Q}_i$, and the LMI conditions for W_{ir1}, W_{ir2}, W_{c1ir}, W_{c2irk_i}, W_{c3ir}, W_{c4ir}, and W_{c6ir} for $i = 1,...,n,\ r = 1,...,M$, as described by equations (9)-(13), and (15), respectively.

Proof: The proof follows along the lines given in this section. ■

IV. $\mathcal{H}_\infty$ Control Strategy for Mobile Networks Routing with Variable Destination Nodes

In mobile networks, occasionally for some destination nodes no external traffic has to be routed in certain periods. However, given the closed-loop system dynamics still some messages may be present in queues that should be routed to their destinations as quickly as possible. Moreover, in the dynamical model (3) the states are defined as queueing length at each node corresponding to a destination node. Therefore, the number of states depends on the active destination nodes. On the other hand, simply neglecting and deleting the corresponding states associated with the inactive destinations can lead to loss of integrity and stability of the overall system. This will also lead to ignoring the leftover messages that are kept in the eliminated queues. To cope with these issues we propose to model and represent the behavior of the network as a singular MJLS which is given below

$$\begin{aligned} E(r(t))\dot{x}_i(t) &= B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &+ \sum_{j\in\wp_{r(t)}(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \end{aligned} \quad (17)$$

where $E(r(t))$ is a diagonal matrix that is specified according to the following two scenarios:

(a) Regular mobile networks: In this case we have $E(r(t)) := I$;

(b) Varying number of destination nodes: In this case some destination nodes become inactive. Therefore, $E(r(t)) := diag\{e_j(r(t))\}$, where

$$e_j(r(t)) = \begin{cases} 1 & \text{if the queue is associated with an active destination node} \\ 0 & \text{otherwise} \end{cases}$$

Hence, when a destination node becomes inactive (active), the dynamics switch from regular to singular (singular to regular). On the other hand, to ensure the existence of a unique solution for singular MJLSs, piecewise regularity and piecewise impulse-free conditions should be investigated at each switching mode. The following lemma provides a necessary and sufficient condition for satisfying the regularity and piecewise impulsive-free conditions.

***Lemma** 2:* [10] System (17) with the state feedback control law $u_i = K_{ir}x_i$ satisfies the piecewise regularity and piecewise impulse-free conditions if and only if $A_{cli} = B_{ir}K_{ir}$ and $A_{cli} + A_{dcli}$ are nonsingular, where $A_{dcli} = \sum_{j\in\wp(i)} B_{dij}(r(t))K_{jr}$.

In the following subsequent section a decentralized routing controller for system (17) is proposed.

A. A Decentralized $\mathcal{H}_\infty$ Control of Singular Time-Varying Delay Systems with Markovian Jump Dynamics

***Theorem** 3:* The fluid flow model of a traffic network governed by (17) with $w \in L_2[0,\infty)$ is stochastically stabilizable, piecewise regular, and piecewise impulse-free if the decentralized state feedback routing control is designed as $u_i = K_{ir}x_i$ with an L_2-gain that is less than γ, and provided that there exist matrices M_{ir}, nonsingular matrices Y_{ir} and symmetric positive definite matrices $\bar{R}_{ir}$, $\bar{Q}_i$ for $i = 1,...,n,\ r = 1,...,M$ such that the following LMI conditions hold $E_rY_{ir}^T = Y_{ir}E_r^T > 0$

$$W_{ir1} = \quad (18)$$
$$\begin{bmatrix} \theta_{ir1} & \theta_{ir2} & B_{w_{ir}} & Y_{ir}^TC_{ir}^T & \theta_{ir3} & \theta_{ir4} & \theta_{ir5} \\ * & \theta_{ir6} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{ir7} & 0 & 0 \\ * & * & * & * & * & \theta_{ir8} & 0 \\ * & * & * & * & * & * & \theta_{ir9} \end{bmatrix} < 0$$

$$W_{ir2} = \begin{bmatrix} M_{ir}^TB_{ir}^T + B_{ir}M_{ir} & \tilde{B}_{dir}\tilde{M}_{jr} & Y_{ir} \\ * & -2Y_{jr}+I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (19)$$

$$W_{ir3} = \begin{bmatrix} \theta_{ir10} & \tilde{\pi}_r \\ * & \tilde{R}_{ir} \end{bmatrix} \geq 0 \quad (20)$$

where $\theta_{ir1} = M_{ir}^TB_{ir}^T + B_{ir}M_{ir} + \pi_{rr}E_rY_{ir}$, $\theta_{ir2} = \tilde{B}_{dir}\tilde{R}_{ir}$, $\theta_{ir3} = (\bar{\pi}_rY_{ir})^T$, $\theta_{ir4} = m_{ik}M_{ir}^T$, $\theta_{ir5} = h_{ji}M_{ir}^T$, $\theta_{ir6} = -(1-d_{ji})\tilde{R}_{ir}$, $\theta_{ir8} = -m_{ik}\bar{R}_{ik}$, $\theta_{ir7} = -diag\{Y_{i1},...,Y_{i(r-1)},Y_{i(r+1)},...,Y_{iM}\}$, $\theta_{ir9} = -h_{ji}\bar{Q}_i$, $\theta_{ir10} = 2(1-m_{ir}\pi_{rr})I - \bar{Q}_i + m_{ir}\pi_{rr}\bar{R}_{ir}$, $\bar{\pi}_r = [\sqrt{\pi_{r1}}E_1...\sqrt{\pi_{r(r-1)}}E_{(r-1)}\ \sqrt{\pi_{r(r+1)}}E_{(r+1)}...\sqrt{\pi_{rM}}E_M]^T$, $\tilde{\pi}_r = [\sqrt{m_{i1}\pi_{r1}}\ ...\ \sqrt{m_{i(r-1)}\pi_{r(r-1)}}\ \sqrt{m_{i(r+1)}\pi_{i(r+1)}}\ ...\ \sqrt{m_{iM}\pi_{rM}}]^T$, m_{ir} = number of subsystems that subsystem i belongs to their $\wp_r(.)$ in mode r, $\tilde{R}_{ir} = diag_{j\in\wp_r(i)}\{\bar{R}_{jr}\}$, $\tilde{M}_{jr} = diag j \in \wp(i)\{M_{jr}\}$, and $\tilde{B}_{dir} = vec\{B_{dijr}\}$, $\ for\ j \in \wp_r(i)$.

Moreover, the robust decentralized state feedback controller gain is given by $K_{ir} = M_{ir}Y_{ir}^{-1}$.

Proof: The proof is provided in Appendix II.

B. LMI Conditions for the Physical Constraints

1) Capacity Constraint: To guarantee the capacity constraint for each subsystem, i.e., (7), let us consider the following ellipsoid for a selected $\varrho_i > 0$, namely $\Sigma_i = \{x_i(t)|\int_{t-\tau_{ij}}^{t} x_i^T(s)K_{ir}^T\bar{R}_{ir}^{-1}K_{ir}x_i(s)ds \leq \varrho_{ir}, \bar{R}_{ir} = \bar{R}_{ir}^T > 0\}$. By performing some manipulations that are mentioned in details in [12], the capacity constraints for the subsystem i can be expressed as the following LMI conditions

$$W_{c1ir} \triangleq \gamma \leq \max_{i,r}\{(\varrho_{ir} - L_2)/L_1\}, r = 1,...,M \quad (21)$$

$$W_{c2irk_i} \triangleq \begin{bmatrix} 2I - \bar{R}_{ir} & G_{kir}^T \\ G_{kir} & h_ic_{kir}^2/\varrho_{ir} \end{bmatrix} \geq 0, k_i = 1,...,li \quad (22)$$

where $L_1 = \int_0^\infty w_i^T(t)w_i(t)dt$ is an upper bound on the energy of the external input $w_i(t)$, and $L_2 = V(x_0, r_0)$.

2) Upper Bound on the Buffer Size: Following along the lines indicated in [12], for each subsystem the following LMI conditions are obtained to guarantee that the queues do not exceed the upper bound of the buffer size, namely

$$\begin{aligned} W_{c3ir} &\triangleq \begin{bmatrix} \alpha_1 & (\bar{M}_{ir}M_{ir})^{-1}\bar{M}_{ir} \\ \bar{M}_{ir}^T(\bar{M}_{ir}M_{ir})^{-T} & 2I - \bar{R}_{ir} \end{bmatrix} \geq 0 \quad (23) \\ \alpha_1 &= 4I - 2Y_{ir} - (Q_{di}^TQ_{di})\varrho_{ir}/(h_ix_{max_{di}}^2), d = 1,...,\bar{d}, \end{aligned}$$

where $\bar{M}_{ir}$ is a selected matrix satisfying $(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir}u_i = x_i$.

3) Non-Negative Orthant Stability: In switching modes when the matrix E_r is full rank, i.e., corresponding to the regular dynamics, the associated LMI conditions for guaranteeing non-negativeness of the states are defined similar to the conditions (13) and (14). However, if E_r is a singular matrix, the state x_i is partitioned into two components as follows: $x_i = [x_{i1}^T\ x_{i2}^T]^T$, where x_{i1} is the queue associated with the active destination nodes and x_{i2} is the queue associated with the inactive destination nodes. We furthermore partition the state vector gain into $K_{ir} = [K_{ir1}\ \ K_{ir2}]$ with appropriate dimensions for K_{ir1} and K_{ir2} according to x_{i1} and x_{i2}, respectively. It should be noted that the queueing dynamics corresponding to the inactive destinations do not receive any external stimuli, i.e., $w_{i2} = 0$. Consequently, the closed-loop dynamics of (17) can be expressed according to

$$\begin{aligned} \dot{x}_{i1} &= B_{ir1}K_{ir1}x_{i1}(t) + \sum_{i=1}^{n} B_{dijr1}K_{jr1}x_{j1}(t-\tau(t)) + B_{iw1}w_{i1}(t) \\ 0 &= B_{ir2}K_{ir2}x_{i2}(t) + \sum_{i=1}^{n} B_{dijr2}K_{jr2}x_{j2}(t-\tau(t)) \end{aligned}$$

By invoking the non-negative orthant theorem indicated in [13], non-negativity of the queueing length can be guaranteed by the following LMI conditions:

$$W_{c4ir} \triangleq (B_{ir1}M_{ir1})_{sm} \geq 0, \quad s \neq m, i = 1, ..., n \tag{24}$$

$$W_{c5ir} \triangleq (B_{dijr1}M_{jr1})_{sm} \geq 0, \; m, s = 1, ..., \bar{d}, j \in \wp_r(i) \tag{25}$$

$$W_{c6ir} \triangleq \begin{cases} (B_{ir2}M_{ir2})_{sm} \leq 0 & s, m = 1, ..., \bar{d}, r \in \bar{\mathcal{S}} \\ (B_{ir2}M_{ir2})_{sm} \geq 0 & s, m = 1, ..., \bar{d}, r \in \mathcal{S} - \bar{\mathcal{S}}, \end{cases} \tag{26}$$

$$W_{c7ir} \triangleq (B_{dijr2}M_{jr2})_{sm} \geq 0, i, s, m = 1, ..., \bar{d}, j \in \wp_r(i) \tag{27}$$

where $\bar{\mathcal{S}}$ is the set of modes in which E_r is singular.

Provided that the non-negativity condition $x_i \geq 0$ is satisfied, $u_i \geq 0$ is guaranteed by the following LMI conditions

$$W_{c8ir} \triangleq M_{ir(sm)} \geq 0, \; s, m = 1, ..., \bar{d}, \tag{28}$$

To summarize, the following theorem presents our robust routing control strategy that corresponds to the mobile network dynamics (17) and satisfies the constraints (5)-(8).

Theorem 4: An $\mathcal{H}_\infty$ routing control design for a traffic network that is governed by the queueing model (17) is obtained by solving the following optimization problem:

$$\min_{M_{ir}, Y_{ir}, \bar{R}_{ir}, \bar{Q}_i} \gamma \tag{29}$$

subject to the selection of positive definite matrices $\bar{R}_{ir}$, $\bar{Q}_i$, and the LMI conditions for $W_{ir1} - W_{ir3}$, W_{c1ir}, W_{c2irk}, W_{c3ir}, W_{c4ir}, W_{c6ir} and W_{c8ir} for $i = 1, ..., n$, $r = 1, ..., M$, as described by equations (18)-(24), (26), and (28), respectively.

Proof: The proof follows along the constructive lines that were derived in this section. ■

V. Simulation Results

Example 1: Consider a scenario of an unmanned network having 50 nodes that are partitioned into three teams covering an area of $8000m \times 12000m$. The first team which includes the nodes $1-10$ is fixed, the second team includes the nodes $11-30$ moves towards north-east, and the third team which contains the nodes $31-50$ moves towards north. It is assumed that the network remains connected at all times. The nominal communication range for each node is 484 m, the capacity is 1 Mbps, and the maximum buffer size is 450 kbit. The transition mode is selected as $\pi_{rj} = 0.002$ for $r = j \pm 1$. The total simulation duration is 700 s for each run. The destination nodes are 7 and 10. For each input flow, the delay function is specified by $\tau(t) = 3 + 0.8|sin(t)|$ s.

I: Percentage of messages that are lost for 139680 kbit traffic load corresponding to different node speeds

Second team max speed (m/s)	20	40	60
% for our proposed method	13.49	22.75	25.32
% for OLSR	15.92	23.17	28.98
% for AODV	18.12	23.67	26.7

The traffic load for each node is based on Poisson distribution with the rate of $\lambda = 300$ bytes$/s$ for 600 s. We assume that the maximum speeds are 0, 10 m/s and 20 m/s for the mobile nodes in teams one, two and three, respectively. The simulations are repeated when the maximum speeds are increased by factors of two and three times of the above values. The maximum speed of the second team is used as an index for comparison. Percentage of total messages that are lost for 139680 kbit traffic load using our proposed routing algorithm, Ad hoc On Demand Distance Vector (AODV) [14], and Optimized Link State Routing protocol (OLSR) [15] corresponding to different node speeds are illustrated in Table I. The results confirm that by increasing the speed of nodes the proportion of dropped messages is also increased. It also follows that our proposed scheme can route messages with fewer losses when compared with the AODV and the OLSR methods.

Appendix I
Proof of Theorem 1

Denote $\mathcal{C}[-h_{ji}, 0]$ as the space of continuous functions on the interval $[-h_{ji}, 0]$. Let us define a process in $\mathcal{C}[-h_{ji}, 0]$ by $x_{is}(t) = x_i(s+t), \; t - \tau_{ij} \leq s \leq t$ to cast the dynamics (3) with time-delays into the framework of Markov process [7]. Consider the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned} V(x_t, r_t) &= V_1 + V_2 + V_3 \\ V_1 &= \sum_{i=1}^{n} x_i^T(t) P_{ir_t} x_i(t) \\ V_2 &= \sum_{i=1}^{n} \sum_{j \in \wp_{r_t}(i)} \int_{t-\tau_{ji}}^{t} u_j^T(s) R_{jr_t} u_j(s) ds \\ V_3 &= \sum_{i=1}^{n} \int_{0}^{h_{ij}} (h_{ij} - \sigma) u_i^T(t-\sigma) Q_i u_i(t-\sigma) d\sigma \end{aligned} \tag{30}$$

where $r(t) = (r_s, t - 2\tau \leq s \leq t)$. To achieve the $\mathcal{H}_\infty$ objective function (4), one should show

$$J_1 = \mathcal{A}V(x_t, r_t) + z^T(t)z(t) - \gamma w^T(t)w(t) < 0 \tag{31}$$

where $\mathcal{A}$ is the infinitesimal generator of $\{(x_{it}, r_t), t \geq 0\}$ [10]. Now assuming

$$\begin{aligned} &\sum_{i=1}^{n} \int_{t-\tau_{ij}(t)}^{t} u_i^T(s) Q_i u_i(s) ds \geq \\ &\sum_{i=1}^{n} \sum_{k=1}^{M} \pi_{r_t k} \sum_{j \in \wp_k(i)} \int_{t-\tau_{ji}(t)}^{T} u_j^T(s) R_{ik} u_j^T(s) ds \end{aligned} \tag{32}$$

and by following some manipulations, one gets

$$J_1 \leq \sum_{i=1}^{n} X_i^T(t)\bar{L}_{ir_t}X_i(t)$$

where

$$X_i = [x_i^T(t) \quad U_j^T(t-\tau_i(t)) \quad w_i^T(t)]^T,$$

$$U_j^T(t-\tau_i(t)) := vec\{u_j^T(t-\tau_{ji}(t))\} \quad for \quad j \in \wp_{r_t}(i)$$

$$\bar{L}_{ik} = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & P_{ik}B_{w_{ik}} & C_{ik}^T & \Omega_{i3} \\ * & \Omega_{i4} & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Omega_{i5} \end{bmatrix} < 0 \quad (33)$$

$\Omega_{i1} = (B_{ik}K_{ik})^TP_{ik} + P_{ik}(B_{ik}K_{ik}) + K_{ik}^T[h_{ji}Q_i + m_{ik}R_{ik}]K_{ik} + \pi_{kk}P_{ik}$, $\Omega_{i2} = P_{ik}\tilde{B}_{dik}$, $\Omega_{i3} = [\sqrt{\pi_{k1}}P_{i1} \ \cdots \ \sqrt{\pi_{k(k-1)}}P_{i(k-1)} \ \sqrt{\pi_{k(k+1)}}P_{i(k+1)} \ \cdots \ \sqrt{\pi_{kM}}P_{iM}]^T$, $\Omega_{i4} = -diag_{j\in\wp_k(i)}\{(1-d_{ji})R_{jk}\}$, $\Omega_{i5} = -diag\{P_{i1},...,P_{i(k-1)},P_{i(k+1)},...,P_{iM}\}$.

Now, let $K_{ik} = M_{ik}Y_{ik}^{-1}$, $P_{ik} = Y_{ik}^{-1}$, $R_{ik} = \bar{R}_{ik}^{-1}$, $Q_i = \bar{Q}_i^{-1}$, $\bar{\bar{R}}_{ik} = vec\{\bar{R}_{jk}\}$ for $j \in \wp_k(i)$, $\bar{Y}_i = vec\{Y_{il}\}$ for $l = 1,...,(p-1),(p+1),...,M$. Then, pre and post multiplying (33) by $\Delta_{ik} = diag\{Y_{ik}, \ \bar{\bar{R}}_{ik}, \ I, \ I, \ \bar{Y}_i\}$ and Δ_{ik}^T respectively, and applying the Schur complements lead to L_{ik} in (9). Therefore, (9) guarantees negative definiteness of J_1 in (31). Now using Dynkin's formula results in [10]

$$\begin{aligned} J_T &= \mathbb{E}[\int_0^T [J_1 - \mathcal{A}V(x_t,r_t)dt] \\ &\leq \mathbb{E}\int_0^T \sum_{i=1}^{n} X_i^T(t)L_{ir_t}X_i(t)dt - \mathbb{E}[V(x_T,r_T) + V(x_0,r_0)] \end{aligned}$$

Using the fact that $L_{ir_t} < 0$ and $\mathbb{E}[V(x_T,r_T)] > 0$ yields $J_T \leq V(x_0,r_0)$, and therefore the $\mathcal{H}_\infty$ objective function (4) is satisfied according to $\|z\|_{\mathbb{E}_2} - \gamma^2\|w(t)\|_2 \leq V(x_0,r_0)$.

The stability and convergence properties of the network states in the absence of the external input w_i, is guaranteed by eliminating the $(n+2)$th and the $(n+3)$th rows and columns of (9). In other words, negative definiteness of the resulting LMI guarantees the stochastic stability of the unforced system (3). In view of the above, the condition (32) should now be expressed according to the new LMI parameters $\bar{Q}_i$ and $\bar{R}_{il}$. Substituting Q_i and R_{il} by $\bar{Q}_i$ and $\bar{R}_{il}$ in (32) and using the fact that $\pi_{kk} = -\sum_{l=1,l\neq k}^{N}\pi_{kl}$, where $\pi_{kl} > 0$, and performing some manipulations, one can get that to guarantee (32), it suffices to satisfy the second LMI condition of (9). This completes the proof of the theorem. ■

The proof of this theorem is provided in details in [12].

APPENDIX II
PROOF OF THEOREM 3

To achieve the $\mathcal{H}_\infty$ objective function (4), it suffices to establish the inequality (31) where $V(x_t,r_t)$ is similar to (30). Whereas, V_1 should be modified properly for singular dynamics to $V_1 = \sum_{i=1}^{n} x_i^T(t)E_{r_t}P_{ir_t}x_i(t)$. By following along the similar lines as those given in proof of Theorem 1, one can show that the LMI condition (18) can guarantee negative definiteness of J and also stochastic stability of the unforced system in absence of w_i.

Moreover, consider that the following condition is satisfied

$$\begin{bmatrix} (B_{ik}K_{ik})^TP_{ik} + P_{ik}(B_{ik}K_{ik}) + I & P_{ik}\tilde{B}_{dik}\tilde{K}_{jk} \\ * & -I \end{bmatrix} < 0 \quad (34)$$

where $\tilde{K}_{jk} = diagj \in \wp_k(i)\{K_{jk}\}$. The condition (34) implies that $(B_{ik}K_{ik})^TP_{ik} + P_{ik}(B_{ik}K_{ik}) < 0$. Therefore, A_{cli} is nonsingular. Furthermore, by applying the Schur compliment it results in $A_{cli} + A_{dcli}$ being nonsingular. Therefore, the closed-loop system satisfies the piecewise regularity and the piecewise impulsive mode free conditions. Now, by substituting $P_{ik} = Y_{ik}^{-1}$ and pre and post multiply (34) by $diag\{Y_{ik}, \bar{\bar{Y}}_{ik}\}$ and its transpose, respectively, where $\bar{\bar{Y}}_{ik} = vec\{\bar{Y}_{jk}\} \ for \ j \in \wp_k(i)$, one can obtain (19). This completes the proof of the theorem. ■

The proof of this theorem is provided in details in [12].

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