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Synchronization of a class of dynamical complex networks with nonsymmetric coupling based on decentralized control

Haiqing Zheng, Yuanwei Jing, Xiuping Zheng, Nan Jiang

Abstract—**In this paper, the synchronization problem for dynamical complex networks composed of general Lur'e systems is dealt with. Based on the Jordan canonical transformation method and a Lur'e-Postnikov function, the global synchronization criteria for dynamical complex networks with non-symmetric coupling are established and formulated as LMI. A decentralized control strategy based on projection lemma and Lur'e-Postnikov function is proposed in order to reduce the conservativeness. A dynamical complex network composed of identical Chua's oscillators is adopted as a numerical example to demonstrate the effectiveness of the proposed results.**

I. Introduction

DYNAMICAL complex networks have attracted increasing attention from physicists, biologists, social scientists and control scientists in recent years [1]-[4]. From a system-theoretic point of view, a dynamical complex network can be considered as a large-scale system with special interconnections among its dynamical nodes. The large-scale system theory has been extensively studied in the last three decades, and many interesting results have been established, on such basic issues as decentrally fixed modes, decentralized controllers design etc.

During the past decades, synchronization of general dynamical complex networks with diffusely couplings has been extensively studied due to its theoretical importance and practical applications [5]-[8]. One unified approach to synchronization analysis is to linearize the network at certain homogeneous state to get the master stability equations and then judge the stability of these resulting equations [9]-[11]. Because of the linearization technique, the synchronization criteria obtained by this way are local. Moreover, additional requirements are often added to couplings of the network in order to get easily verifiable conditions. For instance, the topology matrix is assumed to be symmetric and the inner-linking matrix is assumed to be positive definite.

Many nonlinear physical systems can be represented as a feedback connection of a linear dynamical system and a nonlinear element which is satisfied a sector condition. Since Lur'e and Postnikov first proposed the concept of absolute stability as early as the 1940s, systems of this type have been extensively studied and Lur'e systems with sector bound are such cases. Because in various fields of theory and engineering applications, a large class of nonlinear systems such as Goodwin model, repressilator, toggle switch, swarm model and Chua's circuit can be represented as Lur'e systems, synchronization of dynamical complex networks consisting of Lur'e systems has gained a lot of attentions of the researchers [12]-[15].

Different from general dynamical complex networks, by using the Lur'e system method in control theory, different synchronization conditions for dynamical complex networks composed of Lur'e systems can be derived. For example, in [3], the Lyapunov function method was applied to derive the global synchronization criterion for dynamical networks composed of Escherichia coli cells which can be expressed as Lur'e systems coupled indirectly through intercellular signaling and in [12], the authors discuss the synchronization problem for a class of dynamical complex networks with each node being a Lur'e system. Nevertheless, the given condition in [12] requires the topology matrix being symmetric. Considering that many realistic networks are non-symmetric, it is worthy of studying synchronization problem for dynamical complex networks with non-symmetric coupling.

In this paper, based on the work of Liu and Wang [12], we extend the synchronization problem for dynamical complex networks composed of general Lur'e systems from symmetric coupling to non-symmetric coupling by using the Jordan canonical transformation method. The approach taken in the paper is to transform the synchronization problem of dynamical networks into absolute stability problem of corresponding error system. By using a Lur'e-Postnikov function and the Jordan canonical transformation method, sufficient conditions represented by LMI are given such that the error system is absolutely stable and consequently all states of the network are globally stabilized onto an expected homogeneous state. In order to reduce the conservativeness, a controller design method based on Lur'e-Postnikov function is proposed. It should be pointed out that no linearization technique is involved through derivation of all the synchronization criteria. In addition, in contrast to centralized control methods, decentralized control has many advantages, such as lower dimensionality, easier implementation, lower cost etc.

The rest of the paper is organized as follows. In Section 2, some preliminary definitions and lemmas necessary for successive development are presented. In Section 3, based on

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Haiqing Zheng is with the School of Information Science and Engineering, Northeastern University, Shenyang, China. (E-mail: zhenghaiqing_1983 @yahoo.cn).

Yuanwei Jing, Xiuping Zheng and Nan Jiang are with the School of Information Science and Engineering, Northeastern University, Shenyang, China. (E-mail: ywjjing@mail.neu. edu.cn).

Lur'e-Postnikov function, global synchronization conditions for dynamical complex networks with non-symmetric coupling are given. Moreover, the controller design method in terms of Lur'e-Postnikov function is also introduced to reduce the conservativeness in Section 3. A numerical example is provided to illustrate the efficiency of the given results in Section 4 and concluding remarks are given in Section 5.

II. PRELIMINARIES

Consider the following nonlinear system

$$\dot{x} = Ax + Bf(y_0,t),\ \ y_0 = Cx, \tag{1}$$

where $x \in R^n$, $y_0 \in R^m$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{m\times n}$, f is a memoryless, possibly time-varying nonlinear function,

$$f(0,t)=0,\ \ y_0 = \begin{bmatrix} y_{01} \\ \vdots \\ y_{0m} \end{bmatrix} \text{ and } f(y_0,t) = \begin{bmatrix} f_1(y_{01},t) \\ \vdots \\ f_m(y_{0m},t) \end{bmatrix}.$$

The functions $f_l(y_{0l},t)$, $l = 1,\cdots,m$ are assumed to satisfy the following inequalities

$$0 \le f_l(y_{0l},t)y_{0l} \le \delta_l y_{0l}^2,\ \ l = 1,2,\cdots,m,$$

for all $y_{0l} \in R$ and $t \in R_+$, where $\delta_l \in R$ and $\delta_l \ge 0$. Taking $\Delta_0 = diag(\delta_1,\delta_2,\cdots\delta_m)$, it can be easily seen that

$$f^T(y_0,t)(f(y_0,t) - \Delta_0 y_0) \le 0, \tag{2}$$

for all $y_0 \in R^m$ and $t \in R_+$.

Definition 1. The nonlinearity $f(y_0,t)$ is said to be in the sector $[0,\Delta_0]$ if it satisfies (2).

Definition 2. System (1) is said to be absolutely stable with respect to the sector $[0,\Delta_0]$ if for the nonlinearity $f(y_0,t)$ satisfying (2), the equilibrium point $x = 0$ is globally asymptotically stable.

Lemma 1. (Wu [16]). The eigenvalues of an irreducible matrix $G_0 = (g_{0ij}) \in R^{N\times N}$ with $\sum_{j=1,j\ne i}^{N} g_{0ij} = -g_{0ii}$ satisfy the following properties:

(i) 0 is an eigenvalue of G_0 associated the eigenvector $(1,1,\cdots,1)^T$.

(ii) If $g_{0ij} \ge 0$ for all $1 \le i,j \le N, i \ne j$, then the real parts of all eigenvalues of G_0 are less than or equal to 0 and all possible eigenvalues with zero part are 0. In fact, 0 is an eigenvalue of multiplicity 1.

Lemma 2. (Boyd *et al.* [17] Schur complement). For any blocked matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

the following statements are equivalent:

(i) $S < 0$.

(ii) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$.

(iii) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 3. (Boyd *et al.* [17]). Let matrices $B_0 \in R^{n\times m}$, $C_0 \in R^{k\times l}, Q_0 \in R^{n\times n}$ be given and suppose $rank(B_0) < n$, $rank(C_0) < n$ and $Q_0 = Q_0^T$. Then there is a matrix K_0 of compatible dimension such that

$$B_0K_0C_0 + (B_0K_0C_0)^T + Q_0 < 0,$$

if and only if $B_0^{\perp} Q_0 B_0^{\perp T} < 0$ and $C_0^{T\perp} Q_0 C_0^{T\perp T} < 0$.

Lemma 4. (Wu *et al.* [18]). Let $G \in M_N(C)$ be a given complex matrix. There is a nonsingular matrix $\Phi \in M_N(C)$ such that

$$G = \Phi \begin{bmatrix} J_{n_1}(\lambda_1) & & & 0 \\ & J_{n_2}(\lambda_2) & & \\ & & \ddots & \\ 0 & & & J_{n_k}(\lambda_k) \end{bmatrix} \Phi^{-1} = \Phi J \Phi^{-1}.$$

The Jordan matrix J of G is unique up to permutation of the diagonal Jordan blocks. The eigenvalues λ_i, $i = 1,2,\cdots,k$ are not necessarily distinct. $J_{n_i}(\lambda_i)$ are the Jordan blocks.

$$J_{n_i}(\lambda_i) = \begin{bmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{bmatrix},\ i = 1,\cdots,k.$$

III. MAIN RESULTS

Consider a class of dynamical complex networks with each node being a general Lur'e system

$$\dot{x}_i = Ax_i + Bf(y_i) + \sum_{j=1}^{N} g_{ij} Dx_j,\ y_i = Cx_i,\ i = 1,2,\cdots,N, \tag{3}$$

where $x_i \in R^n$, $y_i \in R^m$, A, B, C have the same meanings as those in (1), $D \in R^{n\times n}$ defines the coupling between two nodes, $G = (g_{ij}) \in R^{N\times N}$ is the coupling matrix of the network, where $g_{ij} \in R$ is defined as follows: if there is a connection from node j to node i $(i \ne j)$, then the coupling strength $g_{ij} \ne 0$; otherwise $g_{ij} = 0$, and the diagonal elements of G are defined by

$$g_{ii} = -\sum_{j=1,j\ne i}^{N} g_{ij},\ i = 1,2,\cdots,N.$$

In network model (3), the coupling matrix G is nonsymmetric and we always assume that G is irreducible and all of its off-diagonal elements are nonnegative.

Denote

$$y_i = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{im} \end{bmatrix} \in R^m \text{ and } f(y_i) = \begin{bmatrix} f_1(y_{i1}) \\ \vdots \\ f_m(y_{im}) \end{bmatrix} \in R^m,$$

where $f_l(y_{il})$, $i = 1,\cdots,N$, $l = 1,\cdots,m$ satisfy the following inequalities:

$$0 \le f_l(y_{il})y_{il} \le \delta_{1l} y_{il}^2,\ 0 \le f_l'(y_{il}) \le \delta_{1l},$$

for all $y_{il} \in R$ and $\delta_{1l} \geq 0$.

The dynamical complex network (3) is said to achieve synchronization if

$$x_i \to s \ (i = 1, 2, \cdots, N), \quad t \to \infty, \tag{4}$$

where s is a solution of an isolated node, satisfying

$$\dot{s} = As + Bf(Cs).$$

To realize (4), the following decentralized control strategy is applied:

$$\dot{x}_i = Ax_i + Bf(y_i) + \sum_{j=1}^{N} g_{ij} Dx_j + B_1 u_i, \ y_i = Cx_i, \ i = 1, \cdots, N \tag{5}$$

where $u_i = K(x_i - s)$ and $K \in R^{m \times n}$ is the control gain matrix.

Defining $e_i = x_i - s$ and using the Kronecker product, the error dynamical system can be given by

$$\dot{e} = (M_1 + M_3 K_1 + G \otimes D)e + M_2 \eta(y_e; S), \ \ y_e = He \tag{6}$$

where $e = x - S = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} - \begin{bmatrix} s \\ \vdots \\ s \end{bmatrix} \in R^{Nn}, \ F(y) = \begin{bmatrix} f(y_1) \\ \vdots \\ f(y_N) \end{bmatrix}$,

$$M_1 = I_N \otimes A, \ M_2 = I_N \otimes B, \ M_3 = I_N \otimes B_1,$$

$$H = I_N \otimes C, \ y_e = (y_{e11}, \cdots, y_{e1m}, \cdots, y_{eN1}, \cdots, y_{eNm})^T,$$

$$K_1 = I_N \otimes K, \ \eta(y_e; S) = F(y_e + y_S) - F(y_S).$$

Denote

$$\eta(y_e; S) = (\eta_1(y_{e11}; s), \cdots, \eta_m(y_{e1m}; s), \cdots, \eta_m(y_{eNm}; s))^T.$$

It is obvious that the nonlinearities $\eta_l(y_{eil}; s)$ satisfy

$$\eta_l(y_{eil}; s)(\eta_l(y_{eil}; s) - \delta_{1l} y_{eil}) \leq 0, \ \ i = 1, \cdots, N, l = 1, \cdots, m. \tag{7}$$

Take $\Delta_1 = diag(\delta_{11}, \cdots, \delta_{1m})$ and $\Delta = diag(\Delta_1, \Delta_1, \cdots, \Delta_1)$. We deduce that the nonlinearity $\eta(y_e; S)$ belongs to the sector $[0, \Delta]$. It is shown from above representations that the synchronization problem of the dynamical complex network (3) is transformed into an equivalent absolute stability problem of the error dynamical system (6).

From Lemma 4, there is a nonsingular matrix $\Phi \in M_N(C)$, such that $G = \Phi J \Phi^{-1}$. Let $e = (\Phi \otimes I_n)\theta$, then, one has

$$\dot{e} = (\Phi \otimes I_n)\dot{\theta}, \tag{8}$$

and

$$\dot{\theta} = (M_1 + M_3 K_1 + J \otimes D)\theta + M_2 \eta(y_e; S), \tag{9}$$

where $\theta = (\theta_1, \theta_2, \cdots, \theta_N)^T \in R^{nN}$, $\theta_i \in R^n$. The variable θ is a linear combination of all the errors e_i, and the synchronization state s of the network (5) is globally stable if the origin of (9) is globally stable.

Proposition 1. The system (9) is globally stable with respect to the sector $[0, \Delta]$ and the controlled network (5) is globally stabilized onto s if there exist: $P > 0, \Lambda_1 = \mathrm{diag}(\lambda_{11}, \cdots, \lambda_{1m}) \geq 0, T_1 = \mathrm{diag}(t_{11}, \cdots, t_{1m}) > 0$, and $T_1' = \mathrm{diag}(t_{11}', \cdots, t_{1m}') > 0$ such that

$$\Omega = \begin{bmatrix} \Sigma_1 & PM_2 + \frac{1}{2}(\Phi \otimes \Delta_1 T_1 C)^T & \Sigma_2 \\ * & -I_N \otimes T_1 & M_2^T (\Phi \otimes \Lambda_1 C)^T \\ * & * & -I_N \otimes T_1' \end{bmatrix} < 0, \tag{10}$$

where $(*)$ denotes the terms induced by symmetry,

$$\Sigma_1 = P(M_1 + M_3 K_1 + J \otimes D) + (M_1 + M_3 K_1 + J \otimes D)^T P,$$

$$\Sigma_2 = (M_1 + M_3 K_1 + J \otimes D)^T (\Phi \otimes \Lambda_1 C)^T + \tfrac{1}{2}(\Phi \otimes \Delta_1 T_1' C)^T.$$

Proof: We choose the following Lur'e-Postnikov function

$$V = \theta^T P \theta + 2\sum_{i=1}^{N} \sum_{l=1}^{m} \lambda_{il} \int_0^{y_{eil}} f_l(\tilde{y}_{eil}) \mathrm{d}\tilde{y}_{eil},$$

where $P > 0, \Lambda = \mathrm{diag}(\lambda_{11}, \cdots, \lambda_{1m}, \cdots, \lambda_{N1}, \cdots, \lambda_{Nm}) \geq 0$ are to be determined.

Taking the derivative of V with respect to (9) yields

$$\begin{aligned} \dot{V} &= \theta^T P \dot{\theta} + \dot{\theta}^T P \theta + 2[\lambda_{11} f_1(y_{e11}) \dot{y}_{e11} + \cdots + \lambda_{1m} f_m(y_{e1m}) \dot{y}_{e1m} \\ &\quad + \cdots + \lambda_{N1} f_1(y_{eN1}) \dot{y}_{eN1} + \cdots + \lambda_{Nm} f_m(y_{eNm}) \dot{y}_{eNm}] \\ &= \theta^T P \dot{\theta} + \dot{\theta}^T P \theta + 2F^T(y_e) \Lambda H \dot{e}. \end{aligned}$$

Considering the sector conditions on the nonlinearities $F(y_e)$ and $\eta(y_e; S)$, we have

$$\begin{aligned} \dot{V} &\leq \dot{V} - \sum_{i=1}^{N} \sum_{l=1}^{m} t_{il} \eta_l(y_{eil}; s)(\eta_l(y_{eil}; s) - \delta_{1l} y_{eil}) \\ &\quad - \sum_{i=1}^{N} \sum_{l=1}^{m} t_{il}' f_l(y_{eil})(f_l(y_{eil}) - \delta_{1l} y_{eil}), \end{aligned} \tag{11}$$

where $t_{il} > 0, t_{il}' > 0, \ i = 1, \cdots, N, \ l = 1, \cdots, m$ are introduced to reduce the conservativeness.

Denote

$$T = \mathrm{diag}(t_{11}, \cdots, t_{1m}, \cdots, t_{N1}, \cdots, t_{Nm}) > 0,$$

$$T' = \mathrm{diag}(t_{11}', \cdots, t_{1m}', \cdots, t_{N1}', \cdots, t_{Nm}') > 0,$$

and it is easy to see that

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{l=1}^{m} t_{il} \eta_l(y_{eil}; s)(\eta_l(y_{eil}; s) - \delta_{1l} y_{eil}) \\ &= \eta^T(y_e; S) T \eta(y_e; S) - \eta^T(y_e; S) \Delta T H e, \end{aligned} \tag{12}$$

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{l=1}^{m} t_{il}' f_l(y_{eil})(f_l(y_{eil}) - \delta_{1l} y_{eil}) \\ &= F^T(y_e) T' F(y_e) - F^T(y_e) \Delta T' H e. \end{aligned} \tag{13}$$

It is noted that the matrices Λ, T and T' are introduced to reduce the conservativeness. So in the following, we choose $\lambda_{il} = \lambda_{jl} \geq 0, \ t_{il} = t_{jl} > 0$ and $t_{il}' = t_{jl}' > 0$.

Denote $T_1 = \mathrm{diag}(t_{11}, \ t_{12}, \cdots, \ t_{1m}), \ T_1' = \mathrm{diag}(t_{11}', \ t_{12}', \cdots, \ t_{1m}')$ and $\Lambda_1 = \mathrm{diag}(\lambda_{11}, \lambda_{12}, \cdots, \lambda_{1m})$. Substituting $\dot{V}$ and above (12) and (13) into the right hand of (11), we obtain

$$\dot{V} \leq q^T \Omega q,$$

where

$$q = \begin{bmatrix} \theta \\ \eta(y_e; S) \\ F(y_e) \end{bmatrix}.$$

If $\Omega < 0$, there exists a sufficiently small number

$\varepsilon > 0$ such that $\dot{V} \le q^T \Omega q \le -\varepsilon q^T q < 0$ for $q \ne 0$. This completes the proof.

In order to get a common gain matrix K , P should have the form of $P = \text{diag}\ (P_1,\ P_1,\ \cdots, P_1) > 0$, where $P_1 > 0$. To reduce the conservativeness of designing controller, we choose the similar method used in [12], and derive the following results.

Theorem 1. The system (9) is globally stable with respect to the sector $[0, \Delta]$ and the controlled network (5) is globally stabilized onto s if there exist:
$Y > 0,\ \Lambda_1 = \text{diag}(\lambda_{11},\ \cdots,\ \lambda_{1m}) \ge 0,\ T_1 = \text{diag}(t_{11}, \cdots, t_{1m}) > 0,$
$T_1' = \text{diag}\ (t_{11}',\ \cdots,\ t_{1m}') > 0$, a nonsingular matrix X_1 and any matrix X_2 such that

$$\begin{bmatrix} -I_N \otimes X_1^T - I_N \otimes X_1 & \Psi_1 & I_N \otimes B & 0 & I_N \otimes X_1^T \\ \Psi_1^T & -I_N \otimes Y & \tfrac{1}{2}\Phi^T \otimes X_1 C^T T_1 \Delta_1 & \Psi_2 & 0 \\ I_N \otimes B^T & \tfrac{1}{2}\Phi \otimes \Delta_1 T_1 C X_1^T & -I_N \otimes T_1 & \Phi^T \otimes B^T C^T \Lambda_1 & 0 \\ 0 & \Psi_2^T & \Phi \otimes \Lambda_1 CB & -I_N \otimes T_1' & 0 \\ I_N \otimes X_1 & 0 & 0 & 0 & -I_N \otimes Y \end{bmatrix} < 0, \quad (14)$$

where $\Psi_1 = I_N \otimes (AX_1^T + B_1 X_2) + J \otimes DX_1^T + I_N \otimes Y,$

$$\Psi_2 = (\Phi \otimes \Lambda_1 C(AX_1^T + B_1 X_2))^T + (\Phi J \otimes \Lambda_1 CDX_1^T)^T + \tfrac{1}{2}\Phi^T \otimes X_1 C^T T_1' \Delta_1.$$

Moreover, the control gain matrix is given by $K = X_2 X_1^{-T}$.

Proof: Based on the Schur complement and Lemma 3, we get (10) is equivalent to the following matrix inequality (15).

$$\begin{bmatrix} -I_N \otimes V - I_N \otimes V^T & \Xi_1 & (I_N \otimes V) M_2 & 0 & I_N \otimes V \\ \Xi_1^T & -I_N \otimes P_1 & \tfrac{1}{2}(\Phi \otimes \Delta_1 T_1 C)^T & \Xi_2 & 0 \\ M_2^T (I_N \otimes V^T) & \tfrac{1}{2}(\Phi \otimes \Delta_1 T_1 C) & -I_N \otimes T_1 & M_2^T (\Phi \otimes \Lambda_1 C)^T & 0 \\ 0 & \Xi_2^4 & (\Phi \otimes \Lambda_1 C) M_2 & -I_N \otimes T_1' & 0 \\ I_N \otimes V^T & 0 & 0 & 0 & -I_N \otimes P_1 \end{bmatrix} < 0, \quad (15)$$

Here, the matrix V is not necessary to be symmetric and positive definite. Pre- and post-multiplying both sides of (15) by $W = \text{diag}\ (I_N \otimes V^{-1},\ I_N \otimes V^{-1}, I_N \otimes I_m, I_N \otimes I_m, I_N \otimes V^{-1})$ and W^T , and letting $X_1 = V^{-1},\ X_2 = KX_1^T,\ Y = X_1 P_1 X_1^T$, we can derive the matrix inequality (14). This completes the proof.

Remark 1. The diagonal matrices $\Lambda_1 \ge 0, T_1 > 0, T_1' > 0$ are introduced to reduce the conservativeness. The change of them does not affect the feasibility of (14). Choosing the matrices Λ_1, T_1, T_1' firstly, the inequality (14) is turned into LMI and the control gain matrix K can be constructed via the feasible solutions of the derived LMI.

IV. NUMERICAL EXAMPLES

To test the effectiveness of our results, we investigate a network composed of 6 identical chaotic Chua's oscillators with global unidirectional coupling. The entire network can be described by the following equations:

$$\begin{bmatrix} \dot{v}_{i1} \\ \dot{v}_{i2} \\ \dot{i}_{i3} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{C_1}(\dfrac{v_{i2} - v_{i1}}{R} - g(v_{i1})) + \sum_{j=1}^{6} \dfrac{g_{ij}}{R_1 C_1} v_{j1} \\ \dfrac{1}{C_2}(\dfrac{v_{i1} - v_{i2}}{R} + i_{i3}) \\ -\dfrac{1}{L}(v_{i2} + R_0 i_{i3}) \end{bmatrix}, i = 1, \cdots, 6 \quad (16)$$

where $R_0 = 0.0011\Omega,\ R = 1\Omega,\ C_1 = 0.1096F,\ C_2 = 1F,$
$L = 0.0680H,\ R_1 = 0.3\Omega,\ G_{a1} = -1.1384S,\ G_{b1} = -0.7225S,$
$g(v_{i1}) = G_{a1} v_{i1} + \tfrac{1}{2}(G_{a1} - G_{b1})\big[|v_{i1} + 1| - |v_{i1} - 1|\big],$
and the coupling matrix is

$$G = (g_{ij}) = \begin{bmatrix} -5 & 1 & 1 & 1 & 1 & 1 \\ 0 & -4 & 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 1 & 1 & 1 \\ 0 & 0 & 0 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

which is nonsymmetric.

Obviously, system (16) can be reformulated as the following forms

$$\dot{x}_i = Ax_i + Bf(y_i) + \sum_{j=1}^{6} g_{ij} Dx_j,\ y_i = Cx_i,\ i = 1, \cdots, 6 \quad (17)$$

where

$$x_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ i_{i3} \end{bmatrix} = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix},\ A = \begin{bmatrix} 1.2628 & 9.1241 & 0 \\ 1 & -1 & 1 \\ 0 & -14.7059 & -0.0162 \end{bmatrix},$$

$$B = \begin{bmatrix} -9.1241 \\ 0 \\ 0 \end{bmatrix},\ D = \begin{bmatrix} 30.4136 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$C = [1\ \ 0\ \ 0],\ f(y_i) = g(v_{i1}) - G_{a1} v_{i1}$ and $f(y_i)$ satisfies $0 \le f(y_i) y_i \le (G_{b1} - G_{a1}) y_i^2$.

If the network (17) is expected to be stabilized on the origin, we choose the decentralized control strategy proposed in the paper to realize the purpose. In the case, $B_1 = B$ and $u_i = Kx_i,\ i = 1, \cdots, 6$ are applied, where K is the control gain matrix to be solved. Choosing $T_1 = 20$, $T_1' = 20$ and $\Lambda_1 = 0.1$ firstly, the following feasible solutions

$$X_1 = \begin{bmatrix} 19.4381 & -0.0764 & -0.3390 \\ 0.2519 & 24.1525 & 193.2668 \\ -0.6528 & -248.4727 & 248.5840 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 62.1719 & 22.7753 & 196.3574 \end{bmatrix},$$

are derived and then $K = \begin{bmatrix} 3.1994 & -0.5848 & 0.1868 \end{bmatrix}$ is followed. The states x_{i1}, x_{i2} and x_{i3} of (17) with $u_i = Kx_i$ are presented in Fig. 1-Fig. 3, from which we observe that the network (17) realizes synchronization.

V. Conclusions

In this paper, we investigate the synchronization problem for a class of dynamical complex networks composed of general Lur'e systems with non-symmetric coupling. This problem is converted into an equivalent absolute stability problem of corresponding error system. Based on a Lur'e-Postnikov function and the Jordan canonical transformation method, the problem of designing a linear feedback controller such that states of all nodes are globally stabilized onto an expected homogeneous state is addressed and the synchronization criteria are established and formulated as LMI. Since the criteria are constructed on the Jordan canonical transformation method instead of the matrix diagonalization method, the obtained results are much sharper. In order to reduce the conservativeness, a decentralized control strategy based on the projection lemma in LMI method is proposed by introducing a freedom matrix which is not necessary to be symmetric and positive definite. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed results.

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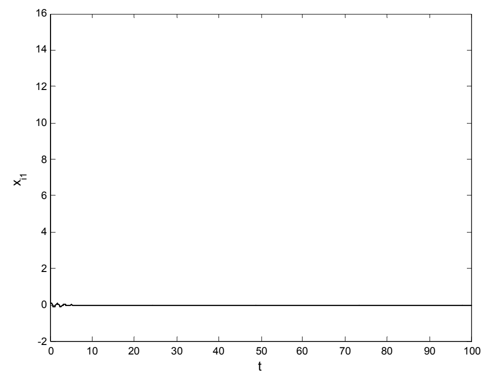

Fig. 1. The states x_{i1} of the network (17)

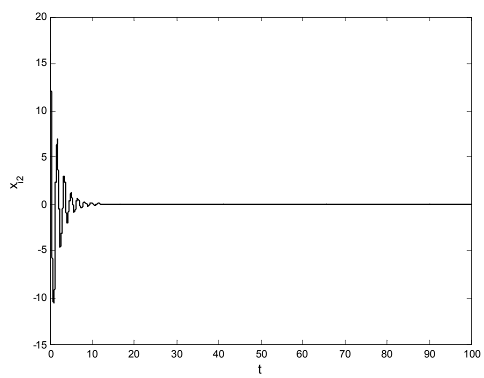

Fig. 2. The states x_{i2} of the network (17)

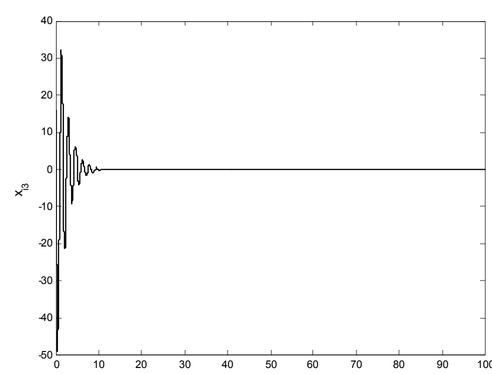

Fig. 3. The states x_{i3} of the network (17)