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Adaptive PID Controller Design based on Output Feedback Passivity for Discrete-Time Nonlinear Systems

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Abstract— **This paper deals with the design of an adaptive PID control system for discrete-time SISO non-linear systems. The proposed method is based on the output feedback strictly passive (OFSP) property of the controlled system. A fundamental design scheme of an adaptive PID control system with a parallel feedforward compensator (PFC) introduced in order to realize an OFSP augmented controlled system will be provided.**

I. INTRODUCTION

The PID control is one of the most common control schemes in industrial processes and of mechanical systems. However since most PID parameters are tuned off line, in cases where the system has complex uncertainties and/or there are some changes of system properties, it is hard to tune the PID parameters adequately and difficult to maintain the desired control performance and stability during operation. Furthermore, the control plays a very important role in the improvement of production quality, accuracy and in reducing production costs. From the facts, in recent decades, a great deal of attention has been turned to automatic tuning or self tuning of PID controllers [1] and several kinds of auto-tuning PIDs including self-tuning schemes and adaptive control strategies have been proposed [2], [3], [4], [5], [6]. Unfortunately, in most PID auto-tuning methods, the tuned PID parameters did not guarantee the stability of the control system after any change of the systems. It is also noted that a lack in the number of tuned parameters in the PID control makes it difficult to maintain stability over the whole range of the considered controlled system. Recently, auto-tuning and adaptive PID control strategies based on the almost strictly positive real (ASPR) property of the controlled system have been proposed [4],[5],[9] for linear continuous-time and also discrete-time systems. These adaptive PID schemes based on the ASPR property of the system can guarantee the asymptotic stability of the resulting PID control system. The ASPR property of the linear system is recognized as the output feedback strictly passive (OFSP) property [10],[11] of non-linear systems. This means that there is a possibility of realizing an adaptive PID control for non-linear systems.

In this paper, we consider the design of an adaptive PID control system for discrete-time SISO non-linear systems. As many processes are sampled data non-linear systems, it is very important to consider the controller design method for discrete-time non-linear systems. The proposed method is based on the OFSP property of the controlled system, so the stability of the resulting adaptive control system can be guaranteed with certainty. However, since most practical systems do not satisfy OFSP conditions and the fact that the OFSP discrete-time system must have a direct feedthrough term of the input, i.e. the discrete OFSP system has to have a relative degree of zero, difficulties such as causality problems will appear in the controller design. By considering the equivalent PID controller, we propose an adaptive PID control system design with a parallel feedforward for non-OFSP systems that has no causality problems. The proposed adaptive PID controller can guarantee the stability of the control system, and by adjusting PID parameters adaptively, the method can maintain a better control performance even if there are some changes of the system properties.

II. OUTPUT FEEDBACK STRICT PASSIVITY

Consider the following n-th order discrete-time SISO nonlinear system with a relative of 0.

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k)) + \boldsymbol{g}(\boldsymbol{x}(k))u(k) \tag{1}$$

$$y(k) = h(\boldsymbol{x}(k)) + J(\boldsymbol{x}(k))u(k) \tag{2}$$

where $\boldsymbol{x}(k) \in R^n$ is a state vector, $u(k)$, $y(k) \in R$ are the input and output of the system. $\boldsymbol{f}(\boldsymbol{x}(k)) : R^n \to R^n$, $\boldsymbol{g}(\boldsymbol{x}(k)) : R^n \to R^n$, $h(\boldsymbol{x}(k)) : R^n \to R$ and $J(\boldsymbol{x}(k)) : R^n \to R$ are smooth in $x(k)$, and we assume that $f(0) = 0$, $h(0) = 0$.

The strict passivity of the system (1),(2) is defined as follows [13]:

Definition 1: (Strict Passivity) The system (1),(2) is said to be strictly passive if there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$ and a positive definite function $S(x(k)) : R^n \to R$ such that

$$V(\boldsymbol{x}(k+1)) - V(\boldsymbol{x}(k)) \leq y(k)u(k) - S(\boldsymbol{x}(k)) \tag{3}$$

for all $u(k) \in R$, $\forall k \geq 0$.

The property of a the strict passive has been investigated in [11], and the strict passivity by means of the discrete-time nonlinear version of the KYP-Lemma has been derived as follows:

Theorem 1: The system (1),(2) is strictly passive if and only if, there exists a non-negative function $V(x(k)) : R^n \to R$ with $V(0) = 0$ such that

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A1-1) There exist functions $l(\boldsymbol{x})$, $W(\boldsymbol{x})$ and a positive definite function $S(x)$ such that

$$V(\boldsymbol{f}(\boldsymbol{x})) - V(\boldsymbol{x}) = -l(\boldsymbol{x})^2 - S(\boldsymbol{x}) \quad (4)$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x) = h(\boldsymbol{x}) - 2l(\boldsymbol{x})W(\boldsymbol{x}) \quad (5)$$

$$g^T(\boldsymbol{x}) \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(\boldsymbol{x}) = 2J(\boldsymbol{x}) - 2W(\boldsymbol{x})^2. \quad (6)$$

A1-2) $V(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u)$ is quadratic in u.

Remark 1: This theorem can also be found in [14] concerning the general QS(quadratic storage)-Passivity.

Further we define an output feedback strict passivity and strong output feedback strict passivity for the system (1),(2) as follows:

Definition 2: (Output feedback strictly passive: OFSP) The system (1),(2) is said to be output feedback strictly passive (OFSP) if there exists an output feedback:

$$u(k) = \alpha(y(k)) + \beta(y(k))v(k) \quad (7)$$

such that the resulting closed loop system is strictly passive.

Definition 3: (Strongly OFSP) The system (1),(2) is said to be strongly OFSP if there exists a static output feedback:

$$u(k) = -\theta^* y(k) + v(k), \theta^* > 0 \quad (8)$$

such that the resulting closed loop system from $y(k)$ to $v(k)$,

$$x(k+1) = \bar{f}(x(k)) + \bar{g}(x(k))v(k) \quad (9)$$

$$y(k) = \bar{h}(x(k)) + \bar{J}(x(k))v(k) \quad (10)$$

with

$$\bar{f}(x(k)) = f(x(k)) - \frac{\theta^*}{1+\theta^* J(x(k))} h(x(k))g(x(k)) \quad (11)$$

$$\bar{g}(x(k)) = \frac{1}{1+\theta^* J(x(k))} g(x(k)) \quad (12)$$

$$\bar{h}(x(k)) = \frac{1}{1+\theta^* J(x(k))} h(x(k)) \quad (13)$$

$$\bar{J}(x(k)) = \frac{1}{1+\theta^* J(x(k))} J(x(k)) \quad (14)$$

is strictly passive and, in addition, a transformed closed loop system with

$$\bar{v}(k) = \frac{1}{1+\theta^* J(x(k))} v(k) \quad (15)$$

as input,

$$x(k+1) = \bar{f}(x(k)) + g(x(k))\bar{v}(k) \quad (16)$$

$$y(k) = \bar{h}(x(k)) + J(x(k))\bar{v}(k) \quad (17)$$

is also strictly passive.

Sufficient conditions for the system (1),(2) to be OFSP has been provided in the following theorem [11].

Theorem 2: The system (1),(2) is OFSP with a static output feedback (8) and a C^2 positive definite storage function if

A2-1) The system has relative degree of 0 and $J(x(k)) > 0$, $\forall x(k)$.

A2-2) The zero dynamics of the system:

$$x(k+1) = f^*(x(k)) \quad (18)$$

is stable with the following C^2 positive definite function V satisfying

a) $V(f^*(x)) - V(x) = -\zeta(x)$ (19)
with a positive definite function $\zeta(x)$.

b) $V(f^*(x) + g(x)u)$ is quadratic in u.

c) There exist positive definite matrices Γ_m, Γ_M such that

$$0 < \Lambda_m \le \left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=\bar{f}(x(k))} \le \Lambda_M$$

A2-3) $\frac{g(x(k))}{J(x(k))}$ is bounded.

Moreover, we have the following lemma concerning the strongly OFSP conditions[11].

Lemma 1: Assumptions A2-1), A2-2) and A2-3) in Theorem 2 are satisfied with $J(x(k)) = d > 0$ then the system $(1), (2)$ is strongly OFSP.

III. Adaptive PID Control System

A. Problem statement

Consider the following system with $J(x) = 0$, but with disturbances in (1), (2):

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k)) + \boldsymbol{g}(\boldsymbol{x}(k))u(k) + \boldsymbol{D}_1(\boldsymbol{x}(k), k)w(k) \quad (20)$$

$$y(k) = h(\boldsymbol{x}(k)) + D_2(\boldsymbol{x}(k), k)w(k). \quad (21)$$

Where, $\boldsymbol{D}_i(\boldsymbol{x}(k), k)w(k)$ $(i = 1, 2)$ denote unmodelled dynamics and/or disturbances in which $\boldsymbol{D}_i(\boldsymbol{x}(k), k) : R^n \to R^n$ is smooth in $\boldsymbol{x}(k)$, and $w(k)$ is an external signal.

We impose the following assumptions on the system $(20), (21)$.

Assumption 1: (1) $\boldsymbol{g}(\boldsymbol{x}(k))$, $\boldsymbol{D}_1(\boldsymbol{x}(k), k)$, $D_2(\boldsymbol{x}(k), k)$ and $w(k)$ are bounded.

(2) There exists a parallel feedforward compensator (PFC):

$$\boldsymbol{x}_f(k+1) = \boldsymbol{A}_f \boldsymbol{x}_f(k) + \boldsymbol{b}_f u(k) \quad (22)$$

$$y_f(k) = \boldsymbol{c}_f^T \boldsymbol{x}_f(k) + d_f u(k) \quad (23)$$

such that the resulting augmented system with the PFC (22), (23) in parallel:

$$\boldsymbol{x}_a(k+1) = \boldsymbol{f}_a(\boldsymbol{x}_a(k)) + \boldsymbol{g}_a(\boldsymbol{x}_a(k))u(k) + \tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a(k), k)w(k) \quad (24)$$

$$y_a(k) = \tilde{y}(k) + d_f u(k) \quad (25)$$

is Strongly OFSP with $w(k) \equiv 0$. Where,

$$\boldsymbol{x}_a(k) = \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}_f(k) \end{bmatrix} \tag{26}$$

$$\boldsymbol{f}_a(\boldsymbol{x}_a(k)) = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}(k)) \\ \boldsymbol{A}_f\boldsymbol{x}_f(k) \end{bmatrix} \tag{27}$$

$$\boldsymbol{g}_a(\boldsymbol{x}_a(k)) = \begin{bmatrix} \boldsymbol{g}(\boldsymbol{x}(k)) \\ \boldsymbol{b}_f \end{bmatrix} \tag{28}$$

$$\tilde{y}(k) = h_a(\boldsymbol{x}_a(k)) + \tilde{D}_2(\boldsymbol{x}_a(k), k)w(k) \tag{29}$$

$$h_a(\boldsymbol{x}_a(k)) = h(\boldsymbol{x}(k)) + \boldsymbol{c}_f^T\boldsymbol{x}_f(k) \tag{30}$$

$$\tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a(k), k) = \begin{bmatrix} \boldsymbol{D}_1(\boldsymbol{x}(k), k) \\ \boldsymbol{0} \end{bmatrix} \tag{31}$$

$$\tilde{D}_2(\boldsymbol{x}_a(k), k) = D_2(\boldsymbol{x}(k), k). \tag{32}$$

(3) There exists a positive constant ζ_0 such that $h(\boldsymbol{x}(k))$ in (21) can be evaluated as

$$|h(\boldsymbol{x}(k))| \le \zeta_0 \,||\boldsymbol{x}(k)||\,. \tag{33}$$

That is, for the augmented system, there exists a positive constant ζ_1 such that $h_a(\boldsymbol{x}_a(k))$ can be evaluated by

$$|h_a(\boldsymbol{x}_a(k))| \le \zeta_1 \,||\boldsymbol{x}_a(k)||\,. \tag{34}$$

The objective here is to design a stable adaptive PID control system for uncertain nonlinear systems under Assumption 1.

B. Adaptive PID controller

Under Assumption 1, for a strong OFSP augmented system (24), (25), let's consider an ideal PID control input given as follows:

$$u^*(k) = -\theta_p^* y_a(k) - \theta_i^* y_{ai}(k) - \theta_d^* y_{ad}(k), \tag{35}$$

with $\theta_p^* > 0, \theta_i^* > 0, \theta_d^* > 0$. Where

$$\begin{aligned} y_{ai}(k) &= y_{ai}(k-1) + Ty_a(k) - \sigma_i y_{ai}(k) \\ &= \bar{\sigma}_i y_{ai}(k-1) + \bar{\sigma}_i T y_a(k) \end{aligned} \tag{36}$$

$$\bar{\sigma}_i = \frac{1}{1+\sigma_i} \;\;, \;\; \sigma_i > 0 \;\; (1 > \bar{\sigma}_i > 0) \tag{37}$$

$$y_{ad}(k) = \frac{1}{T}\left\{y_a(k) - y_a(k-1)\right\}, \tag{38}$$

and T is a sampling time.

We adopt a pseudo-integral signal $y_{ai}(k)$ in the controller. In this case, the resulting control system with the input (35) will be stabilized by setting a sufficiently large θ_p^*, and this can be easily shown using the strongly OFSP properties of the augmented system. Unfortunately, since the controlled system is unknown, one can not design ideal PID gains, and since the output of the augmented system $y_a(k)$ consists of the control input $u(k)$, a causality problem will appear.

To solve these problems, we consider an equivalent control input and adaptively adjusting the obtained equivalent PID gains.

Now consider an ideal control input (35) with an ideal gain θ_p^* which renders the closed loop system strictly passive. The ideal control input (35) is expressed as follows:

$$\begin{aligned} u^*(k) =& -\theta_p^* y_a(k) - \theta_i^* y_{ai}(k) - \theta_d^* y_{ad}(k) \\ =& -\theta_p^*\{\tilde{y}(k) + d_f u^*(k)\} \\ & -\theta_i^*\left\{\bar{\sigma}_i y_{ai}(k-1) + \bar{\sigma}_i T y_a(k)\right\} \\ & -\theta_d^*\left\{\frac{1}{T}\left\{y_a(k) - y_a(k-1)\right\}\right\}. \end{aligned} \tag{39}$$

From (39), we have the following equivalent input $u_1^*(k)$:

$$\begin{aligned} u^*(k) = u_1^*(k) =& -\tilde{\theta}_{p1}^*\tilde{y}(k) - \tilde{\theta}_{i1}^*\bar{\sigma}_i y_{ai}(k-1) \\ & + \tilde{\theta}_{d1}^*\frac{1}{T}y_a(k-1) + \mu^*(k) \\ =& -\tilde{\boldsymbol{\theta}}_1^{*T}\tilde{\boldsymbol{z}}(k) + \mu^*(k), \end{aligned} \tag{40}$$

where

$$\tilde{\boldsymbol{\theta}}_1^* = \begin{bmatrix} \tilde{\theta}_{p1}^* \\ \tilde{\theta}_{i1}^* \\ \tilde{\theta}_{d1}^* \end{bmatrix}, \tilde{\boldsymbol{z}}(k) = \begin{bmatrix} \tilde{y}(k) \\ \bar{\sigma}_i y_{ai}(k-1) \\ -\frac{1}{T}y_a(k-1) \end{bmatrix}$$

$$\mu^*(k) \;\;=\;\; -(\bar{\sigma}_i T\tilde{\theta}_{i1}^* + \frac{1}{T}\tilde{\theta}_{d1}^*)y_a(k), \tag{41}$$

with

$$\tilde{\theta}_{p1}^* = (1 + d_f\theta_p^*)^{-1}\theta_p^*, \tilde{\theta}_{i1}^* = (1 + d_f\theta_p^*)^{-1}\theta_i^*,$$
$$\tilde{\theta}_{d1}^* = (1 + d_f\theta_p^*)^{-1}\theta_d^*.$$

Further expanding (39), we have

$$\begin{aligned} u^*(k) =& -\theta_{pid}^*\left\{\tilde{y}(k) + d_f u^*(k)\right\} \\ & -\theta_i^*\bar{\sigma}_i y_{ai}(k-1) + \theta_d^*\frac{1}{T}y_a(k-1), \end{aligned} \tag{42}$$

with

$$\theta_{pid}^* = \theta_p^* + \bar{\sigma}_i T\theta_i^* + \frac{1}{T}\theta_d^*, \tag{43}$$

and then the following equivalent input $u_2^*(k)$ can also be obtained.

$$\begin{aligned} u^*(k) = (t)u_2^*(k) \;=&\; -\tilde{\theta}_{p2}^*\tilde{y}(k) - \tilde{\theta}_{i2}^*\bar{\sigma}_i y_{ai}(k-1) \\ & +\tilde{\theta}_{d2}^*\frac{1}{T}y_a(k-1) \\ =&\; -\tilde{\boldsymbol{\theta}}_2^{*T}\tilde{\boldsymbol{z}}(k), \end{aligned} \tag{44}$$

where

$$\tilde{\boldsymbol{\theta}}_2^* = \begin{bmatrix} \tilde{\theta}_{p2}^* \\ \tilde{\theta}_{i2}^* \\ \tilde{\theta}_{d2}^* \end{bmatrix}, \tilde{\boldsymbol{z}}(k) = \begin{bmatrix} \tilde{y}(k) \\ \bar{\sigma}_i y_{ai}(k-1) \\ -\frac{1}{T}y_a(k-1) \end{bmatrix} \tag{45}$$

with

$$\tilde{\theta}_{p2}^* = (1 + d_f\theta_{pid}^*)^{-1}\theta_{pid}^*, \tilde{\theta}_{i2}^* = (1 + d_f\theta_{pid}^*)^{-1}\theta_i^*,$$
$$\tilde{\theta}_{d2}^* = (1 + d_f\theta_{pid}^*)^{-1}\theta_d^*$$

It follows that $u^*(k) \equiv u_1^*(k) \equiv u_2^*(k)$.

The actual control input is designed by adjusting the equivalent input gains $\tilde{\theta}_{p2}^*$, $\tilde{\theta}_{i2}^*$ and $\tilde{\theta}_{d2}^*$ of $u_2^*(k)$ as follows:

$$u(k) = -\tilde{\theta}_p(k)\tilde{y}(k) - \tilde{\theta}_i(k)\bar{\sigma}_i y_{ai}(k-1) + \tilde{\theta}_d(k)\frac{1}{T}y_a(k-1) = -\tilde{\boldsymbol{\theta}}^T(k)\tilde{\boldsymbol{z}}(k), \tag{46}$$

where

$$\tilde{\boldsymbol{\theta}}(k) = \left[\tilde{\theta}_p(k),\ \tilde{\theta}_i(k),\ \tilde{\theta}_d(k)\right]^T. \tag{47}$$

The parameter adjusting law is given by

$$\tilde{\boldsymbol{\theta}}(k) = \tilde{\boldsymbol{\theta}}(k-1) + \boldsymbol{\Gamma}\tilde{\boldsymbol{z}}(k)y_a(k) - \sigma\tilde{\boldsymbol{\theta}}(k) = \bar{\sigma}\tilde{\boldsymbol{\theta}}(k-1) + \bar{\sigma}\boldsymbol{\Gamma}\tilde{\boldsymbol{z}}(k)y_a(k) \tag{48}$$

$$\bar{\sigma} = \frac{1}{1+\sigma}, \quad \sigma > 0 \quad (1 > \bar{\sigma} > 0) \tag{49}$$

$$\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^T > 0. \tag{50}$$

In this case, the augmented output $y_a(k)$ can be obtained from (25), (46) and (48) as

$$y_a(k) = \frac{\tilde{y}(k) - \bar{\sigma}d_f\tilde{\boldsymbol{\theta}}(k-1)^T\tilde{\boldsymbol{z}}(k)}{1 + \bar{\sigma}d_f\tilde{\boldsymbol{z}}^T(k)\boldsymbol{\Gamma}\tilde{\boldsymbol{z}}(k)} \tag{51}$$

by using available signals. This means that the proposed adaptive PID controller can be designed without causality problems.

IV. Stability Analysis

Since $u(k) = u(k) - u_2^*(k) + u_1^*(k)$, the obtained closed loop system with the input (46) can be expressed as

$$\begin{aligned}\boldsymbol{x}_a(k+1) &= \boldsymbol{f}_a(\boldsymbol{x}_a(k)) + \boldsymbol{g}_a(\boldsymbol{x}_a(k))u(k) + \tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k)w(k)\\ &= \boldsymbol{f}_a(\boldsymbol{x}_a(k)) + \boldsymbol{g}_a(\boldsymbol{x}_a(k))\{u(k) - u_2^*(k) + u_1^*(k)\} + \tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k)w(k)\\ &= \bar{\boldsymbol{f}}_a(x_a(k)) + \boldsymbol{g}_a(x_a(k))\Delta\bar{u}(k) + \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k)w(k)\end{aligned} \tag{52}$$

$$y_a(k) = \bar{y}(k) + d_f\Delta\bar{u}(k) + \bar{D}_2(\boldsymbol{x}_a(k),k)w(k), \tag{53}$$

where

$$\Delta\bar{u}(k) = \Delta u(k) + \tilde{u}(k) + \mu^*(k) \tag{54}$$

$$\Delta u(k) = u(k) - u_2^*(k) - (\tilde{\boldsymbol{\theta}}(k)^T - \tilde{\boldsymbol{\theta}}_2^{*T})\boldsymbol{z}(\tilde{k}) = -\Delta\tilde{\boldsymbol{\theta}}(k)^T\boldsymbol{z}(\tilde{k}) \tag{55}$$

$$\tilde{u}(k) = -\tilde{\theta}_{i1}^*\bar{\sigma}_i y_{ai}(k-1) + \tilde{\theta}_{d1}^*\frac{1}{T}y_a(k-1) \tag{56}$$

and

$$\bar{\boldsymbol{f}}_a(x_a(k)) = \boldsymbol{f}_a(x_a(k)) - \frac{\theta_p^*}{1+d_f\theta_p^*}\boldsymbol{g}_a(x_a(k))h_a(\boldsymbol{x}_a(k)) \tag{57}$$

$$\bar{y}(k) = \frac{1}{1+d_f\theta_p^*}h_a(\boldsymbol{x}_a(k)) \tag{58}$$

$$\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k) = \tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k) - \frac{\theta_p^*}{1+d_f\theta_p^*}\boldsymbol{g}_a(x_a(k))\tilde{D}_2(\boldsymbol{x}_a(k),k) \tag{59}$$

$$\bar{D}_2(\boldsymbol{x}_a(k),k) = \frac{1}{1+d_f\theta_p^*}\tilde{D}_2(\boldsymbol{x}_a(k),k). \tag{60}$$

Under Assumption 1(2), the resulting closed loop system with $w(k) = 0$ is strictly passive with a C^2 positive definite function. Therefore, there exists a C^2 positive definite function $V_1(\boldsymbol{x}_a(k))$ such that

C1)

$$V_1(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)) - V_1(\boldsymbol{x}_a) = -l_1(\boldsymbol{x}_a)^2 - S_1(\boldsymbol{x}_a) \tag{61}$$

$$\left.\frac{\partial V_1(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)}\boldsymbol{g}_a(\boldsymbol{x}_a) = \bar{y}(k) - 2l_1(\boldsymbol{x}_a)W_1(\boldsymbol{x}_a)$$

$$\boldsymbol{g}_a^T(\boldsymbol{x}_a)\left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)}\boldsymbol{g}_a(\boldsymbol{x}_a) = 2d_f - 2W_1(\boldsymbol{x}_a)^2$$

with functions $l_1(\boldsymbol{x})$, $W_1(\boldsymbol{x})$ and a positive definite function $S_(x)$

C2) $V_1(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a) + \boldsymbol{g}_a(\boldsymbol{x}_a)u(k))$ is quadratic in $u(k)$.

Further we suppose the following assumptions are satisfied.

Assumption 2:

(1) $V_1(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a(k)) + \boldsymbol{g}_a(\boldsymbol{x}_a(k))u(k) + \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k)w(k))$ is quadratic in $u(k)$.

(2) $V_1(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a(k)) + \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a(k),k)w(k))$ is quadratic in $w(k)$.

Assumption 3: There exist constants ζ_2 and ζ_3 such that $V_1(\boldsymbol{x}_a(k))$ and $S_1(\boldsymbol{x}_a(k))$ can be evaluated as

$$V_1(\boldsymbol{x}_a(k)) \le \zeta_2\,||\boldsymbol{x}_a(k)||^2 \tag{62}$$

$$S_1(\boldsymbol{x}_a(k)) \le \zeta_3\,||\boldsymbol{x}_a(k)||^2 \tag{63}$$

Then we have the following theorem concerning the stability of the obtained control system.

Theorem 3: Under Assumptions 1, 2, 3, all the signals in the resulting closed loop system are bounded. Further, in the case where $w(k) \equiv 0$, we have $\lim_{k\to\infty} y(k) = 0$ by setting $\sigma = 0$, $\sigma_i = 0$

Proof 1: Consider a positive definite function $V_1(\boldsymbol{x}_a(k))$ which satisfies conditions C1), C2) and Assumptions 2 and 3. $V_1(\boldsymbol{x}_a(k))$ can be expanded as

$$\begin{aligned}V_1(\boldsymbol{x}_a(k+1)) &= V_1(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a) + \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k) + \boldsymbol{g}_a(\boldsymbol{x}_a)\Delta\bar{u}(k))\\ &= A_u(\boldsymbol{x}_a) + B_u(\boldsymbol{x}_a)\Delta\bar{u}(k) + C_u(\boldsymbol{x}_a)\Delta\bar{u}(k)^2.\end{aligned} \tag{64}$$

Applying the Taylor expansion formula at $u(k) = 0$ and $w(k) = 0$, we have from the condition C1) that

$$\begin{aligned}A_u(\boldsymbol{x}_a) &= V_1(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a) + \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k))\\ &= V_1(\boldsymbol{x}_a) - l_1(\boldsymbol{x}_a)^2 - S_1(\boldsymbol{x}_a) + \left.\frac{\partial V_1(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)}\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k)\\ &\quad + \frac{1}{2}\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k)\left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)}\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w^2(k)\end{aligned} \tag{65}$$

$$B_u(\boldsymbol{x}_a) = \left.\frac{\partial V_1(\boldsymbol{\beta})}{\partial\boldsymbol{\beta}}\right|_{\boldsymbol{\beta}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)+\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k)}\boldsymbol{g}_a(\boldsymbol{x}_a)$$

$$
\begin{aligned}
&= \left.\frac{\partial V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \boldsymbol{g}_a(\boldsymbol{x}_a) \\
&\quad +\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \boldsymbol{g}_a(\boldsymbol{x}_a) w(k) \\
&= \bar{y}(k) - 2l_1(\boldsymbol{x}_a) W_1(\boldsymbol{x}_a) \\
&\quad +\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \boldsymbol{g}_a(\boldsymbol{x}_a) w(k) \quad (66)
\end{aligned}
$$

$$
\begin{aligned}
&C_u(\boldsymbol{x}_a) \\
&= \frac{1}{2}\boldsymbol{g}_a^T(\boldsymbol{x}_a) \left.\frac{\partial^2 V_1(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^2}\right|_{\boldsymbol{\beta}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)+\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k)} \boldsymbol{g}_a(\boldsymbol{x}_a) \\
&= \frac{1}{2}\boldsymbol{g}_a^T(\boldsymbol{x}_a) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \boldsymbol{g}_a(\boldsymbol{x}_a) \\
&= d_f - W_1(\boldsymbol{x}_a)^2 \quad (67)
\end{aligned}
$$

Therefore $V_1(\boldsymbol{x}_a(k+1))$ can be expressed by

$$
\begin{aligned}
&V_1(\boldsymbol{x}_a(k+1)) \\
&= V_1(\boldsymbol{x}_a) - S_1(\boldsymbol{x}_a) - (l_1(\boldsymbol{x}_a) + W_1(\boldsymbol{x}_a)\Delta\bar{u}(k))^2 \\
&\quad + \{\bar{y}(k) + d_f \Delta\bar{u}(k)\}\Delta\bar{u}(k) \\
&\quad + \left.\frac{\partial V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k) w(k) \\
&\quad +\frac{1}{2}\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k) w^2(k) \\
&\quad +\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \boldsymbol{g}_a(\boldsymbol{x}_a) w(k)\Delta\bar{u}(k). \quad (68)
\end{aligned}
$$

Further along with this expansion, considering a positive constant δ such as $0 < \delta < 1$ we have

$$
\begin{aligned}
&\frac{\delta}{1-\delta} V_1\left(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a) + \boldsymbol{g}_a(\boldsymbol{x}_a)\Delta\bar{u}(k) + \bar{\delta}\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k)\right) \\
&= \frac{\delta}{1-\delta}\Big\{ V_1(\boldsymbol{x}_a) - (l_1(\boldsymbol{x}_a) - W_1(\boldsymbol{x}_a)\Delta\bar{u}(k))^2 - S_1(\boldsymbol{x}_a) \\
&\qquad + \{\bar{y}(k) + d_f\Delta\bar{u}(k)\}\Delta\bar{u}(k) \Big\} \\
&\quad - \left.\frac{\partial V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k) w(k) \\
&\quad -\frac{1}{2}\bar{\delta}\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k) w^2(k) \\
&\quad -\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \boldsymbol{g}_a(\boldsymbol{x}_a) w(k)\Delta\bar{u}(k), \quad (69)
\end{aligned}
$$

where $\bar{\delta} = 1 - \frac{1}{\delta}$.

From (68) and (69), the difference of $V_1(\boldsymbol{x}_a(k))$ can be represented by

$$
\begin{aligned}
&V_1(\boldsymbol{x}_a(k+1)) - V_1(\boldsymbol{x}_a(k)) \\
&= -\rho\{S_1(\boldsymbol{x}_a) - \delta V_1(\boldsymbol{x}_a)\} - \rho(l_1(\boldsymbol{x}_a) - W_1(\boldsymbol{x}_a)\Delta\bar{u}(k))^2 \\
&\quad +\rho y_a(k)\Delta\bar{u}(k) - \rho\bar{D}_2(\boldsymbol{x}_a,k)w(k)\Delta\bar{u}(k) \\
&\quad +\frac{1}{2\delta}\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k) \left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)} \bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k) w^2(k) \\
&\quad -\rho\delta V_1\left(\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a) + \boldsymbol{g}_a(\boldsymbol{x}_a)\Delta\bar{u}(k) + \bar{\delta}\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w(k)\right) \quad (70)
\end{aligned}
$$

with $\rho = \frac{1}{1-\delta}$.

Here we consider the following positive definite function $V(\boldsymbol{x}_a)$:

$$
\begin{aligned}
V(\boldsymbol{x}_a) &= V_1(\boldsymbol{x}_a) + \rho V_2(\boldsymbol{x}_a) + \rho V_3(\boldsymbol{x}_a) \quad (71) \\
V_2(\boldsymbol{x}_a) &= \frac{\bar{\sigma}}{2}\Delta\tilde{\boldsymbol{\theta}}^T(k-1)\boldsymbol{\Gamma}^{-1}\Delta\tilde{\boldsymbol{\theta}}(k-1) \quad (72) \\
V_3(\boldsymbol{x}_a) &= \frac{\bar{\sigma}_i}{2T}\tilde{\theta}_{i1}^* y_{ai}^2(k-1) + \frac{1}{2T}\tilde{\theta}_{d1}^* y_a^2(k-1) \quad (73)
\end{aligned}
$$

and denote the difference as $\Delta V(\boldsymbol{x}_a) = V(\boldsymbol{x}_a(k+1)) - V(\boldsymbol{x}_a)$. The difference $\Delta V_2(\boldsymbol{x}_a)$ can be expressed from the parameter adjusting law (48) by

$$
\begin{aligned}
&\Delta V_2(\boldsymbol{x}_a) \\
&= \frac{\bar{\sigma}}{2}\Big\{\Delta\tilde{\boldsymbol{\theta}}^T(k)\boldsymbol{\Gamma}^{-1}\Delta\tilde{\boldsymbol{\theta}}(k) - \Delta\tilde{\boldsymbol{\theta}}^T(k-1)\boldsymbol{\Gamma}^{-1}\Delta\tilde{\boldsymbol{\theta}}(k-1)\Big\} \\
&= -\frac{1}{2}\left(\frac{1}{\bar{\sigma}} - \bar{\sigma}\right)\Delta\tilde{\boldsymbol{\theta}}^T(k)\boldsymbol{\Gamma}^{-1}\Delta\tilde{\boldsymbol{\theta}}(k) \\
&\quad -\Delta u(k) y_a(k) - \sigma\Delta\tilde{\boldsymbol{\theta}}^T(k)\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_2^* \\
&\quad -\frac{\bar{\sigma}}{2}\left\{\sigma\tilde{\boldsymbol{\theta}}_2^* - \boldsymbol{\Gamma}\tilde{\boldsymbol{z}}(k)y_a(k)\right\}^T\boldsymbol{\Gamma}^{-1}\left\{\sigma\tilde{\boldsymbol{\theta}}_2^* - \boldsymbol{\Gamma}\tilde{\boldsymbol{z}}(k)y_a(k)\right\}, \quad (74)
\end{aligned}
$$

and the difference $\Delta V_3(\boldsymbol{x}_a)$ can be expressed from (36) to (38) that

$$
\begin{aligned}
\Delta V_3(\boldsymbol{x}_a) &= \frac{\bar{\sigma}_i}{2T}\left\{\tilde{\theta}_{i1}^*\left\{y_{ai}^2(k) - y_{ai}^2(k-1)\right\}\right\} \\
&\quad +\frac{1}{2T}\left\{\tilde{\theta}_{d1}^*\left\{y_a^2(k) - y_a^2(k-1)\right\}\right\} \\
&= -\frac{\tilde{\theta}_{i1}^*}{2T}\left(\frac{1}{\bar{\sigma}_i} - \bar{\sigma}_i\right)y_{ai}^2(k) - \{\tilde{u}(k) + \mu^*(k)\}y_a(k) \\
&\quad -\frac{T}{2}\left\{\bar{\sigma}_i\tilde{\theta}_{i1}^* y_a^2(k) + \tilde{\theta}_{d1}^* y_{ad}^2(k)\right\} \quad (75)
\end{aligned}
$$

Consequently the difference $\Delta V(\boldsymbol{x}_a)$ can be evaluated as

$$\begin{aligned}
&\Delta V(\boldsymbol{x}_a)\\
&\le -\rho\left\{S_1(\boldsymbol{x}_a)-\delta V_1(\boldsymbol{x}_a)\right\}\\
&\quad -\rho\left\{\frac{1}{2}\left(\frac{1}{\bar{\sigma}}-\bar{\sigma}\right)-\delta_1-\delta_2-\delta_3\right\}\Delta\tilde{\boldsymbol{\theta}}^T(k)\boldsymbol{\Gamma}^{-1}\Delta\tilde{\boldsymbol{\theta}}(k)\\
&\quad -\rho\left\{\frac{\tilde{\theta}^*_{i1}}{2T}\left(\frac{1}{\bar{\sigma}_i}-\bar{\sigma}_i\right)-\delta_4-\delta_8\right\}y_{ai}^2(k)\\
&\quad -\rho\left\{\frac{T}{2}\bar{\sigma}_i\tilde{\theta}^*_{i1}-\delta_6-\delta_7\right\}y_a^2(k)\\
&\quad -\rho\left\{\frac{T}{2}\tilde{\theta}^*_{d1}-\delta_5-\delta_9\right\}y_{ad}^2(k)\\
&\quad +\frac{1}{2\delta}\bar{\boldsymbol{D}}_1^T(\boldsymbol{x}_a,k)\left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)}\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)w^2(k)\\
&\quad +\frac{\rho\sigma^2}{4\delta_1}\tilde{\boldsymbol{\theta}}_2^{*T}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_2^*+\frac{\rho}{4\delta_2\lambda_{\Gamma m}}\bar{D}_2(\boldsymbol{x}_a,k)^2w^2(k)h_a(\boldsymbol{x}_a)^2\\
&\quad +\frac{\rho}{4\delta_3\lambda_{\Gamma m}}\bar{D}_2(\boldsymbol{x}_a,k)^2w^4(k)\tilde{D}_2(\boldsymbol{x}_a,k)^2\\
&\quad +\frac{\rho}{4}\bar{D}_2(\boldsymbol{x}_a,k)^2w^2(k)\\
&\qquad \times\left(\frac{1}{\delta_4}+\frac{1}{\delta_5}+\frac{\bar{\sigma}_i^2T^2}{\delta_6}+\frac{1}{T^2\delta_7}\right)\left|\left|\ \Delta\tilde{\boldsymbol{\theta}}(k)\right|\right|^2\\
&\quad +\frac{\rho}{4}\bar{D}_2(\boldsymbol{x}_a,k)^2w^2(k)\left(\frac{\tilde{\theta}^{*2}_{i1}}{\delta_8}+\frac{\tilde{\theta}^{*2}_{d1}}{\delta_9}\right)
\end{aligned}\tag{76}$$

with any positive constants δ_1 to δ_9. Where, $\lambda_{\Gamma m}=\lambda_{min}[\Gamma]$ denotes the minimum value of eigenvalues of Γ.

Finally, taking into consideration the fact that there exist positive constants λ_M, g_M, D_{1M} and D_{2M} such as

$$\left|\left|\left.\frac{\partial^2 V_1(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}^2}\right|_{\boldsymbol{\alpha}=\bar{\boldsymbol{f}}_a(\boldsymbol{x}_a)}\right|\right|\le||\boldsymbol{\Lambda}_M||\le\lambda_M \tag{77}$$

$$||\boldsymbol{g}_a(\boldsymbol{x}_a)||\le g_M \tag{78}$$

$$\left|\left|\tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)\right|\right|\le D_{1M} \tag{79}$$

$$\left|\tilde{D}_2(\boldsymbol{x}_a,k)\right|\le D_{2M} \tag{80}$$

$$|w(k)|\le w_M \tag{81}$$

and the fact that

$$\begin{aligned}
&\left|\left|\bar{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)\right|\right|\\
&\le\ \left|\left|\tilde{\boldsymbol{D}}_1(\boldsymbol{x}_a,k)\right|\right|-\frac{\theta_p^*}{1+d_f\theta_p^*}||\boldsymbol{g}_a(x_a(k))||\left|\tilde{D}_2(\boldsymbol{x}_a,k)\right|\\
&\le\ D_{1M}-\frac{\theta_p^*}{1+d_f\theta_p^*}g_MD_{2M}
\end{aligned}\tag{82}$$

$$\left|\bar{D}_2(\boldsymbol{x}_a,k)\right|\le\frac{1}{1+d_f\theta_p^*}\left|\tilde{D}_2(\boldsymbol{x}_a,k)\right|\le\frac{1}{1+d_f\theta_p^*}D_{2M},\tag{83}$$

we have

$$\begin{aligned}
&\Delta V(\boldsymbol{x}_a)\\
&\le\ -\rho\left\{\zeta_3-\delta\zeta_2-\frac{\zeta_1^2}{4\delta_2}\left(\frac{1}{1+d_f\theta_p^*}\right)^2\right.\\
&\qquad\qquad\left.\times D_{2M}^2w_M^2\,||\boldsymbol{\Gamma}||\right\}||\boldsymbol{x}_a(k)||^2\\
&\quad -\rho\left[\left\{\frac{1}{2}\left(\frac{1}{\bar{\sigma}}-\bar{\sigma}\right)-\delta_1-\delta_2-\delta_3\right\}\lambda_{\Gamma m}-\frac{1}{4}\left(\frac{1}{1+d_f\theta_p^*}\right)^2\right.\\
&\quad\left.\times\left(\frac{1}{\delta_4}+\frac{1}{\delta_5}+\frac{\bar{\sigma}_i^2T^2}{\delta_6}+\frac{1}{T^2\delta_7}\right)D_{2M}^2w_M^2\right]\left|\left|\ \Delta\tilde{\boldsymbol{\theta}}(k)\right|\right|^2\\
&\quad -\rho\left\{\frac{\tilde{\theta}^*_{i1}}{2T}\left(\frac{1}{\bar{\sigma}_i}-\bar{\sigma}_i\right)-\delta_4-\delta_8\right\}y_{ai}^2(k)\\
&\quad -\rho\left\{\frac{T}{2}\bar{\sigma}_i\tilde{\theta}^*_{i1}-\delta_6-\delta_7\right\}y_a^2(k)\\
&\quad -\rho\left\{\frac{T}{2}\tilde{\theta}^*_{d1}-\delta_5-\delta_9\right\}y_{ad}^2(k)+\bar{R}
\end{aligned}\tag{84}$$

with

$$\begin{aligned}
\bar{R}=&\ \frac{1}{2\delta}\lambda_M\left(D_{1M}-\frac{\theta_p^*}{1+d_f\theta_p^*}g_MD_{2M}\right)^2w_M^2\\
&+\frac{\rho\sigma^2}{4\delta_1}\lambda_{\Gamma M}\left|\left|\tilde{\boldsymbol{\theta}}_2^*\right|\right|^2+\frac{\rho}{4\delta_3\lambda_{\Gamma m}}\left(\frac{1}{1+d_f\theta_p^*}\right)^2||\boldsymbol{\Gamma}||\,D_{2M}^4w_M^4\\
&+\frac{\rho}{4}\left(\frac{1}{1+d_f\theta_p^*}\right)^2D_{2M}^2w_M^2\left(\frac{\tilde{\theta}^{*2}_{i1}}{\delta_8}+\frac{\tilde{\theta}^{*2}_{d1}}{\delta_9}\right).
\end{aligned}$$

Where, $\lambda_{\Gamma M}=\lambda_{max}[\Gamma]$ denotes the maximum value of eigenvalues of Γ.

Since there exist a sufficiently large ideal PID gains and sufficiently small constants δ and δ_1 to δ_9 such that

$$\begin{aligned}
&\left\{\zeta_3-\delta\zeta_2-\frac{\zeta_1^2}{4\delta_2}\left(\frac{1}{1+d_f\theta_p^*}\right)^2D_{2M}^2w_M^2\,||\boldsymbol{\Gamma}||\right\}>0\\
&\left\{\frac{1}{2}\left(\frac{1}{\bar{\sigma}}-\bar{\sigma}\right)-\delta_1-\delta_2-\delta_3\right\}\left|\left|\boldsymbol{\Gamma}^{-1}\right|\right|-\frac{1}{4}\left(\frac{1}{1+d_f\theta_p^*}\right)^2\\
&\qquad\times\left(\frac{1}{\delta_4}+\frac{1}{\delta_5}+\frac{\bar{\sigma}_i^2T^2}{\delta_6}+\frac{1}{T^2\delta_7}\right)D_{2M}^2w_M^2>0\\
&\frac{\tilde{\theta}^*_{i1}}{2T}\left(\frac{1}{\bar{\sigma}_i}-\bar{\sigma}_i\right)-\delta_4-\delta_8>0\\
&\frac{T}{2}\bar{\sigma}_i\tilde{\theta}^*_{i1}-\delta_6-\delta_7>0\\
&\frac{T}{2}\tilde{\theta}^*_{d1}-\delta_5-\delta_9>0,
\end{aligned}$$

we can conclude that all the signals in the control system are bounded. Further, in the case where $w(k)=0$ and $\sigma=0$,

$\sigma_i = 0$, we have

$$\begin{aligned}
&\Delta V(\boldsymbol{x}_a(k)) \\
&= -S_1(\boldsymbol{x}_a(k)) - (l_1(\boldsymbol{x}_a(k)) + W_1(\boldsymbol{x}_a(k))\Delta\bar{u}(k))^2 \\
&\quad -\frac{1}{2}\tilde{\boldsymbol{z}}^T(k)\boldsymbol{\Gamma}\tilde{\boldsymbol{z}}(k)y_a^2(k) \\
&\quad -\frac{T}{2}\left\{\tilde{\theta}_{i1}^{*}y_a^2(k) + \tilde{\theta}_{d1}^{*}y_{ad}^2(k)\right\} \\
&\leq -S_1(\boldsymbol{x}_a(k)) \leq 0. \qquad (85)
\end{aligned}$$

Thus we obtain $\lim_{k\to\infty} y(k) = 0$.

V. Illustrative Example

In order to confirm the effectiveness of the proposed method, we here show a simple example.

Let's consider a tracking control of the following nonlinear system with the sampling period of 0.1[sec].

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k)) + \boldsymbol{g}(\boldsymbol{x}(k))u(k) + \boldsymbol{D}_1 w(k) \quad (86)$$

$$y(k) = h(\boldsymbol{x}(k)) \quad (87)$$

where $\boldsymbol{x}(k) = [x_1(k)\ x_2(k)]^T$ and

$$\boldsymbol{f}(\boldsymbol{x}(k)) = \begin{bmatrix} \dfrac{x_1(k)\,(1+0.1sinx_1(k))}{5} + 0.6(x_1(k)+x_2(k)) \\ \dfrac{x_1(k)\,(1+0.1cosx_2(k))}{5} + 0.6x_2(k) \end{bmatrix} \quad (88)$$

$$\boldsymbol{g}(\boldsymbol{x}(k)) = \begin{bmatrix} 1+0.1cosx_2(k) \\ 1+0.1cosx_2(k) \end{bmatrix} \quad (89)$$

$$h(\boldsymbol{x}(k)) = x_1(k) \quad (90)$$

and the disturbance is given by

$$w(k) = \begin{cases} 0\ ,\ t < 10[\mathrm{sec}] \\ 1\ ,\ t \geq 10[\mathrm{sec}] \end{cases} \quad (91)$$

with $\boldsymbol{D}_1 = [1\ 1]^T$.

The reference signal $r(k)$ which the system's output is required to follow is given by an output of the following reference model with a transfer function:

$$G(z) = \frac{0.9516}{z - 0.9048} \quad (92)$$

This system is not OFSP, so we have to design a PFC. In order to deduce the affect from the PFC output to the real output, we first introduce a pre-filter (93) with a integral action:

$$\text{Filter:}\frac{\alpha z - \beta}{z - 1}\ ,\ \alpha = 1\ ,\ \beta = 0.8 \quad (93)$$

and then for a extended system with the pre-filter, the PFC designed as

$$y_f = J(\boldsymbol{x}(k))u(k) = d_f u(k)\ ,\ d_f = 0.5 \quad (94)$$

The controller is designed by replacing $\tilde{y}(k)$ to $\tilde{e}(k) = \tilde{y}(k) - r(t)$ in (46) to (51).

Fig. 1 shows the simulation result. A good control performance was obtained.

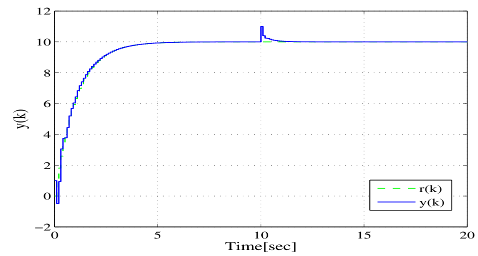

Fig. 1. Simulation result

VI. Conclusions

In this paper, we proposed a design scheme of an adaptive PID control system for discrete-time nonlinear SISO systems. The proposed method utilizes the OFSP properties of the controlled system, so that the stability of the resulting adaptive control system can be guaranteed with certainty. A robust adaptive PID design scheme for the OFSP augmented system with a parallel feedforward compensator without causality problems was developed by considering the equivalent PID controller.

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