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A Neural-Fuzzy Sliding Mode Observer for Robust Fault Diagnosis

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***Abstract*— A robust fault diagnosis (FD) scheme using Takagi-Sugeno (T-S) neural-fuzzy model and sliding mode technique is presented for a class of nonlinear systems that can be described by T-S fuzzy models. A neural-fuzzy observer and neural-fuzzy sliding mode observer are constructed respectively. A modified back-propagation (BP) algorithm is used to update the parameters of the two observers. Stability of the observers are analyzed as well. Finally, the proposed FD scheme using these observers is applied to a point mass satellite orbital control system example. Numerical simulation results show that this robust fault diagnosis strategy is effective for the considered class of nonlinear systems.**

I. INTRODUCTION

In the last three decades, mathematical model-based fault diagnosis (FD) schemes have received a great deal of investigations, e.g., see [1], [2], [3] and [4]. This is to some extent due to the increasing complexity of modern engineering systems and increased attention to safety, reliability, and economics factors. However, model-based fault detection, isolation, and estimation for nonlinear systems in presence of uncertainties is still a challenging task.

A number of researchers have recently explored the fault diagnosis for nonlinear systems using learning methodologies, where they use various online approximation techniques to estimate the deviation of system dynamics caused by faults. These online estimation techniques include adaptive observers [5], [6], neural networks [7], [8], [9], neural adaptive observers [10], [11] and iterative learning observers (ILO) [12], [13], etc. In spite of these advances, several issues still need further research. Among these are: i) The FD algorithm should be easily implementable to alleviate computational tasks. ii) The FD scheme should be able to specify the fault as precisely and quickly as possible to provide helpful information for a fault tolerant strategy.

Fuzzy logic/model-based state observation and fault diagnosis have been subject of several studies as well, e.g., [14], [15], [16], etc. One class of methods is to build a group of local linear models using Takagi-Sugeno (T-S) fuzzy models to describe the original nonlinear systems. As a result, the fault diagnosis schemes for linear systems are extended to nonlinear systems [14]. The second class of methods treats the fuzzy model in the same way as neural networks, since both of them possess the same approximation capability of nonlinear functions in a compact set. The third class of approaches apply fuzzy logic/reasoning to the fault evaluation and classification [17].

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Some learning methods use dead-zone operators to update parameters in robust fault diagnosis [8]. However, the drawback in doing so is that fault estimation accuracy may also be affected by the dead-zone operator. Additionally, a projection operator is needed to avoid parameter drift during the update process and the design of the projection operator is not straightforward. Due to the inherent robustness of sliding mode, sliding mode observer-based robust FD methods were proposed by several researchers, e.g., [18], [19], [20]. One class of FD method based on sliding mode maintains the sliding motion even in the presence of fault. The fault is then reconstructed by manipulating the equivalent output injection signal. Another approach designs the observer in such a way that the sliding motion is destroyed in the presence of fault [21]. Then, other online estimators are needed to approximate the fault.

This research is motivated by extending previous work on T-S fuzzy observer, neural network, and sliding mode observer to fault diagnosis for a class of nonlinear systems. In this work, a neural-fuzzy observer (NFO) and a neural-fuzzy sliding mode observer (NFSMO) are proposed for the purpose of fault detection, isolation and estimation for a class of nonlinear systems that can be represented by T-S fuzzy models. When no fault is present, a fuzzy controller and a fuzzy observer are used to stabilize the system and estimate its states, respectively. Then, a three-layer neural network is used to isolate and estimate fault its occurrence. In order to achieve robust fault diagnosis, a sliding mode term is utilized to deal with the effect of modeling uncertainties and approximation error. A modified back-propagation (BP) algorithm is used to update the parameters of the observer so that the stability of the proposed observer-based system can be analyzed by Lyapunov's direct method. In the simulation example, we apply the proposed FD scheme to a satellite orbital control system to demonstrate its performance.

II. PROBLEM FORMULATION

Consider the nominal dynamics of a class of nonlinear systems

$$\begin{aligned} \dot{x} &= f(x,u,t) \\ y &= g(x,t) \end{aligned} \tag{1}$$

where $x \in \Re^n$ is the state vector, $y \in \Re^p$ is the output vector, and $u \in \Re^m$ is the control input vector of the system. The state function $f : \Re^n \times \Re^m \times \Re^+ \to \Re^n$ and the measurement function $g : \Re^n \times \Re^+ \to \Re^p$ are both smooth vector field.

In this study, we assume that (1) can be represented or sufficiently approximated by a T-S fuzzy system. The T-S

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system consists of a set of fuzzy rules, where the ith rule is

$$\text{Rule } i: \text{ If } z_1 \text{is } \mu_1^i(z_1), \ldots \text{and } z_r \text{ is } \mu_r^i(z_r)$$
$$\text{Then} \begin{cases} \dot{x} = A_i x + B_i u \\ y = \sum_{i=1}^{l} h_i(z) C_i x \end{cases} \quad (2)$$

where the vector of premise variables $z \in \Re^r$ is a subset of y and $\mu_j^i : \Re \to [0, 1]$. The function $\mu_j^i(z_j)$ is the jth membership function in the ith rule which is applied to the jth premise variable.

The global T-S fuzzy system is then written as

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{l} h_i(z)(A_i x + B_i u) \\ y &= \sum_{i=1}^{l} h_i(z) C_i x \end{aligned} \quad (3)$$

where l is the number of fuzzy rules, and

$$h_i(z) = \frac{\omega_i(z)}{\sum_{k=1}^{l} \omega_k(z)} \quad \omega_k(z) = \prod_{j=1}^{r} \mu_j^i(z_j) \quad (4)$$

Thus, the nonlinear system (1) with modeling uncertainties and process fault can be described as

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{l} h_i(z)(A_i x + B_i u) + \eta(t) + \mathcal{B}(t - T_f) f_a(t) \\ y &= \sum_{i=1}^{l} h_i(z) C_i x \end{aligned} \quad (5)$$

where $\eta(t) \in \Re^n$ represents the system modeling uncertainties, which is assumed to be bounded by a constant, i.e., $\|\eta(t)\| < \bar{\eta}$. The function $f_a(t) \in \Re^n$ denotes the process fault in the system, which is composed of actuator fault and/or component fault. The time function $\mathcal{B}(t - T_f)$ is 1, when $t \geq T_f$; otherwise is zero. Time T_f denotes the time at which a fault occurs.

III. Main Result

A. Neural-Fuzzy Observer

For the faulty system (5), a neural-fuzzy Luenberger observer is designed as

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{l} h_i(z)\big\{A_i \hat{x} + B_i u + L_i(y(t) - \hat{y}(t))\big\} \\ &\quad + \mathcal{B}(t - T_n)\hat{M}(t) \\ \hat{y}(t) &= \sum_{i=1}^{l} h_i(z) C_i \hat{x}(t) \end{aligned} \quad (6)$$

where $\hat{x} \in \Re^n$ and $\hat{y} \in \Re^p$ are the state vector and output vector of the observer, respectively. The term $L_i \in \Re^{n \times p}$ is the gain for the local linear observer in the center of the ith fuzzy region. The observer input $\hat{M}(t) \in \Re^n$ is designed to estimate the fault, and T_n is the time when this estimator starts its function. In order to separately demonstrate the properties of the fuzzy model and neural network-based fault estimator, we assume that the neural network is not activated until the observation of the state by T-S fuzzy Luenberger observer, and $T_n < T_f$.

The recurrent dynamic neural network-based fault estimator is of the following structure

$$\hat{M}(t) = \hat{W} \sigma(\hat{V} \bar{x}(t)) \quad (7)$$

where $\bar{x}(t) = [\tilde{y}(t - \tau)^\top \ \hat{M}(t - \tau)^\top]^\top$ is the input of the neural network, $\tilde{y} = y - \hat{y}$ is the output estimation error, and τ is the sampling interval. The activation function is selected to be a sigmoidal function$\sigma(\hat{V}_i \bar{x}) = \frac{1 - \mathrm{e}^{-2\hat{V}_i \bar{x}}}{1 + \mathrm{e}^{-2\hat{V}_i \bar{x}}}$ where $\hat{V}_i$ is the ith row of $\hat{V}$, and $\sigma_i(\hat{V}_i \bar{x})$ is the ith element of $\sigma(\hat{V}\bar{x})$.

After the occurrence of a fault, the estimation error dynamics become

$$\begin{aligned} \dot{\tilde{x}}(t) &= \sum_{i=1}^{l} \sum_{j=1}^{l} h_i(z) h_j(z)(A_i - L_i C_j)\tilde{x} \\ &\quad + \eta(t) + f_a(t) - \hat{W}\sigma(\hat{V}\bar{x}(t)) \\ \tilde{y}(t) &= \sum_{i=1}^{l} h_i(z) C_i \tilde{x}(t) \end{aligned} \quad (8)$$

Since L_i is designed to guarantee the stability of the estimation error dynamics without a fault, the input of the neural network is zero at T_n. Thus, $\hat{M}(t)$ will remain zero during the time interval $t \in [T_n, T_f)$. When $t \geq T_f$, the fault $f_a(t)$ breaks the stability of the estimation error dynamics and the neural network is triggered to approximate the fault.

B. Parameter Update Law

A learning strategy is established to update the observer's parameters. The parameter update law is defined in such a way that the stability of the observer can be guaranteed.

Defining a cost function $J = \frac{1}{2}\tilde{y}^2$, we design a similar parameter update law as [22]

$$\dot{\hat{W}}_{i,j} = -\rho_1 \frac{\partial J}{\partial \hat{W}_{i,j}} - \rho_2 \|\tilde{y}\| \hat{W}_{i,j} \quad (9)$$

$$\dot{\hat{V}}_{i,j} = -\rho_3 \frac{\partial J}{\partial \hat{V}_{i,j}} - \rho_4 \|\tilde{y}\| \hat{V}_{i,j} \quad (10)$$

where $\hat{W}_{i,j}$ and $\hat{V}_{i,j}$ are the (i, j)th element of $\hat{W}$ and $\hat{V}$, ρ_1, $\rho_3 > 0$ are the learning rates, and ρ_2 and ρ_4 are small positive numbers.

Based on chain rules of derivative, the cost function, and (8), we obtain

$$\frac{\partial J}{\partial \hat{W}_{i,j}} = (\tilde{y}^\top \tilde{C} d_{xM})_{1 \times i} \cdot \sigma_j \quad (11)$$

$$\frac{\partial J}{\partial \hat{V}_{i,j}} = (\tilde{y}^\top \tilde{C} d_{xV})_{1 \times i} \cdot \bar{x}_j \quad (12)$$

where $\tilde{C} = \sum_{i=1}^{l} h_i(z) C_i$, and

$$d_{xM} = \frac{\partial \tilde{x}}{\partial \hat{M}} \qquad d_{xV} = \frac{\partial \tilde{x}}{\partial \mathrm{net}_{\hat{V}}} \quad (13)$$

where $\mathrm{net}_{\hat{V}} = \hat{V}\bar{x}$.

Using (11)-(13), we re-formulate the update laws as

$$\dot{\hat{W}} = -\rho_1(\tilde{y}^\top \tilde{C} d_{xM})^\top (\sigma(\hat{V}\bar{x}))^\top - \rho_2\|\tilde{y}\|\hat{W} \quad (14)$$

$$\dot{\hat{V}} = -\rho_3(\tilde{y}^\top \tilde{C} d_{xV})^\top \bar{x}^\top - \rho_4\|\tilde{y}\|\hat{V} \quad (15)$$

Instead of using the static approximation of the gradients (13) in [22], d_{xM} and d_{xV} can be derived based on (8) as

$$\dot{d}_{xM} = \tilde{A} d_{xM} - I \quad (16)$$

$$\dot{d}_{xV} = \tilde{A} d_{xV} - \hat{W}(I - \Lambda(\hat{V}\bar{x})) \quad (17)$$

where $\tilde{A} = \sum_{i=1}^{l}\sum_{j=1}^{l} h_i(z)h_j(z)(A_i - L_iC_j)$, and $\Lambda(\hat{V}\bar{x}) = \mathrm{diag}\{\sigma_i^2(\hat{V}_i\bar{x})\}$.

During the updating process of the neural network parameters, we first initialize d_{xM} and d_{xV} to be zero matrices, and then dynamically update d_{xM} and d_{xV} using equations (16) and (17). After that, we substitute their values into (14) and (15) to compute the parameters $\hat{W}$ and $\hat{V}$.

C. Stability of NFO-based Systems

The fault $f_a(t)$ can be treated as a nonlinear function of the state estimation error $\tilde{x}$ and time t, therefore, there exist parameters W and V such that any continuous fault function on the compact set can be represented as

$$f_a(t) = W\sigma(V\bar{x}) + \epsilon_1(\tilde{x}) \quad (18)$$

where $\epsilon_1(\tilde{x})$ is the bounded neural network approximation error. We assume that the upper bounds on the fixed ideal parameters W and V satisfy

$$\|W\|_F \leq W_M \quad (19)$$

$$\|V\|_F \leq V_M \quad (20)$$

where $\|\cdot\|_F$ is the Frobenius norm of a matrix.

Substituting (18) into (8), we get

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x} + \tilde{W}\sigma(\hat{V}\bar{x}) + \epsilon_2(t) + \eta(t) \quad (21)$$

$$\tilde{y}(t) = \tilde{C}\tilde{x}(t) \quad (22)$$

where $\tilde{W} = W - \hat{W}$, and $\epsilon_2(t) = W[\sigma(V\bar{x}) - \sigma(\hat{V}\bar{x})] + \epsilon_1(t)$ is a bounded disturbance term, i.e., $\|\epsilon_2(t)\| \leq \bar{\epsilon}_2$, due to the boundedness of W, the boundedness of sigmoidal function, and the boundedness of uncertainty and approximation error.

By using the proposed modified back-propagation algorithm to update its parameters, the stability of the neural-fuzzy observer is guaranteed in the following theorem.

Theorem 1: Consider the T-S fuzzy system (3) and its neural-fuzzy observer (6). If the parameters of the neural network model are updated according to (14)-(17), then the state estimation error $\tilde{x}$, parameter estimation error $\tilde{W}$, $\tilde{V}$, and output estimation error $\tilde{y}$ are all bounded.

Proof: let's first prove the boundedness of $\tilde{x}$ and $\tilde{W}$. Consider a positive definite Lyapunov function candidate:

$$V_s = \frac{1}{2}\tilde{x}^\top P_1\tilde{x} + \frac{1}{2}tr(\tilde{W}^\top\tilde{W}) \quad (23)$$

where P_1 is a symmetric positive definite matrix satisfying

$$\tilde{A}^\top P_1 + P_1\tilde{A} = -Q$$

in which Q is a positive definite matrix, and $\tilde{W}$ can be further written as

$$\dot{\tilde{W}} = \rho_1(\tilde{y}^\top \tilde{C} d_{xM})^\top (\sigma(\hat{V}\bar{x}))^\top + \rho_2\|\tilde{y}\|\hat{W} \quad (24)$$

Since $\tilde{A}$ is designed to be Hurwitz using the LMI method, according to (16), d_{xM} is stable and converges to $\tilde{A}^{-1}$.

Based on (21) and (24), the time derivative of V_s is

$$\begin{aligned}\dot{V}_s &= \frac{1}{2}\dot{\tilde{x}}^\top P_1\tilde{x} + \frac{1}{2}\tilde{x}^\top P_1\dot{\tilde{x}} + tr(\tilde{W}^\top\dot{\tilde{W}}) \\ &= -\frac{1}{2}\tilde{x}^\top Q\tilde{x} + \tilde{x}^\top P_1(\tilde{W}\sigma(\hat{V}\bar{x}) + \epsilon_2 + \eta) \\ &\quad + tr[\tilde{W}^\top l_1\tilde{x}\sigma(\hat{V}\bar{x})^\top + \tilde{W}^\top\rho_2\|\tilde{C}\tilde{x}\|(W - \tilde{W})] \end{aligned} \quad (25)$$

where $l_1 = \rho_1 d_{xM}^\top \tilde{C}^\top\tilde{C}$.

Using the properties of matrix trace and sigmoidal function in [22], we have

$$tr[\tilde{W}^\top l_1\tilde{x}\sigma^\top] \leq \|\tilde{W}\|\|l_1\|\|\tilde{x}\|\sigma_m \quad (26)$$

$$tr[\tilde{W}^\top\rho_2\|\tilde{C}\tilde{x}\|\hat{W}] \leq (W_M\|\tilde{W}\| - \|\tilde{W}\|^2)\rho_2\|\tilde{C}\|\|\tilde{x}\| \quad (27)$$

where σ_m is defined such that $\|\sigma^\top\| \leq \sigma_m$.

Therefore, (25) can be further written as

$$\begin{aligned}\dot{V}_s &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{x}\|^2 + \|\tilde{x}\|\|P_1\|(\|\tilde{W}\|\sigma_m + \bar{\epsilon}_2 + \bar{\eta}) \\ &\quad + \sigma_m\|\tilde{W}\|\|l_1\|\|\tilde{x}\| + (W_M\|\tilde{W}\| - \|\tilde{W}\|^2)\rho_2\|\tilde{C}\|\|\tilde{x}\| \\ &= -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{x}\|^2 - \beta_1\|\tilde{x}\|\|\tilde{W}\|^2 \\ &\quad + \beta_2\|\tilde{x}\|\|\tilde{W}\| + \beta_3\|\tilde{x}\| \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{x}\|^2 + (\frac{\beta_2^2}{4\beta_1} + \beta_3)\|\tilde{x}\| \end{aligned} \quad (28)$$

where

$$\beta_1 = \rho_2\|\tilde{C}\| \quad (29)$$

$$\beta_2 = \sigma_m(\|P_1\| + \|l_1\|) + \rho_2 W_M\|\tilde{C}\| \quad (30)$$

$$\beta_3 = \|P_1\|(\bar{\epsilon}_2 + \bar{\eta}) \quad (31)$$

Thus, from (28), we can see that when

$$\|\tilde{x}\| > \frac{\beta_2^2 + 4\beta_1\beta_3}{2\lambda_{\min}(Q)\beta_1} = b_1 \quad (32)$$

$\dot{V}_s < 0$, which means $\dot{V}_s$ is negative definite outside the ball with radius b_1 described as $\chi_1 = \{\tilde{x} \mid \|\tilde{x}\| > b_1\}$. When $\tilde{x}$ is increased outside of the ball χ_1, the negative of $\dot{V}_s$ results in reducing V_s and $\tilde{x}$. This analysis shows the ultimate boundedness of $\tilde{x}$.

Then, we consider the boundedness of the weight error $\tilde{W}$, which can be rewritten as

$$\begin{aligned}\dot{\tilde{W}} &= \rho_1(\tilde{y}^\top\tilde{C}d_{xM})^\top(\sigma(\hat{V}\bar{x}))^\top + \rho_2\|\tilde{y}\|W - \rho_2\|\tilde{y}\|\tilde{W} \\ &= -\rho_2\|\tilde{y}\|\tilde{W} + \rho_2\|\tilde{y}\|W + \kappa_1(\tilde{x}_1, \hat{V}) \end{aligned} \quad (33)$$

where

$$\kappa_1(\tilde{x}_1, \hat{V}) = \rho_1(\tilde{y}^\top\tilde{C}d_{xM})^\top(\sigma(\hat{V}\bar{x}))^\top \quad (34)$$

We can see that $\kappa_1(\cdot)$ is bounded since $\tilde{x}$, $\sigma(\cdot)$ and $\tilde{C}$ are all bounded, and d_{xM} is bounded, because $\tilde{A}$ is a stable matrix. Given the ideal weight W is fixed, (33) can be treated as a linear system with bounded input $\rho_2\|\tilde{y}\|W+\kappa_1(\tilde{x}_1,\hat{V})$. (33) is stable since ρ_2 is positive and input is bounded. Therefore, the boundedness of $\tilde{W}$ is guaranteed.

The boundedness of $\tilde{W}$ implies the boundedness of $\hat{W}$. From (17), we can see that d_{xV} is also bounded since $\sigma_i^2(\cdot)$ is a bounded function, and $\tilde{A}$ is a stable matrix.

The dynamic equation of $\tilde{V}$ is

$$\begin{aligned}\dot{\tilde{V}} &= \rho_3(\tilde{y}^\top \tilde{C} d_{xV})^\top \bar{x}^\top + \rho_4\|\tilde{y}\|\hat{V} \\ &= -\rho_4\|\tilde{y}\|\tilde{V} + \rho_3(\tilde{y}^\top \tilde{C} d_{xV})^\top \bar{x}^\top + \rho_4\|\tilde{y}\|V \end{aligned} \quad (35)$$

The second and third terms on the right hand side of above equation are both finite, since $\tilde{x}$, $\hat{W}$, $\sigma(\cdot)$, $\tilde{C}$, d_{xV} are all bounded, and ρ_3 and ρ_4 are both positive finite values. Consequently, we can conclude that the boundedness of $\tilde{V}$ is also ensured.

D. Neural-Fuzzy Sliding Mode Observer

From above stability analysis, we see that the fault estimation accuracy might be affected by the system modeling uncertainty, neural network approximation error, etc. Therefore, we modify the neural-fuzzy observer (6) by adding a signum function

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{l} h_i(z)\big\{A_i\hat{x} + B_i u + L_i(y(t)-\hat{y}(t))\big\} \\ &\quad + \mathcal{B}(t-T_n)\hat{M}(t) + \gamma \mathrm{sign}(F\tilde{y}) \\ \hat{y}(t) &= C_i\hat{x}(t) \end{aligned} \quad (36)$$

where the sliding mode gain $\gamma \geq \bar{\eta}$, and $P_1^\top = F\tilde{C}$.

Then, the estimation error dynamics become

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x} + \tilde{W}\sigma(\hat{V}\bar{x}) + \epsilon_2(t) + \eta(t) - \gamma \mathrm{sign}(F\tilde{y}) \quad (37)$$

$$\tilde{y}(t) = \tilde{C}\tilde{x}(t) \quad (38)$$

Regarding the stability of above dynamics, we have the following theorem.

Theorem 2: Consider the T-S fuzzy system (3) and the neural-fuzzy sliding mode observer (36). If the parameters of the neural network model are updated according to (14)-(17), then $\tilde{x}$, $\tilde{W}$, $\tilde{V}$, and $\tilde{y}$ are all bounded, and $\tilde{x}$ can converge to a small bound.

Proof: The proof procedure is similar to that in theorem 1. We again use the Lyapunov function (23), and its time derivative is rewritten as

$$\begin{aligned}\dot{V}_s &= \frac{1}{2}\dot{\tilde{x}}^\top P_1\tilde{x} + \frac{1}{2}\tilde{x}^\top P_1\dot{\tilde{x}} + tr(\tilde{W}^\top\dot{\tilde{W}}) \\ &= -\frac{1}{2}\tilde{x}^\top Q\tilde{x} + \tilde{x}^\top P_1(\tilde{W}\sigma(\hat{V}\bar{x})+\epsilon_2) \\ &\quad + \tilde{x}^\top P_1\eta - \tilde{x}^\top P_1\gamma\mathrm{sign}(F\tilde{C}\tilde{x}) \\ &\quad + tr[\tilde{W}^\top l_1\tilde{x}\sigma(\hat{V}\bar{x})^\top + \tilde{W}^\top\rho_2\|\tilde{C}\tilde{x}\|(W-\tilde{W})] \end{aligned} \quad (39)$$

Still using the inequalities (26) and (27), we have

$$\begin{aligned}\dot{V}_s &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{x}\|^2 + \|\tilde{x}\|\|P_1\|(\|\tilde{W}\|\sigma_m + \bar{\epsilon}_2) \\ &\quad + (\bar{\eta}-\gamma)\|\tilde{x}^\top P_1\| + \sigma_m\|l_1\|\|\tilde{W}\|\|\tilde{x}\| \\ &\quad + (W_M\|\tilde{W}\| - \|\tilde{W}\|^2)\rho_2\|\tilde{C}\|\|\tilde{x}\| \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{x}\|^2 - \beta_1\|\tilde{x}\|\|\tilde{W}\|^2 \\ &\quad + \beta_2\|\tilde{x}\|\|\tilde{W}\| + \beta_3'\|\tilde{x}\| \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{x}\|^2 + (\frac{\beta_2^2}{4\beta_1} + \beta_3')\|\tilde{x}\| \end{aligned} \quad (40)$$

where β_1 and β_2 are still (29) and (30), $\beta_3' = \|P_1\|\bar{\epsilon}_2 < \beta_3$.

Thus, when

$$\|\tilde{x}\| > \frac{\beta_2^2 + 4\beta_1\beta_3'}{2\lambda_{\min}(Q)\beta_1} = b_2 < b_1 \quad (41)$$

$\dot{V}_s < 0$, $\tilde{x}$ is ultimately bounded by a ball with a smaller radius b_2, i.e., $\chi_2 = \{\tilde{x} \mid \|\tilde{x}\| \leq b_2\}$.

When the sliding mode term just counteracts the effect of modeling uncertainty, it results in the convergence of $\|\tilde{x}\|$ to a smaller bound which implies a more accurate fault estimation. If the sliding mode gain γ is sufficiently large, the sliding mode may eliminate the effect of fault and uncertainties which are both treated as an unknown input. Therefore, it is concluded that γ should be carefully selected. According to above analysis, the tight bound of the modeling uncertainty would be a preferable choice for γ.

E. Robust Fault Diagnosis Scheme

In this work, after the T-S fuzzy model observes all the states, we use the output error or output estimation error to detect the fault, i.e.

$$\begin{cases} \text{No fault occurs} & \text{if } \|e_y(t)\| < \epsilon_f \\ \text{Fault occurs, and } \hat{M}(t) \text{ works} & \text{if } \|e_y(t)\| \geq \epsilon_f \end{cases} \quad (42)$$

or

$$\begin{cases} \text{No fault occurs} & \text{if } \|\tilde{y}(t)\| < \epsilon_f' \\ \text{Fault occurs, and } \hat{M}(t) \text{ works} & \text{if } \|\tilde{y}(t)\| \geq \epsilon_f' \end{cases} \quad (43)$$

where $e_y(t) = y_d - y$ is the output error, y_d is the reference trajectory, and ϵ_f and ϵ_f' are thresholds for robust fault detection. The choice of ϵ_f and ϵ_f' replies on the system characteristics and the diagnosis scheme in use. In this work, the output of the neural network $\hat{M}(t)$ is selected for the process fault isolation and fault estimation.

IV. Simulation Example

In this section, we apply the proposed neural-fuzzy observer and neural-fuzzy sliding mode observer to a point mass satellite dynamic system [23].

The fourth-order satellite model is considered in [23] as

$$\begin{aligned} \dot{r} &= v & r(0) &= r_0 \\ \dot{v} &= rw^2 - \frac{k}{mr^2} + \frac{u_1}{m} & v(0) &= 0 \\ \dot{\phi} &= w & \phi(0) &= 0 \\ \dot{\omega} &= -\frac{2v\omega}{r} + \frac{u_2}{mr} & \omega(0) &= \omega_0 \end{aligned} \quad (44)$$

where $m = 200\text{kg}$ is the mass of the satellite, (r, ϕ) are the polar coordinates of the satellite, v is the radial speed, and ω is the angular speed. Control inputs u_1 and u_2 are the radial and tangential thrust forces, respectively. Since the control purpose is to track the output r and ω to their constant reference trajectory r_r and ω_r, the equation $\dot{\phi} = \omega$ is omitted.

When we choose $x = [x_1\ x_2\ x_3]^\top = [r\ v\ \omega]^\top$, and $y = [r\ \omega]^\top$, the reduced-order system is written as

$$
\begin{aligned}
\dot{x}_1 &= x_2 & x_1(0) &= r_0 \\
\dot{x}_2 &= x_1 x_3^2 - \frac{k}{m x_1^2} + \frac{u_1}{m} & x_2(0) &= 0 \\
\dot{x}_3 &= -\frac{2 x_2 x_3}{x_1} + \frac{u_2}{m x_1} & x_3(0) &= \omega_0
\end{aligned}
\tag{45}
$$

The parameter $k = K_E m$, where $K_E = 3.986 \times 10^5 \text{km}^3/\text{s}^2$ is derived from the parameters of the earth ($M_E = 5.974 \times 10^{24}\text{kg}$, $R_E = 6.378 \times 10^3\text{km}$). The satellite is first observed in perigee 375 km above the surface of the earth $r_0 = R_E + 375\text{km}$. The initial angular speed ω_0 is computed using the orbital mechanics $\omega_0 = \sqrt{(e_{orbit}+1)K_E/r_0^3}$, where $e_{orbit} = 0.162$ is the eccentricity.

In the design of fuzzy control and observer, we define the nonlinear terms as $z_1(x_1, x_3) = x_3^2 - \frac{k}{m x_1^3}$, $x_2(x_1, x_3) = \frac{x_3}{x_1}$, and $z_3(x_1, x_3) = \frac{1}{x_1}$.

We assume that the outputs satisfy $x_1 \in [r_{\min}, r_{\max}]$ and $x_3 \in [\omega_{\min}, \omega_{\max}]$, where $r_{\min} = 0.9 r_0$, $r_{\max} = 1.1 r_0$, $\omega_{\min} = -4$, and $\omega_{\max} = 4$ in simulation. Thus,

$$
\begin{aligned}
z_1^{\max} &= \omega_{\max}^2 - \frac{k}{m r_{\max}^3} & z_1^{\min} &= -\frac{k}{m r_{\min}^3} \\
z_2^{\max} &= \frac{\omega_{\max}}{r_{\min}} & z_2^{\min} &= \frac{\omega_{\min}}{r_{\min}} \\
z_3^{\max} &= \frac{1}{r_{\min}} & z_3^{\min} &= \frac{1}{r_{\max}}
\end{aligned}
\tag{46}
$$

The nonlinear terms z_1 can be represented by

$$
\begin{aligned}
\mu_1^1 z_1^{\max} + \mu_1^2 z_1^{\min} &= z_1 \\
\mu_1^1 + \mu_1^2 &= 1
\end{aligned}
\tag{47}
$$

So, the membership functions μ_1^1 and μ_1^2 are

$$
\mu_1^1 = \frac{z_1 - z_1^{\min}}{z_1^{\max} - z_1^{\min}} \quad \mu_1^2 = \frac{-z_1 + z_1^{\max}}{z_1^{\max} - z_1^{\min}} \tag{48}
$$

and μ_2^1, μ_2^2, μ_3^1, and μ_3^2 can be derived in a similar way.

There are a total of eight fuzzy rules. The membership functions for these eight fuzzy rules are computed using (4). The output tracking controller is designed using the approach in [24].

In this simulation, the three-layer neural network is of a structure $5 \times 5 \times 3$. In the parameter update law (16) and (17), the learning rates are set to be $\rho_1 = \rho_3 = 20$, and the damping coefficients are $\rho_2 = \rho_4 = 0.1$. The initial values of d_{xM} and d_{xV} are zero vector and matrix, respectively. The sliding mode gain γ is set to be 0.0025.

The simulation results are shown in Fig. 1 to 4. Fig. 1 illustrates the performance of output tracking and state observation using T-S fuzzy model when there is no fault.

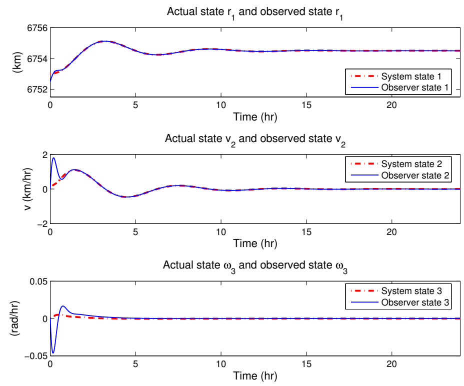

Fig. 1. Time behaviors of system states and observer states using T-S fuzzy control and observer in the case of no fault

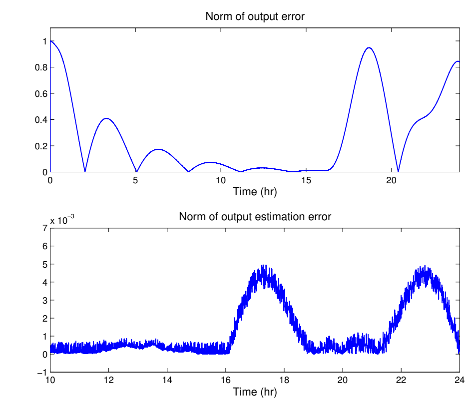

Fig. 2. Time behaviors of the norm of the output estimation error

In the simulation, we assume that only an incipient fault occurs which disturbs the second state at the 16th hour, and the neural network is enabled in the 15th hour. Fig. 2 shows the norm of output error and the norm of output estimation error, which are both useful for detecting fault. After a fault occurs, $\|e_y\|$ and $\|\tilde{y}\|$ both quickly exceed the thresholds. However, in order to isolate and estimate the fault, we need to use other signals.

Fig. 3 portrays the characteristics of the fault functions and the three outputs of the neural network when using the neural-fuzzy observer. When a fault occurs, only the neural network output that corresponds to the faulty state specifies the dynamics of the fault, and the other neural network outputs associated with the healthy states remain close to zero. Due to the approximation error and system modeling uncertainties, there exists fault estimation error.

Fig. 4 exhibits the same fault functions and the three outputs of the neural network when using the neural-fuzzy sliding mode observer. Comparing the fault diagnosis results with those using neural-fuzzy observer in Fig. 3, a better performance is achieved in fault estimation using the NFSMO, though the chattering caused by sliding mode might increase as well.

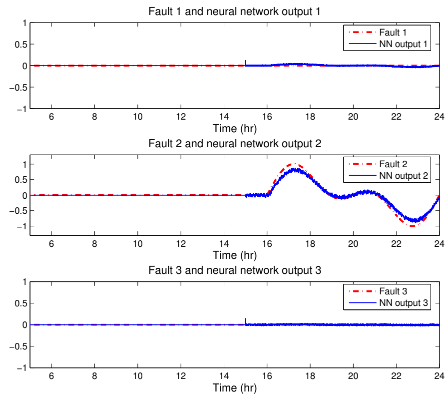

Fig. 3. Time behaviors of the output of the neural-fuzzy observer under an incipient fault

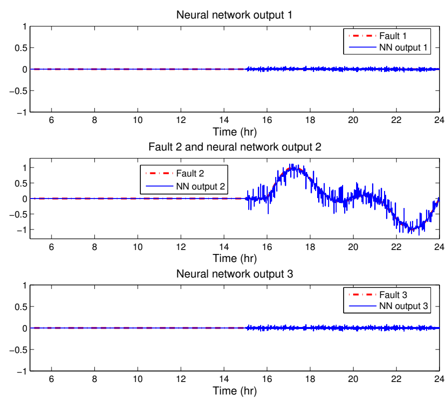

Fig. 4. Time behaviors of the output of the neural-fuzzy sliding mode observer under an incipient fault

V. CONCLUSIONS

In this work, a neural-fuzzy observer and a neural-fuzzy sliding mode observer were proposed for the purpose of robust fault diagnosis in a class of nonlinear systems. Using the modified back-propagation algorithm to update the observer parameters, the stability of these two observer-based systems were rigorously analyzed. Following the theoretical analysis, this robust fault diagnosis scheme was applied to a point mass satellite orbital control system, and numerical simulation demonstrates its satisfactory performance.

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