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A Hierarchical Architecture for Cooperative Fault Accommodation of Formation Flying Satellites in Deep Space

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Abstract— **In this paper a new fault accommodation algorithm based on a multi-level hierarchical architecture is proposed for satellite formation flying missions. This framework introduces a high level (HL) supervisor and two recovery modules: low level fault recovery (LLFR) and formation level fault recovery (FLFR). In the LLFR module, conventional recovery controller (RC) is implemented using fault severity estimation techniques. Due to imprecise fault estimation and the resulting ineffective recovery controller, the HL supervisor alerts violation of error bounds that are imposed by the mission specifications. The FLFR module is activated to compensate for the performance degradation of the faulty satellite by requiring the healthy satellites to allocate additional resources. Consequently, fault is cooperatively recovered by our proposed architecture, and the formation flying mission specifications are satisfied. Simulation results confirm the validity and effectiveness of our analytical work.**

I. Introduction

Formation flying is a new concept proposed for a cluster of satellites that calls for development of novel technologies. This new field has been surveyed in detail in [1] and [2], where five architectures are introduced for formation flying control (FFC), namely Multiple-Input Multiple-Output (MIMO), Leader/Follower (L/F), Virtual Structure (VS), Cyclic and Behavioral. Due to the strict high-precision control requirements that are necessary in position and attitude of satellites in formation flying missions, the problem of fault diagnosis has become critically significant in this field of study.

Considerable research has already been devoted to fault diagnosis and recovery for the satellite's attitude and orbital control systems (AOCS). In [3] adaptive control is applied to systems with actuator uncertainty and failure. In [4], a fault is assumed to belong to a finite set of parameters (modes), and a sliding mode controller is designed for accommodation of each mode in a hierarchical framework. In [5], by solving a Lyapunov equation, a robust state-space observer is proposed to simultaneously estimate descriptor system states, actuator faults, their finite time derivatives, and attenuate input disturbances in any desired accuracy. Moreover, a fault-tolerant control scheme is worked out using the estimates of descriptor states and faults, and the linear matrix inequality (LMI) technique. In [6], [7] an adaptive Kalman filtering algorithm is developed to estimate the reduction of control effectiveness in a closed-loop setting. The state estimate is fed back to achieve steady-state regulation, while the control effectiveness estimate is used for on-line tuning of the control law. In [8], [9] the authors designed an iterative learning observer (ILO), which uses a learning mechanism instead of using integrators that are commonly used in classical adaptive observers.

Various methods have been proposed in the literature for the problem of fault recovery of a single satellite. However, none of these methods have properly introduced and investigated the concept of cooperative fault recovery and accommodation in formation flying satellites. In this paper, the fault accommodation problem in formation flying control (FFC) is investigated by using a cooperative scheme. This cooperative scheme is proposed by the authors in [10] for the special case of a two-satellite formation with absolute state measurements. In this paper, our cooperative fault accommodation framework is formulated for the general case of multiple-satellite formation with relative measurements in deep space. In our cooperative fault accommodation framework, once a fault is recovered by the low-level fault recovery (LLFR) controller using the fault parameter estimate, the performance of the LL-recovered satellite with respect to the mission specifications is determined by the high level (HL) "supervisor", which is represented as a discrete-event system (DES) [11]. In case that the LL-recovered satellite violates the mission error specifications, the supervisor activates the formation-level fault recovery (FLFR) module, in which all other satellites will try to compensate for the performance degradation of the partially LL-recovered satellite. Hence, the fault is cooperatively accommodated by the LLFR and FLFR modules. This is the main idea behind our multi-level cooperative fault accommodation scheme proposed in this work.

II. Problem Formulation

Consider the four-satellite formation depicted in Fig. 1. Assume that the satellite #2 is faulty and partially recovered by the low-level fault recovery (LLFR) system. Therefore, it tracks the desired trajectory within an error bound of radius r. The entire purpose of formation-level fault recovery (FLFR) is to show that by restricting (decreasing) the input effort of satellite #2, at the expense of more input effort from other satellites #1, #3 and #4, one can reduce the error radius r to meet the error specifications of the formation

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mission. The main objective is to propose a framework and guidelines to optimally achieve the FLFR without resorting to any iterative trial-and-error procedure.

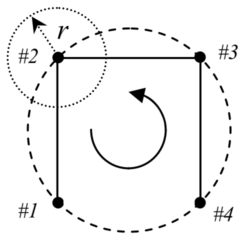

Fig. 1. A four-satellite formation.

In this work, a four-satellite formation in *deep space*, as shown in Fig. 1, is considered. However, the main results can be easily extended to the general case of *n*-satellite formation. The model of the satellite #*i* is approximated by a double integrator on each of the three axes as follow:

$$\ddot{x}_i = b_{x_i} u_{x_i} + d_x$$
$$\ddot{y}_i = b_{y_i} u_{y_i} + d_y$$
$$\ddot{z}_i = b_{z_i} u_{z_i} + d_z$$

where $(x_i, y_i, z_i)^T \in R^3$ is the three-dimensional position coordinates in the local inertial frame, and $(u_{x_i}, u_{y_i}, u_{z_i})^T \in R^3$ is the input force vector. The actuators are modeled by the gains b_{x_i}, b_{y_i} and b_{z_i}, and the environmental disturbances are represented by $(d_x, d_y, d_z)^T$.

For simplicity, the following analysis ignores the effects of disturbances. However, later on in Section IV these effects are analyzed. Since the dynamics on the three axes are decoupled, for sake of simplicity and due to space limitations, we only consider the dynamics on the *x*-axis although the results can be easily extended to the *y*- and *z*-axes dynamics, that is we consider to have $\ddot{x}_i = b_i u_i$.

The control approach implemented in this work is based on a conventional linear design technique. The solid lines in Fig. 1 represent the system outputs, for which a linear controller will be designed. In order to avoid output redundancy, three outputs (corresponding to three solid lines) are chosen. For each solid line, the corresponding output error and its first two derivatives are as follow:

$$e_{ij} = x_{ij} - x_{ij}^d \quad , \quad \dot{e}_{ij} = \dot{x}_{ij} - \dot{x}_{ij}^d \quad , \quad \ddot{e}_{ij} = b_j u_j - b_i u_i - \ddot{x}_{ij}^d$$

where $x_{ij} = x_j - x_i$ is the relative position state between the two satellites #*i* and #*j*, and x_{ij}^d is the desired relative state. Therefore, in compact matrix form the second derivatives of output errors are expressed as follow:

$$\begin{bmatrix} \ddot{e}_{12} \\ \ddot{e}_{23} \\ \ddot{e}_{34} \end{bmatrix} = \begin{bmatrix} -b_1 & b_2 & 0 & 0 \\ 0 & -b_2 & b_3 & 0 \\ 0 & 0 & -b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} -\ddot{x}_{12}^d \\ -\ddot{x}_{23}^d \\ -\ddot{x}_{34}^d \end{bmatrix} \tag{1}$$

Due to availability of high precision autonomous formation flying (AFF) sensors [12] in deep space, instead of imprecise measurements of absolute states x_i and $\dot{x}_i$, relative states x_{ij} and $\dot{x}_{ij}$ are measured and used in the formation feedback loop. Therefore, the control law resulting from the output error e_{ij} is derived as follows:

$$b_j u_j - b_i u_i = \ddot{x}_{ij}^d - \lambda_1 \dot{e}_{ij} - \lambda_0 e_{ij} \overset{\Delta}{=} p_{ij}(t)$$

which results in the closed-loop characteristic polynomial $s^2 + \lambda_1 s + \lambda_0 = 0$. Choosing the parameters $\lambda_1 > 0$ and $\lambda_0 > 0$ properly, one can make the closed-loop system asymptotically stable. The matrix form of the three control laws corresponding to the three solid lines in Fig. 1 is as follow:

$$\begin{bmatrix} -b_1 & b_2 & 0 & 0 \\ 0 & -b_2 & b_3 & 0 \\ 0 & 0 & -b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} p_{12}(t) \\ p_{23}(t) \\ p_{34}(t) \end{bmatrix} \tag{2}$$

III. Low-Level Fault Recovery

In order to determine the four control signals in $u = [u_1 \quad u_2 \quad u_3 \quad u_4]^T$, infinite solutions exist to the matrix equation (2). This shows that there exist many degrees of freedom in choosing the control inputs. In the following, we take advantage of this flexibility to optimize fuel consumption across the formation. The following cost function is considered in the constrained optimization problem

$$\underset{u_i \in R}{Minimize} \ \sum_{i=1}^{4} \frac{\alpha_i}{2} u_i^2 \quad subject \ \ to \ \ (2)$$

Solving the above minimization problem by the Lagrange method results in:

$$u = \begin{bmatrix} -b_1 & b_2 & 0 & 0 \\ 0 & -b_2 & b_3 & 0 \\ 0 & 0 & -b_3 & b_4 \\ \frac{\alpha_1}{b_1} & \frac{\alpha_2}{b_2} & \frac{\alpha_3}{b_3} & \frac{\alpha_4}{b_4} \end{bmatrix}^{-1} \begin{bmatrix} p_{12}(t) \\ p_{23}(t) \\ p_{34}(t) \\ 0 \end{bmatrix} \tag{3}$$

Replacing the original form of equation (3) into (1) we get

$$\begin{bmatrix} \ddot{e}_{12} \\ \ddot{e}_{23} \\ \ddot{e}_{34} \end{bmatrix} = \begin{bmatrix} -\lambda_1 \dot{e}_{12} - \lambda_0 e_{12} \\ -\lambda_1 \dot{e}_{23} - \lambda_0 e_{23} \\ -\lambda_1 \dot{e}_{34} - \lambda_0 e_{34} \end{bmatrix} \tag{4}$$

In the absence of any noise, disturbances and system uncertainties, the above shows that asymptotic stability is achieved by the LLFR module if the fault is accurately estimated. However, when there is only a partial or an inaccurate estimation/recovery of a fault, asymptotic stability is no longer achieved, and instead ultimate boundedness can be guaranteed. These results will be discussed in the next section where the FLFR module is qualitatively and quantitatively formulated to tackle possible violations of the error specifications.

IV. Formation-Level Fault Recovery

In case of a partially LL-recovered satellite due to biased estimates, that is $\hat{b}_2 \neq b_2$ or $\hat{b}_2 = b_2 + \varepsilon$, where ε is unknown but bounded ($|\varepsilon| < B_\varepsilon$) with B_ε known, we get

$$\begin{bmatrix} \ddot{e}_{12} + \lambda_1 \dot{e}_{12} + \lambda_0 e_{12} \\ \ddot{e}_{23} + \lambda_1 \dot{e}_{23} + \lambda_0 e_{23} \\ \ddot{e}_{34} + \lambda_1 \dot{e}_{34} + \lambda_0 e_{34} \end{bmatrix} = D\begin{bmatrix} p_{12} & p_{23} & p_{34} & 0 \end{bmatrix}^T \tag{5}$$

where

$$D = \begin{bmatrix} 0 & -\varepsilon & 0 & 0 \\ 0 & +\varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} -b_1 & \hat{b}_2 & 0 & 0 \\ 0 & -\hat{b}_2 & b_3 & 0 \\ 0 & 0 & -b_3 & b_4 \\ \frac{\alpha_1}{b_1} & \frac{\alpha_2}{\hat{b}_2} & \frac{\alpha_3}{b_3} & \frac{\alpha_4}{b_4} \end{bmatrix}^{-1}}_{\hat{J}} \tag{6}$$

$$= \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \\ -d_1 & -d_2 & -d_3 & -d_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In comparison with equation (4), the matrix $D(t)$ in equation (5) will disturb and affect the asymptotic stability of the closed-loop system. Equation (5) consists of two disturbed subsystems (e_{12} and e_{23}) and one asymptotically stable subsystem (e_{34}). The third subsystem (e_{34}) is asymptotically stable, so that its states approach to zero ($e_{34} \to 0$ and $\dot{e}_{34} \to 0$). Therefore, one is left with the first two disturbed subsystems e_{12} and e_{23}, for which the dynamic equation (5) yields:

$$\dot{X}(t) = A_{clp} X(t) + \overline{D} \quad , \quad A_{clp} = (I + \Delta)A \tag{7}$$

where

$$X(t) = \begin{bmatrix} e_{12} \\ \dot{e}_{12} \\ e_{23} \\ \dot{e}_{23} \end{bmatrix}, \overline{D} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \left(d_1 \ddot{x}_{12}^d + d_2 \ddot{x}_{23}^d + d_3 \ddot{x}_{34}^d \right)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\lambda_0 & -\lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\lambda_0 & -\lambda_1 \end{bmatrix}, \Delta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & d_1 & 0 & d_2 \\ 0 & 0 & 0 & 0 \\ 0 & -d_1 & 0 & -d_2 \end{bmatrix}$$

Our first result is given next.

Lemma 1. Consider the matrix D in equation (6), and assume that the coefficient α_2 in equation (3) corresponds to the faulty satellite with partially estimated/recovered actuator $\hat{b}_k \neq b_k$ ($k = 2$ in this case). The parameters α_k and $\hat{b}_k$ ($k = 2$ in this case) appear in the denominator and not numerator of the nonzero elements of the matrix D, and in order to decrease the norm of D, one should appropriately increase the coefficient α_2.

Proof. Let us start with equation (6) and rewrite it as

$$D = \begin{bmatrix} 0 & -\varepsilon & 0 & 0 \\ 0 & +\varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{DET} M_{co}$$

where

$$DET = \frac{\alpha_1 \hat{b}_2 b_3 b_4}{b_1} + \frac{\alpha_2 b_1 b_3 b_4}{\hat{b}_2} + \frac{\alpha_3 b_1 \hat{b}_2 b_4}{b_3} + \frac{\alpha_4 b_1 \hat{b}_2 b_3}{b_4}$$

is the determinant and M_{co} is the cofactor (adjugate) of matrix $\hat{J}$ in equation (6). We now have

$$D = \frac{1}{DET} \begin{bmatrix} 0 & -\varepsilon & 0 & 0 \\ 0 & +\varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \times & \times & \times & \times \\ f_1(.) & f_2(.) & f_3(.) & f_4(.) \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix}}_{M_{co}}$$

$$= \frac{\varepsilon}{DET} \underbrace{\begin{bmatrix} -f_1(.) & -f_2(.) & -f_3(.) & -f_4(.) \\ +f_1(.) & +f_2(.) & +f_3(.) & +f_4(.) \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{F} = \frac{\varepsilon}{DET} F$$

$$\stackrel{\Delta}{=} \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \\ -d_1 & -d_2 & -d_3 & -d_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix F has elements $f_1(.)$, $f_2(.)$, $f_3(.)$, and $f_4(.)$, which can be shown to be functions of $(\alpha_1, \alpha_3, \alpha_4, b_1, b_3, b_4)$. Therefore, α_2 and $\hat{b}_2$ do not appear in the numerator of elements of matrix D and instead appear in its denominator. Moreover, since the denominator DET is a monotonically increasing function of α_2, the norms $|\varepsilon f_1(.)/DET|$, $|\varepsilon f_2(.)/DET|$, $|\varepsilon f_3(.)/DET|$, and $|\varepsilon f_4(.)/DET|$ are monotonically decreasing functions of α_2. This implies that the norm of D is a monotonically decreasing function of α_2 which completes the proof. ■

We are now in a position to state our main results.

Theorem 1. To stabilize the nominal (disturbance free) system $\dot{X} = (I + \Delta)AX$ given by equation (7), the FLFR module should choose a sufficiently large coefficient α_k in (3) corresponding to the partially estimated/recovered actuator $\hat{b}_k \neq b_k$.

Proof. We rearrange the nominal system $\dot{X} = (I + \Delta)AX$ (A is Hurwitz) into an equivalent closed-loop configuration of the following system S and controller CON:

$$S : \begin{cases} \dot{X} = AX + U \\ Y = AX \end{cases} \quad , \quad CON : U = \Delta Y$$

where the system S is controllable and observable. Let us take

$$\gamma_1 = \sup_{\omega \in R} \sigma_{max}[S(j\omega)]$$

where σ_{max} denotes the maximum singular value of a complex matrix. γ_1 is finite since $S(j\omega)$ is Hurwitz. Taking $||\Delta||_\infty \leq \gamma_2$, the controller CON satisfies:

$$||U||_\infty \leq \gamma_2 ||Y||_\infty$$

According to the small-gain theorem [13], [14] the sufficient condition for the overall closed-loop system stability is $\gamma_1\gamma_2 < 1$. Taking $B_\Delta = 1/\gamma_1$, the sufficient condition becomes $||\Delta||_\infty < B_\Delta$, which is equivalent to appropriately decreasing $||\Delta||_\infty$, or alternatively, appropriately decreasing $||D||_\infty$. The latter can be achieved by using Lemma 1-(b) where the coefficient α_k is selected to be sufficiently large. ■

Theorem 2. Assume that the nominal system $\dot{X} = (I+\Delta)AX$ given in (7) is stable (Theorem 1). In order to decrease the norm of the error vector X in equation (7), the FLFR module should appropriately increase the coefficient α_k in (3) corresponding to the partially estimated/recovered actuator $\hat{b}_k \neq b_k$.

Proof. Since the nominal system (7) is stable, and taking $A_{clp} = (I+\Delta)A$ as in (7), the error vector X is governed by the dynamical system $\dot{X}(t) = A_{clp}X(t) + \overline{D}(t)$ whose Laplace transform is given by

$$X(s) = (sI - A_{clp})^{-1}\overline{D}(s) = G(s)\overline{D}(s)$$

(by neglecting the initial conditions) where $X = [X_i]_{4\times 1}$ and $G(s) = [G_{ij}(s)]_{4\times 4}$ are the error vector and the transfer function matrix, respectively. Using the definition of $\overline{D}(t)$ in (7), and d_i $(i = 1,2,3,4)$ from Lemma 1, and $q_1 > 0$ and $q_2 > 0$ as

$$q_1 \overset{\Delta}{=} \frac{\alpha_1 \hat{b}_2 b_3 b_4}{b_1} + \frac{\alpha_3 b_1 \hat{b}_2 b_4}{b_3} + \frac{\alpha_4 b_1 \hat{b}_2 b_3}{b_4} \quad , \quad q_2 \overset{\Delta}{=} \frac{b_1 b_3 b_4}{\hat{b}_2}$$

we now have

$$X(s) = G(s)\frac{1}{q_2\alpha_2 + q_1}\begin{bmatrix} 0 \\ -\varepsilon \\ 0 \\ \varepsilon \end{bmatrix} \mathcal{L}\{ f_1\ddot{x}_{12}^d + f_2\ddot{x}_{23}^d + f_3\ddot{x}_{34}^d \}$$

or alternatively

$$X_i(s) = (G_{i4} - G_{i2})\frac{\varepsilon}{q_2\alpha_2 + q_1}\mathcal{L}\{ f_1\ddot{x}_{12}^d + f_2\ddot{x}_{23}^d + f_3\ddot{x}_{34}^d \}$$

where $\mathcal{L}\{.\}$ represents the Laplace transform of a given signal. Let us define

$$H_i \overset{\Delta}{=} \sup_{\substack{t \in R^+ \\ |\varepsilon| < B_\varepsilon}} \left| (G_{i4}(t) - G_{i2}(t)) * \left(f_1\ddot{x}_{12}^d + f_2\ddot{x}_{23}^d + f_3\ddot{x}_{34}^d \right) \right|$$

where "$*$" represents the convolution operator. We now express the error vector X_i in the time domain as follows

$$|X_i(t)| \leq \frac{|\varepsilon| H_i}{q_2\alpha_2 + q_1} \overset{\Delta}{=} B_{X_i} \tag{8}$$

where the values of the parameters q_1, q_2, and H_i are known, and the parameter ε is unknown ($|\varepsilon| < B_\varepsilon$ and B_ε is known). It follows that $|X_i(t)|$ is bounded by a monotonically decreasing function of the parameter $\alpha_2 > 0$. This completes the proof of the theorem. ■

Let us now assume that an external (environmental) disturbance D_{ext} that is bounded by B_{ext} (i.e., $||D_{ext}|| < B_{ext}$) is applied to our system. Equation (8) should be modified as follows

$$|X_i(t)| \leq B_{X_i} + T_i$$

where

$$T_i \overset{\Delta}{=} \sup_{\substack{\omega \in D_\omega \\ |\varepsilon| < B_\varepsilon}} |G_{i4} - G_{i2}| B_{ext} > 0$$

One immediate conclusion is that by using the above method, one cannot certainly get a better (smaller) error than T_i.

Since the estimation error ε is unknown, Theorem 1 can not determine how much one should decrease the parameter α_2 in equation (8) to meet the desired mission error specifications that is determined by e_s. However, since ε is bounded ($|\varepsilon| < B_\varepsilon$), and assuming that the bound B_ε is known *a priori*, a non-iterative (one shot) solution can be obtained as

$$|X_i(t)| \leq \frac{B_\varepsilon H_i}{q_2\alpha_2 + q_1} = e_s$$

or equivalently

$$\alpha_2 = \frac{B_\varepsilon H_i}{q_2 e_s} - \frac{q_1}{q_2} \tag{9}$$

which is a rather conservative solution. Therefore, one may prefer to deal with this problem from a probabilistic perspective as discussed next.

From the probabilistic perspective, we assume that the probability distribution function of the estimation error ε is known and is equal to $f_\varepsilon(m)$. The objective is to find the parameter α_2 such that the probability of violation of the error specification (e_s) is less than a predefined probability, namely π ($0 < \pi < 1$), that is

$$P(|X_i(t)| > e_s) < \pi \quad \Rightarrow \quad P(|X_i(t)| < e_s) > 1 - \pi$$

Considering the definition of B_{X_i} in equation (8), it yields

$$P(|X_i(t)|< e_s) \;>\; P\left(\frac{|\varepsilon| H_i}{q_2\alpha_2 + q_1} < e_s\right) = P(\varepsilon^- < \varepsilon < \varepsilon^+)$$

where $\varepsilon^+ = -\varepsilon^- = \dfrac{(q_2\alpha_2 + q_1)e_s}{H_i}$. Therefore, determining the solution reduces to finding the coefficient α_2 which satisfies the following expression

$$\int_{m=\varepsilon^-}^{\varepsilon^+} f_\varepsilon(m)dm \quad = \quad 1 - \pi$$

If the information regarding the probability distribution function of the estimation error ε is not available, one conventional and practical solution would be to assume that it is uniformly distributed in the interval $[-B_\varepsilon \; B_\varepsilon]$ as given by

$$f_\varepsilon(m) = \begin{cases} \dfrac{1}{2B_\varepsilon} & -B_\varepsilon < m < B_\varepsilon \\ 0 & otherwise \end{cases} \tag{10}$$

We now need to solve the following equation for α_2:

$$\int_{m=\varepsilon^-}^{\varepsilon^+} \frac{1}{2B_\varepsilon}dm \quad = \quad 1 - \pi \quad \Rightarrow \quad \frac{\varepsilon^+ - \varepsilon^-}{2B_\varepsilon} = 1 - \pi$$

The solution becomes

$$\alpha_2 = \frac{B_\varepsilon(1-\pi)H_i}{q_2 e_s} - \frac{q_1}{q_2} \tag{11}$$

The solution in equation (9) is a special case (most conservative result, $\pi = 0$) of the solution in equation (11). However, one can improve the performance of the FLFR module by utilizing a more precise probability distribution function instead of the uniform distribution function that is used in equation (10).

V. Simulation Results

Consider the four-satellite deep space formation in the *xy*-plane as shown in Fig. 1. The objective is a counter-clockwise rotation maneuver in the *xy*-plane with the frequency of $\omega = 0.1 \ (rad/s)$, such that the satellites always maintain a square shape with the side lengths of $200 \ (m)$. The desired formation outputs are the relative distances among the neighboring satellites. The major environmental disturbance in deep space is solar pressure [15], which is of the order $10^{-5}(N)$, and the sensor noise is an additive zero-mean white Gaussian process with the variance 10^{-4}.

For simulations, a 20% loss-of-effectiveness fault is applied to the *x*-axis actuator of the satellite *#2*, and the corresponding fault parameter is estimated within a 10% relative error, that is $|\varepsilon_2|/b_2 = |\hat{b}_2 - b_2|/b_2 = 0.1$. Let satellite *#2* be partially recovered by the LLFR module. The objective is to further accommodate this satellite by the FLFR to meet the desired error specification of $e_s = 0.03(m)$. Using the low-level (LL) recovery controller with the design parameters $\lambda_0 = 2$ and $\lambda_1 = 3$, we consider the following three scenarios:

(a) All the satellites are fault-free. In this case, we have $\hat{b}_i = b_i$ ($i = 1,2,3,4$). Moreover, the optimization parameters are set equal to one another, i.e. $\alpha_i = 1$ ($i = 1,2,3,4$). The maximum tracking error obtained is quite acceptable (namely, $error = 0.0001 \ m << 0.03m = e_s$). Fig. 2-(a) shows the *x*-axis cumulative input effort, which is defined according to

$$E_{xi}(t) = \int_{\tau=0}^{t} u_{xi}^2(\tau)d\tau \qquad (i = 1,2,3,4)$$

(b) Satellite #*2* is faulty and the fault is partially recovered by the LLFR module. In this case, the optimization parameters α_i are taken from part (a), and the LL recovery controller is activated. Fig. 2-(b) shows the *x*-axis cumulative input efforts. The maximum tracking error obtained is unacceptable (namely, $error = 0.072 \ m > 0.03m = e_s$) due to the violation of the error specification e_s, and hence the faulty satellite is partially recovered by the LLFR controller. Therefore, the FLFR module is activated by the supervisor.

(c) The partially LL-recovered satellite #*2* is cooperatively accommodated by the FLFR module. In this case, all the optimization parameters α_i are taken from part (a) except for the one corresponding to the partially LL-recovered satellite ($1 < \alpha_2 \le 10$). The simulation results for α_2 versus the disturbance norm $||D||$ and the maximum tracking error e_s are shown in Fig. 3-(a) and Fig. 3-(b), respectively. These figures indicate that the undesirable perturbations and the tracking error are decreased by increasing α_2 at the formation level (FL). The analytical (for brevity, $\pi = 0.00, 0.05, 0.10, ..., 0.40$) and simulation results for α_2 versus the maximum tracking error e_s are simultaneously sketched in Fig. 4. In this figure to satisfy the error specification $e_s = 0.03(m)$, in Fig. 4, the simulation curve indicates that the minimum required parameter for α_2 is $\alpha_2 = 5.6$, whereas the analytical curves estimate it to be $\alpha_2 = 3.4$ ($\pi = 0.40$), $\alpha_2 = 5.4$ ($\pi = 0.20$), $\alpha_2 = 6.0$ ($\pi = 0.15$), and $\alpha_2 = 7.6$ ($\pi = 0.00$, most conservative result). Fig. 4 justifies the validity and effectiveness of our analytically estimated α_2 when compared with the simulation result of the desired α_2. Fig. 2-(c) and Fig. 2-(d) show the *x*-axis cumulative input efforts for the cases $\alpha_2 = 6$ and $\alpha_2 = 10$, respectively. Comparing Fig. 2-(c) and Fig. 2-(d) with Fig. 2-(b), one can conclude that the more one increases the parameter α_2 in the FLFR process, the less satellite *#2* will spend control effort, and the more other satellites will use control efforts to compensate for the deficiency of satellite *#2*. This is an interesting interpretation of the FLFR in favor of the partially LL-recovered satellite *#2*.

VI. Conclusions

The problem of fault accommodation in satellite formation flying was investigated based on a new hierarchical multi-level architecture. Two fault-recovery levels are designed in this framework, namely a low level fault recovery (LLFR) and a formation level fault recovery (FLFR). The LLFR utilizes conventional recovery methods based on fault severity estimation techniques. However, due to inexact estimation of a fault, the high level (HL) supervisor is able to detect potential violations of the mission specifications. Subsequently, the FLFR module is activated. At the formation level, the partially LL-recovered faulty satellite is further accommodated by the entire formation, at the cost of other healthy satellites spending more control efforts to compensate for the deficiency of the faulty satellite. The simulation results presented show that the FLFR module was capable of accommodating the faulty (partially LL-recovered) satellite in the formation and improved the overall formation performance.

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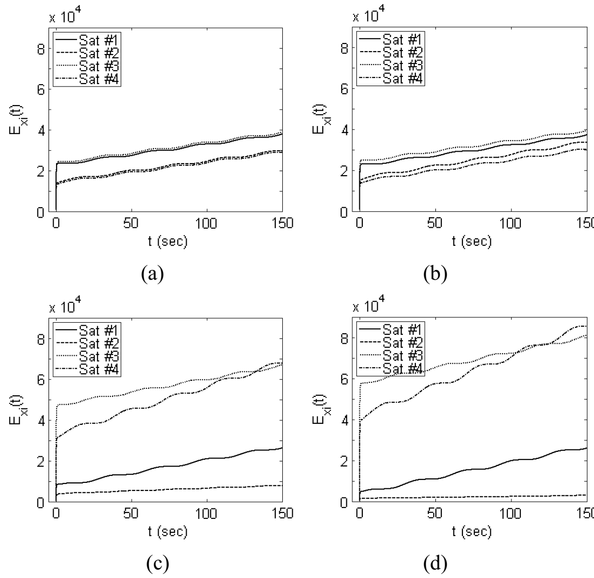

Fig. 2. The *x*-axis cumulative input effort for (a) the case of all satellites fault free, (b) faulty satellite #2 with LLFR: $\alpha_2 = 1$, (c) faulty satellite #2 with HLFR: $\alpha_2 = 6$, and (d) faulty satellite #2 with HLFR: $\alpha_2 = 10$.

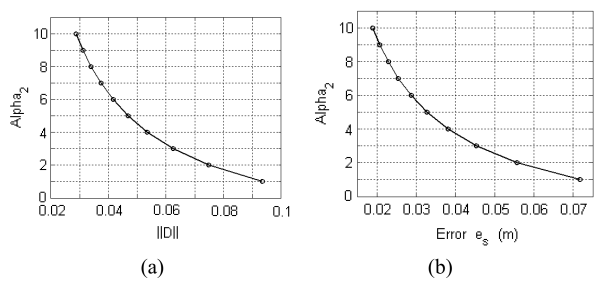

Fig. 3. Simulation results for α_2 versus (a) $||D||$ and (b) e_s.

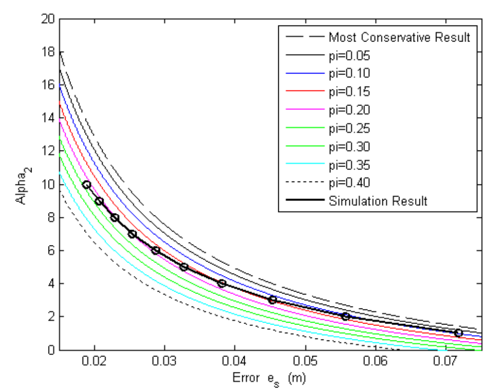

Fig. 4. The analytical and simulation results for α_2 versus the maximum tracking error e_s.