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Stabilization of Complex Switched Networks with Two Types of Delays via Impulsive Control

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Abstract—**This paper presents a general complex switched network (CSN) model that contains switching behaviors in both its nodes and topology configuration. Stabilization of such directed time-varying CSNs with two types of delays is investigated. The two types of delays consist of the system delay at each node and the coupling time-delay between nodes. Based on the Lyapunov stability theory, delay independent stabilizing conditions for CSNs with both types of delays are obtained via impulsive control. A numerical example is provided for illustration.**

I. Introduction

In recent years, complex networks (CNs) have attracted rapidly increasing attention in the scientific community, largely due to the ubiquity of CNs in sciences and societies [1, 2]. A number of important issues such as the stability, synchronization and spread mechanism of complex networks have been the subjects of extensive research in the literature [3-14, 19-22]. Although there is a great deal of existing research on the stabilization of CNs with either identical coupling delays [7-10] or heterogeneous coupling delays [11], relatively few results are available for the stabilization of complex switched networks (CSNs) with delays. In [12] the stabilization of the directed and undirected complex networks with identical coupled nodes was achieved under hybrid impulsive and switching control. Though the model in [12] contains no delay, the control framework presents a way to deal with complex networks with switching behaviors on nodes.

In fact, switching behaviors can exist not only on nodes (through their dynamical behavior), but can also occur on the network topology configuration, see, e.g., [13, 14, 23-26], in which the synchronization problems of CNs with switching topologies were studied. In this paper, a general complex switched network (CSN) model is presented. The model is more general than those in the literatures in that it contains switching behaviors on both its nodes and topology configuration. This type of networks can be found in many evolutionary processes [12], such as bursting rhythm models in pathology, optimal control models in economics and so on. Stabilization of CSNs with both the system delay of each node and the coupling time-delay is investigated under impulsive control. For simplicity, the system delays of all nodes are assumed to be identical. Similarly, the coupling delays are also assumed to be identical.

It is worth noting that in the proposed control scheme of this paper, the impulse effects from the control can be applied not only at intervals coinciding with network mode switching, but also at intervals when there is no network switching. Based on the Lyapunov stability theory, delay independent stabilizing conditions for CSNs with both node and coupling delays are obtained under impulsive control. A numerical example is provided for illustration.

II. Problem Formulation and Preliminaries

Consider a CSN with two types of delays. The CSN consists of N non-identical coupled nodes and thus is described by

$$\dot{x}_i(t)=\sum_{s=1}^{\infty}\Big[A_{i\sigma}x_i(t)+g_{i\sigma}(t,x_i)+B_{i\sigma}x_i(t-\tau_1) + \sum_{j=1}^{N}c_{ij}^{\sigma}(t)\Gamma(t)x_j(t-\tau_2)\Big]\, l_s(t), \quad i=1,2\cdots,N, \tag{1}$$

where $x_i(t)=(x_{i1},x_{i2},\cdots,x_{in})^T\in R^n$ is the state vector of node i, $t\ge t_0$. $A_{i\sigma}$ and $B_{i\sigma}$ are constant $n\times n$ matrices. $g_{i\sigma}:R_+\times R^n\to R^n$ is a nonlinear vector-valued function with $g_{i\sigma}(t,0)\equiv 0$. The constants τ_1 and τ_2 are the system delay at each node and the coupling time-delay between nodes respectively, $\tau_1,\tau_2>0$. $l_s(t)$ is the ladder function. $l_s(t)=1$ for $T_s<t\le T_{s+1}$ with discontinuity points $T_1<T_2\cdots<T_s\cdots$, $\lim_{s\to\infty}T_s=\infty$, where $T_1>t_0$ $s\in Z^+$; otherwise, $l_s(t)=0$. The switching signal

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$\sigma := \sigma(t) : R^+ \to P$, where $P = \{1, \cdots, m\}$ with constant m being the total number of the modes, and $\sigma(t) = p \in P$ for $T_s < t \le T_{s+1}$. $\Gamma(t) = (r_{ij}(t))_{n\times n} : R^+ \to R^{n\times n}$ is the inner-coupling matrix. If two coupled nodes are linked through their i th and j th state variables respectively, $r_{ij}(t) \neq 0 \; (1 \le i, j \le n)$, otherwise, $r_{ij}(t) = 0 (1 \le i, j \le n)$. $C_\sigma(t) = (c_{ij}^\sigma(t))_{N\times N}$ is the coupling configuration matrix. If there is a connection between node i and node j ($i \neq j$), $c_{ij}^\sigma(t) \neq 0$; otherwise, $c_{ij}^\sigma(t) = 0 (i \neq j)$. The diagonal elements of matrix $C_\sigma(t)$ are defined as $c_{ii}^\sigma(t) = -\sum_{j=1, j\neq i}^{N} c_{ij}^\sigma(t)$, $i = 1, \cdots, N$. Notice that the switching matrices $(A_{i\sigma}, B_{i\sigma}, C_\sigma(t))$ are allowed to take values in the finite set $\{(A_{i1}, B_{i1}, C_1(t)), \cdots, (A_{im}, B_{im}, C_m(t))\}$, $i = 1, \cdots, N$.

For the CSN (1), an impulsive controller u is designed as follows:

$$u = \sum_{k=1}^{\infty} E_k x_i(t) \delta(t - t_k), \tag{2}$$

where E_k are $n \times n$ constant matrices, and $\delta(t - t_k)$ is the Dirac impulse function, with discontinuous points $t_1 < t_2 < \cdots < t_k < \cdots$, where $t_1 > t_0$ and $\lim_{k\to\infty} t_k = \infty$.

Suppose that for any switching instant T_s, there exist two positive integers k and p such that $t_k = T_s < t_{k+1} < \cdots < t_{k+p} = T_{s+1}$, where t_k, …, t_{k+p} are the impulsive instants.

From (1) and (2), the controlled CSN can be derived as:

$$\begin{cases} \dot{x}_i(t) = A_{i\sigma} x_i + B_{i\sigma} x_i(t - \tau_1) + g_{i\sigma}(t, x_i) \\ + \sum_{j=1}^{N} c_{ij}^\sigma(t) \Gamma(t) x_j(t - \tau_2), \qquad t \in (t_{k-1}, t_k], \\ \Delta x_i = x_i(t_k^+) - x(t_k) = E_k x_i, \quad t = t_k, \\ x_i(t_0^+) = x_{i0}, \qquad k = 1,2,3,\cdots, i = 1,2,\cdots,N. \end{cases} \tag{3}$$

It is worth noting that (3) is a general form of the networks studied in [7], [8], [12] and [13].

The following lemmas are needed in the subsequent discussion.

Lemma 1 [15]**:** If X and Y are real matrices with appropriate dimensions, there exists a constant $\varepsilon > 0$ such that

$$X^T Y + Y^T X \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y.$$

Lemma 2 [16]**:** If $P \in R^{n\times n}$ is a positive definite matrix, $Q \in R^{n\times n}$ is a symmetric matrix, then

$$\lambda_{\min}(P^{-1}Q) x^T P x \le x^T Q x \\ \le \lambda_{\max}(P^{-1}Q) x^T P x, x \in R^n,$$

where $\lambda_{\min}(*)$ and $\lambda_{\max}(*)$ are the minimum and maximum eigenvalues of $*$, respectively.

III. Stabilization of the Directed CSN with Delays

In this section, sufficient stabilizing conditions for the CSN (3) are derived. The notations $P_{i\sigma}$ denote some positive-definite matrices with $P_{i\sigma} = P_{i\sigma}^T$. $\|x(t)\|_2^2 = x_1^2(t) + \cdots x_n^2(t)$. Similar to references [16] and [17], the following assumptions are made for our result.

Assumption 1: For $t \ge t_0$, $x_i \in R^n$, $i = 1,2,\cdots,N$, there exist continuous functions $\varphi_{i\sigma}(t) \ge 0$, such that

$$g_{i\sigma}^T(t, x_i) P_{i\sigma} x_i(t) \le \varphi_{i\sigma}(t) x_i^T(t) P_{i\sigma} x_i(t).$$

Assumption 2: For $t \ge t_0$, there exist positive constants ρ_i such that $\|x_i(t - \tau)\|^2 \le \rho_i \|x_i(t)\|^2$, $i = 1, \cdots, N$, hold for $\tau = \tau_1$ and for $\tau = \tau_2$.

Remark 1: Though the constants ρ_i, $i = 1, \cdots, N$, in Assumption 2 need to be derived from the simulation in some cases, which might be restrictive for these cases in practice, this assumption is reasonable for many other systems, especially for those of which the bounds or the monotonies of states are known in advance.

Assumption 3: If the system states $x_i(t)$, $i = 1, \cdots, N$, are in the σ mode of node i, the delayed system states $x_i(t - \tau)$ are in the $\sigma - \mathrm{mode}(\tau)$ mode of node i, where $[\sigma - \mathrm{mode}(\tau)] := \sigma(t - \tau)$.

For convenience, define

$$\mu_i = \frac{\min_{1\le\sigma\le m}\{\lambda_{\min}(P_{i\sigma})\}}{\rho_i \max_{1\le\sigma\le m}\{\lambda_{\max}(P_{i(\sigma - \mathrm{mode}(\tau_1))})\}}, \tag{4}$$

$$\mu = \min_{1\le i\le N}(\mu_i), \tag{5}$$

$$\rho = \frac{\max_{1\le i\le N, 1\le\sigma\le m} \lambda_{\max}(P_{i\sigma})}{\min_{1\le i\le N, 1\le\sigma\le m} \lambda_{\min}(P_{i\sigma})}, \tag{6}$$

$$\eta_k = \lambda_{\max}[(I+E_k)^T(I+E_k)],\ k=1,2,\cdots, \tag{7}$$

$$\gamma(t) = \max_{\substack{1\le i\le N\\ 1\le\sigma\le m}}\{\lambda_{\max}\ [P_{i\sigma}^{-1}(P_{i\sigma}A_{i\sigma}+A_{i\sigma}^T P_{i\sigma}+2\varphi_{i\sigma}(t)P_{i\sigma} + \varepsilon_2^{-1}I)]\}, \tag{8}$$

$$\theta = \varepsilon_2 \max_{\substack{1\le i\le N\\ 1\le\sigma\le m}}\{\lambda_{\max}[P_{i(\sigma-\text{mode}(\tau_1))}^{-1}(B_{i\sigma}^T P_{i\sigma}P_{i\sigma}B_{i\sigma})]\}, \tag{9}$$

$$\vartheta(t) = \varepsilon_1 \max_{\substack{1\le i\le N\\ 1\le\sigma\le m}}\left\{\sum_{j=1}^{N}|c_{ji}^{\sigma}(t)|\lambda_{\max}(P_{i(\sigma-\text{mode}(\tau_2))}^{-1})\right\}, \tag{10}$$

where $\xi_l > 0$ and $\varepsilon_l > 0$, $l=1,2$ are constants, and

$$\hat{\gamma}(t) = \gamma(t) + (\theta+\vartheta(t))\mu^{-1}. \tag{11}$$

Theorem 1: Suppose that Assumptions 1-3 are satisfied and $\eta_k \ge 0$ holds, and there exist two positive integers k and p such that $t_k = T_s < t_{k+1} < \cdots < t_{k+p} = T_{s+1}$.

i) If there exist two constants α, β, satisfying $\beta \ge \alpha \ge 0$ and $\hat{\gamma}(t) \le -\beta < 0$, such that

$$\ln(\rho\eta_k) - \alpha(t_k - t_{k-1}) \le 0, k=1,2,\cdots, \tag{12}$$

then, the CSN (1) is globally exponentially stable under the control of (2).

i) If $\hat{\gamma}(t) \ge 0$ and there exists a constant $\alpha \ge 1$ such that

$$\ln(\alpha\rho\eta_k) + \int_{t_k}^{t_{k+1}}\hat{\gamma}(s)ds \le 0, k=1,2,\cdots, \tag{13}$$

then, $\alpha = 1$ implies that the CSN (1) is stable under the control of (2), and $\alpha > 1$ implies that the CSN (1) is asymptotically stable under the control of (2).

Proof. Construct a Lyapunov candidate as

$$V(t) = \sum_{i=1}^{N} x_i^T(t)P_{i\sigma}x_i(t). \tag{14}$$

It is obviously that $P_{i\sigma}$ remains identical for $t \in (t_{k-1}, t_k]$, therefore, for any $t \in (t_{k-1}, t_k]$, the total derivative of $V(t)$ with respect to (3) is

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N}\{\dot{x}_i^T(t)P_{i\sigma}x_i(t) + x_i^T(t)P_{i\sigma}\dot{x}_i(t)\}\\ &= \sum_{i=1}^{N}\{x_i^T(t)(P_{i\sigma}A_{i\sigma}+A_{i\sigma}^T P_{i\sigma})x_i(t)\\ &\quad + 2x_i^T(t)P_{i\sigma}g_{i\sigma}(t,x_i) + x_i^T(t)P_{i\sigma}B_{i\sigma}x_i(t-\tau_1)\\ &\quad + \sum_{j=1}^{N}c_{ij}^{\sigma}(t)[x_i^T(t)P_{i\sigma}\Gamma(t)x_j(t-\tau_2)\\ &\quad + x_i^T(t-\tau_1)B_{i\sigma}^T P_{i\sigma}x_i(t) + x_j^T(t-\tau_2)\Gamma^T(t)P_{i\sigma}x_i(t)]\}\end{aligned}$$

According to Lemma 1 and Assumption 1, there exist two constants $\varepsilon_1, \varepsilon_2 > 0$ and continuous functions $\varphi_{i\sigma}(t) \ge 0$, such that

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\{x_i^T(t)(P_{i\sigma}A_{i\sigma}+A_{i\sigma}^T P_{i\sigma}+2\varphi_{i\sigma}(t)P_{i\sigma})x_i(t)\\ &\quad + \varepsilon_2 x_i^T(t-\tau_1)B_{i\sigma}^T P_{i\sigma}P_{i\sigma}B_{i\sigma}x_i(t-\tau_1) + \varepsilon_2^{-1}x_i^T(t)x_i(t)\\ &\quad + \sum_{j=1}^{N}|c_{ij}^{\sigma}(t)|[\varepsilon_1 x_j^T(t-\tau_2)x_j(t-\tau_2)\\ &\quad + \varepsilon_1^{-1}x_i^T(t)P_{i\sigma}\Gamma(t)\Gamma^T(t)P_{i\sigma}x_i(t)]\}\end{aligned}$$

For the directed CSN, since $\sum_{j=1}^{N}c_{ij}^{\sigma}(t) = 0$, the following equations can be obtained:

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}^{\sigma}(t)\varepsilon_1^{-1}x_i^T(t)P_{i\sigma}^T\Gamma(t)\Gamma^T(t)P_{i\sigma}x_i(t)\\ &\quad = \sum_{i=1}^{N}(\varepsilon_1^{-1}x_i^T(t)P_{i\sigma}\Gamma(t)\Gamma^T(t)P_{i\sigma}x_i(t))\sum_{j=1}^{N}c_{ij}^{\sigma}(t) = 0,\\ &\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}^{\sigma}(t)\varepsilon_1 x_j^T(t-\tau_2)x_j(t-\tau_2)\\ &\quad = \sum_{i=1}^{N}\sum_{j=1}^{N}c_{ji}^{\sigma}(t)\varepsilon_1 x_i^T(t-\tau_2)x_i(t-\tau_2).\end{aligned}$$

Therefore, from Lemma 2,

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\{x_i^T(t)(P_{i\sigma}A_{i\sigma}+A_{i\sigma}^T P_{i\sigma}+2\varphi_{i\sigma}(t)P_{i\sigma}+\varepsilon_2^{-1}I)\\ &\quad \cdot x_i(t) + \varepsilon_2 x_i^T(t-\tau_1)B_{i\sigma}^T P_{i\sigma}P_{i\sigma}B_{i\sigma}x_i(t-\tau_1)\\ &\quad + \sum_{j=1}^{N}|c_{ji}^{\sigma}(t)|\varepsilon_1 x_i^T(t-\tau_2)x_i(t-\tau_2)\}\\ &\le \gamma(t)V(t) + \theta V(t-\tau_1) + \vartheta(t)V(t-\tau_2)\end{aligned}$$

From Assumption 2,

$$\begin{aligned}V(t) &\ge \sum_{i=1}^{N}\lambda_{\min}(P_{i\sigma})x_i^T(t)x_i(t)\\ &\ge \sum_{i=1}^{N}\mu_i \max_{1\le\sigma\le m}\{\lambda_{\max}(P_{i(\sigma-\text{mode}(\tau_1))})\}x_i^T(t-\tau_1)x_i(t-\tau_1)\\ &\ge \sum_{i=1}^{N}\mu_i x_i^T(t-\tau_1)P_{i(\sigma-\text{mode}(\tau_1))}x_i(t-\tau_1)\\ &\ge \mu V(t-\tau_1),\end{aligned}$$

where μ_i and μ are defined in (4) and (5) respectively. Similarly, $V(t) \ge \mu V(t-\tau_2)$. Thus,

$$\dot{V}(t) \le [\gamma(t) + (\theta+\vartheta(t))\mu^{-1}]V(t) \le \hat{\gamma}(t)V(t),$$

which implies that

$$V(t) \le V(t_{k-1}^{+}) \exp \int_{t_{k-1}}^{t} \hat{\gamma}(s)ds, \quad t \in (t_{k-1}, t_k]. \qquad (15)$$

Since $P_{i\sigma}$ remains identical for $t \in (t_{k-1}, t_k]$, from (14) and (15), for $t \in (t_{k-1}, t_k]$,

$$\max_{\substack{1 \le i \le N \\ 1 \le \sigma \le m}} \{\lambda_{\max}(P_{i\sigma})\} \sum_{i=1}^{N} x_i^T(t_{k-1}^{+}) x_i(t_{k-1}^{+}) \exp \int_{t_{k-1}}^{t} \hat{\gamma}(s)ds$$

$$\ge \min_{\substack{1 \le i \le N \\ 1 \le \sigma \le m}} \{\lambda_{\min}(P_{i\sigma})\} \sum_{i=1}^{N} x_i^T(t) x_i(t),$$

or

$$\varpi(t) \le \rho \varpi(t_{k-1}^{+}) \exp \int_{t_{k-1}}^{t} \hat{\gamma}(s)ds, t \in (t_{k-1}, t_k], \qquad (16)$$

where $\varpi(t) = \sum_{i=1}^{N} x_i^T(t) x_i(t)$, and ρ is defined in (6).

On the other hand, when $t = t_k^{+}$,

$$\varpi(t_k^{+}) = \sum_{i=1}^{N} x_i^T(t_k^{+}) x_i(t_k^{+})$$

$$\le \lambda_{\max}[(1+E_k)^T(1+E_k)] \sum_{j=1}^{N} x_i^T(t_k) x_i(t_k)$$

$$\le \eta_k \varpi(t_k), k = 1,2,\cdots \qquad (17)$$

From (16) and (17), for any $t \in (t_0, t_1]$,

$$\varpi(t) \le \rho \varpi(t_0^{+}) \exp \int_{t_0}^{t} \hat{\gamma}(s)ds,$$

which leads to $\varpi(t_1) \le \rho \varpi(t_0^{+}) \exp \int_{t_0}^{t_1} \hat{\gamma}(s)ds$ and $\varpi(t_1^{+}) \le \eta_1 \varpi(t_1) \le \rho \eta_1 \varpi(t_0^{+}) \exp \int_{t_0}^{t_1} \hat{\gamma}(s)ds$.

Similarly, for $t \in (t_1, t_2]$,

$$\varpi(t) \le \rho \varpi(t_1^{+}) \exp \int_{t_1}^{t} \hat{\gamma}(s)ds$$

$$\le \rho^2 \eta_1 \varpi(t_0^{+}) \exp[\int_{t_0}^{t_1} \hat{\gamma}(s)ds] \exp[\int_{t_1}^{t} \hat{\gamma}(s)ds]$$

$$= \rho^2 \eta_1 \varpi(t_0^{+}) \exp \int_{t_0}^{t} \hat{\gamma}(s)ds.$$

In general, for $t \in (t_k, t_{k+1}]$,

$$\varpi(t) \le \rho^{k+1} \varpi(t_0^{+}) \eta_1 \cdots \eta_k \exp \int_{t_0}^{t} \hat{\gamma}(s)ds. \qquad (18)$$

i) If there exist two constants α, β satisfying $\beta \ge \alpha \ge 0$ and $\hat{\gamma}(t) \le -\beta < 0$, for $t \in (t_{k-1}, t_k]$, it follows from (18) that

$$\varpi(t) \le \varpi(t_0^{+}) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp[-\beta(t-t_0)]$$

$$\le \varpi(t_0^{+}) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp[-\alpha(t_{k-1}-t_0)]$$
$$\cdot \exp[-(\beta-\alpha)(t-t_0)]$$
$$= \rho \varpi(t_0^{+}) \rho \eta_1 \exp[-\alpha(t_1-t_0)] \rho \eta_2 \exp[-\alpha(t_2-t_1)]$$
$$\cdots\cdots \rho \eta_{k-1} \exp[-\alpha(t_{k-1}-t_{k-2})]$$
$$\cdot \rho \eta_k \exp[-(\beta-\alpha)(t-t_0)].$$

If (12) holds, then, $\rho \eta_k \exp[-\alpha(t_k - t_{k-1})] \le 1$, $k = 1,2,\cdots$. Therefore,

$$\varpi(t) \le \varpi(t_0^{+}) \rho^2 \eta_k \exp[-(\beta-\alpha)(t-t_0)],$$
$$t \in (t_k, t_{k+1}].$$

Since ρ and η_k are bounded constants, it can be concluded that the trivial solution of (3) is globally exponentially stable.

ii) If $\hat{\gamma}(t) \ge 0$, and there exists a constant $\alpha \ge 1$, from (18), it follows that

$$\varpi(t) \le \varpi(t_0^{+}) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp \int_{t_0}^{t} \hat{\gamma}(s)ds$$

$$\le \varpi(t_0^{+}) \rho^{k+1} \eta_1 \eta_2 \cdots \eta_k \exp \int_{t_0}^{t_{k+1}} \hat{\gamma}(s)ds$$

$$= \varpi(t_0^{+}) \rho \alpha^{-k} \exp[\int_{t_0}^{t_1} \hat{\gamma}(s)ds]$$

$$\cdot \rho \eta_1 \alpha \exp[\int_{t_1}^{t_2} \hat{\gamma}(s)ds] \cdots \rho \eta_k \alpha \exp[\int_{t_k}^{t_{k+1}} \hat{\gamma}(s)ds].$$

If (13) holds, then $\alpha \rho \eta_k \exp\left[\int_{t_k}^{t_{k+1}} \hat{\gamma}(s)ds\right] \le 1$, $k = 1,2,\cdots$, therefore,

$$\varpi(t) \le \varpi(t_0^{+}) \rho \alpha^{-k} \exp[\int_{t_0}^{t_1} \hat{\gamma}(s)ds], t \in (t_k, t_{k+1}].$$

Then, $\alpha = 1$ implies that the trivial solution of the trivial solution of (3) is stable, and $\alpha > 1$ implies that the trivial solution of (3) is asymptotically stable.

The proof is thus completed.

IV. Illustrative Example

Consider a nearest-neighbor coupled network with 100 coupled nodes, in which each node is a Lorenz chaotic system [18] with delays.

In the simulation, let the switching signal $\sigma(s) = 1,2$, and the switching interval $T_s - T_{s-1} = \tau_{switch} = 0.02$. Then, the CNS with two types of delays $\tau_1 = 0.5$ and $\tau_2 = 0.04$ can be described by (1) with $x_i = (x_{i1}, x_{i2}, x_{i3})^T$, $g_{i\sigma}(t, x_i) = (0, -x_{i1}x_{i3}, x_{i1}x_{i2})^T$, and $\Gamma = I$. In the simulation, for any $h = 0,1,2,\cdots$, if $t \in (0.02(2h), 0.02(2h+1)]$, $\sigma(s) = 1$, $C_\sigma(t) = C_1(t)$, each isolated node is a time-delay Lorenz chaos system when

$$A_{i1}=\begin{bmatrix}-10 & 10 & 0\\ 5 & 5.5 & 0\\ 0 & 0 & -8/3\end{bmatrix} \text{ and } B_{i1}=\begin{bmatrix}0\\ 6.5\\ 0\end{bmatrix}; \text{ and if}$$

$t\in(0.02(2h+1),0.02(2h+2)]$, $\sigma(s)=2$, $C_\sigma(t)=C_2(t)$, all isolated nodes are Lorenz chaos systems

when $A_{i2}=\begin{bmatrix}-10 & 10 & 0\\ 28 & -1 & 0\\ 0 & 0 & -8/3\end{bmatrix}$, and $B_{i2}=0$, where

$$C_1(t)=\begin{bmatrix}-1-\sin t & 1 & \sin t & 0 & \cdots & 0\\ 1 & \ddots & 0 & \ddots & \ddots & 0\\ 0 & 1 & \ddots & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & -1-\sin t & 0 & \sin t\\ 0 & \ddots & \ddots & 1 & -2 & 1\\ 1 & 0 & \cdots & 0 & 1 & -2\end{bmatrix}_{100\times 100}$$

$C_2(t)=2\ C_1(t)$.

Notice that when $t\in(0,\tau_1]$ or $t\in(0,\tau_2]$, $x_i(t-\tau_1)=0$ or $x_i(t-\tau_2)=0$. The initial conditions of nodes are increasing from $[0,0.5,1]^T$ to $[148.5,149,149.5]^T$.

For the sake of simplicity, let $E_k=diag\{-0.69,-0.69,-0.69\}$ and $P_{i\sigma}=I$. Thus, $\eta_k=0.0961>0$, $g_{i\sigma}^T(t,x_i)P_{i\sigma}x_i=0$ and $\rho=1$. Through simulation, it can be obtained that $\|x_i(t)\|_2^2\ge 0.7914\|x_i(t-\tau_l)\|_2^2$, $l=1,2$. Then, from Assumption 2, $1/\rho_i\ge 0.7914$. From (5), $\mu=0.7914$. Let $\varepsilon_1=1$, $\varepsilon_2=0.01$, $\xi_1=\xi_2=1$, and then $\gamma(t)=128.0512$, $\theta=0.4225$, $\vartheta(t)=8$, $\hat{\gamma}(t)=138.6938>0$. Choose $\alpha=1.02$, then

$$t_k-t_{k-1}\le-\frac{\ln(\alpha\rho\eta_k)}{\max\limits_{t\in[0,\infty)}\hat{\gamma}(t)}=0.0167. \text{ Let}$$

$t_k-t_{k-1}=\tau_{impulsive}$ be a constant. From Theorem 1, if $p\tau_{impulsive}=\tau_{switch}$, the impulsive controller u defined by (2) can asymptotically stabilize CSN (1). In the simulation, let $t_k-t_{k-1}=0.01$, i.e., $p=2$, the states variations of the 100 nodes are shown in Fig.1. Note that the first impulse is added when $t=0.6$.

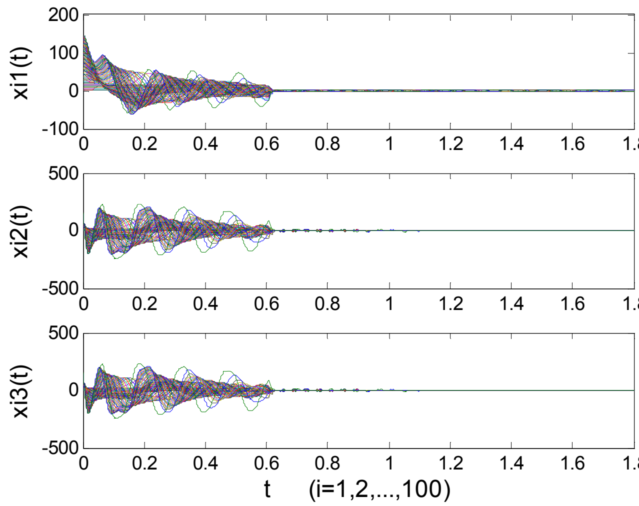

Fig.1. States of the controlled CSN with $\tau_1=0.5$, $\tau_2=0.04$.

From the simulation result, it can be observed that the proposed impulsive scheme is effective to achieve asymptotical stabilization of the CSN with two types of delays.

V. Conclusion

This paper has presented a complex switched network (CSN) model that is more general than those in the literature. The CSN model features switching behaviors in both its nodes and topology configuration. Stabilization of such CSNs with both system delays at nodes and coupling time-delays between nodes has been considered. Based on the Lyapunov stability theory, delay independent stabilizing conditions for CSNs with both types of delays are obtained via an impulsive control framework. A numerical example illustrates the effectiveness of the control method.

References

[1] S. Boccaletti, V. Latora, Y. Morenod, M. Chavez, D.-U. Hwang, "Complex networks: Structure and dynamics," *Physics Reports*, Vol. 424, pp. 175 – 308, 2006.

[2] S.H. Strogatz, "Exploring complex networks," *Nature*, Vol. 410 , pp.268-276, 2001.

[3] I.C. Lestas, G. Vinnicombe, "Scalable robust stability for nonsymmetric heterogeneous networks", *Automatica*, Vol. 43, pp. 714 – 723, 2007.

[4] J. Xiang , G. Chen, "On the V-stability of complex dynamical networks", *Automatica*, Vol. 43, pp. 1049-1057, 2007.

[5] T. Chen, X. Liu, W. Lu, "Pinning Complex Networks by a Single Controller", *IEEE Trans. Cir. & Sys. -I*, Vol. 54, pp. 1317-1326, 2007.

[6] Z. Duan, J. Wang, G. Chen, L. Huang, "Stability analysis and decentralized control of a class of complex dynamical networks," *Automatica*, vol. 44, pp. 1028-1035, 2008.

[7] J. Yao, Z. H. Guan, D. J. Hill, H. O. Wang, "On passivity and impulsive control of complex dynamical networks with coupling delays," *Proceedings of the 44rd IEEE Conference on Decision and Control and European Control Conference*, pp.1595-1600, December. 2005.

[8] Z. Li, G. Chen, "Global Synchronization and Asymptotic Stability of Complex Dynamical Networks", *IEEE Trans. Circuits Syst., II*, Vol. 53, pp. 28–33, 2006.

[9] Z.X. Liu, Z.Q. Chen, Z.Z. Yuan, “Pinning control of weighted general complex dynamical networks with time delay”, *Physica A* Vol. 375 pp. 345–354, 2007.
[10] X. Gong, S. Guan, X. Wang, C.-H. Lai, "Stability of the steady state of delay-coupled chaotic maps on complex networks", *PHYSICAL REVIEW E* Vol. 77, pp. 056212, 2008.
[11] L. Xiang, Z. Liu, Z. Chen, Z. Yuan, “Pinning weighted complex networks with heterogeneous delays by a small number of feedback controllers”, *Science in China Series F*, Vol. 51, pp. 511-523, 2008.
[12] Z-H Guan, H. Zhang, “Stabilization of complex network with hybrid impulsive and switching control,” *Chaos, Solitons & Fractals*, Vol 37, pp.1372-1382, 2008.
[13] J. Yao, D.J. Hill, Z.-H. Guan, H.O. Wang, “Synchronization of Complex Dynamical Networks with Switching Topology via Adaptive Control”, *Proceedings of the 45th IEEE Conference on Decision & Control*, pp.2819 -2824, December. 2006.
[14] A. Papachristodoulou, A. Jadbabaie, “Synchronization in Oscillator Networks: Switching Topologies and Non-homogeneous Delays”, *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005*, Seville, Spain, December 12-15, 2005
[15] Y. Wang, L. Xie, C. E.de Souza , “Robust control of a class of uncertain nonlinear systems,” *Systems and Control Letters*, Vol.19, pp.139-149,1992.
[16] Z-H Guan, D.J. Hill, X. Shen, “On Hybrid Impulsive and Switching Systems and Application to Nonlinear Control,” *IEEE Trans Automatic Control*, Vol. 50, pp. 1058-1062, 2005.
[17] H. Xu, X. Liu, Kok Lay Teo, “Delay independent stability criteria of impulsive switched systems with time-invariant delays,” *Mathematical and Computer Modelling,* Vol 47, pp 372-379, 2008.
[18] J. Lü, G. Chen, D. Cheng, S. Celikovsky, “Bridge the gap between the Lorenz system and the Chen system,” *Int. J. Bifurcation and Chaos*, Vol.12, pp.2917-2926, 2002.
[19] P. Checco, M. Biey, L. Kocarev, “Synchronization in random networks with given expected degree sequences”, *Chaos, Solitons and Fractals* Vol. 35, pp. 562–577, 2008.
[20] M. Chen, “Chaos Synchronization in Complex Networks”, *IEEE Trans. Cir. & Sys. -I*, Vol. 55, pp. 1335-1346, 2008.
[21] X.-J. Xua, Z.-X. Wu, G. Chen, “Epidemic spreading in lattice-embedded scale-free networks”, *Physica A* Vol. 377, pp. 125–130, 2007.
[22] Christopher Griffin and Richard Brooks, “A Note on the Spread of Worms in Scale-Free Networks”, *IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART B: CYBERNETICS*, Vol. 36, pp. 198-202, 2006.
[23] Maurizio Porfiri, Daniel J. Stilwell, Erik M. Bollt, and Joseph D. Skufca. Random talk: Random walk and synchronizability in a moving neighborhood network, Physica D: Nonlinear Phenomena, Vol. 224, pp. 102-113, 2006.
[24] Igor V. Belykh, Vladimir N. Belykh, Martin Hasler. Blinking model and synchronization in small-world networks with a time-varying coupling, Physica D: Nonlinear Phenomena, Volume 195, Issues 1-2, 1 August 2004, Pages 188-206
[25] Paul So, Bernard C. Cotton, and Ernest Barreto. Synchronization in interacting populations of heterogeneous oscillators with time-varying coupling, Chaos, 18, 037114 (2008)
[26] D.J. Stilwell, E.M. Bollt and D.G. Roberson, Sufficient conditions for fast switching synchronization in time-varying network topologies, SIAM J. Appl. Dynam. Syst. 6 (2006) (1), pp. 140–156