2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

H_∞ Closed–Loop Control for Unstable Uncertain Discrete Input–Shaped Systems

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***Abstract*— This article focuses on the design of a discrete robust H_∞ controller for an input–shaped, underdamped, (stable or unstable) system. The system is noiseless, uncertain, time–invariant and its characteristic polynomial coefficients are allowed to vary within predefined interval–sets. A discrete input shaper is used for the nominal system generating a pre–compensated discrete Finite Impulse Response shaped system. The shaped system's description is reformulated in order to be amenable to a robust controller synthesis. The designed H_∞ controller compensates for the uncertainties of the system's parameters while providing the control command for stabilizing the system.**

***Index Terms*— Input shaping, H_∞ robust control**

I. INTRODUCTION

Input shaping is a technique primarily used for the suppression of residual vibration in oscillatory underdamped linear systems [1]. Due to its design simplicity and ease of implementation, it has become a quite popular solution for obtaining vibrationless responses of such systems. Robustness issues have been examined in [2], where only reasonable estimates of the natural frequency and damping factor of the oscillations are needed to be known. Metrics for the system's transient performance are provided in [3].

Modifications have been made to the classical scheme when the input to the system is constrained [4], while input shaping for discrete plants is examined in [5]. Analysis of the time–optimality when using negative shapers has been performed in [6, 7]. Although input shaping technique targets linear time–invariant systems, its application to nonlinear or time–varying ones needs special treatments as proposed in [8–10]. In such cases, the system is linearized in an infinite number of operating points and the shaper is tuned according to the current plant's state.

In cases where the plant under control is completely unknown, adaptive versions of the classical techniques have been proposed. Online identification and tuning schemes are proposed in [11, 12], where the shaper is tuned online according to the currently identified frequency response of the system. For plants controlled in closed loop, modifications have been made in order to tune both the shaper and the controller [13, 14].

The major disadvantage of input shaping is that it acts in open loop; thus the system is very sensitive to: a) effects from disturbances, and b) uncertainties in the model when the plant is unstable. A robust control scheme for continuous–time stable plants has been proposed by [15], where the loop is closed around the shaper, in order to deal with uncertainties in the system model. The case of input shaping control for uncertain systems has also been examined in [16]; however the shaping filter is not inserted in the main loop, but is responsible for vibration suppression of the closed–loop system.

In this paper, a robust control scheme for discrete unstable systems is proposed to handle modeling uncertainties. An H_∞ controller is designed to close the loop around the input shaped plant, while stability of the closed–loop system is guaranteed. The main difference of this work compared to previous ones is the insertion of the shaping filter in the main loop, rather than as a feedforward filter [17]. This issue needs careful attention, since even if the open loop system is stable, the resulting one may be easily driven unstable.

In Section II the relationship between discrete and continuous–time LTI systems is analyzed. In Section III the classical input shaping technique, along with the discrete version are presented, while in Section IV the design of an H_∞ controller for the stabilization of uncertain discrete input–shaped systems is discussed. Finally, in Section V simulation results are provided in order to prove the efficacy of the proposed controller in comparison to those obtained by standard input shaping open–loop control. Concluding remarks are provided in the last section.

II. SYSTEM DESCRIPTION

Consider the discrete SISO LTI plant described as:

$$G\left(z^{-1}\right)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}=\prod_{i=1}^{n}G_i\left(z^{-1}\right)=\prod_{i=1}^{n}\frac{B_i\left(z^{-1}\right)}{A_i\left(z^{-1}\right)}, \quad (1)$$

where

$$B_i\left(z^{-1}\right)=\sum_{j=1}^{m_i}b_{j,i}z^{-j}, \quad A_i\left(z^{-1}\right)=1+a_{1,i}z^{-1}+a_{2,i}z^{-2}. \ (2)$$

The number of modes, n, and the B_i polynomials are assumed to be known a–priori.

If the coefficients $a_{1,i}, a_{2,i}$ are known exactly, then the plant is a nominal one, consisted of $G_i^0(z^{-1})$, where *0* denotes the nominal case. In this paper the A_i–polynomials coefficients are considered to be constant but unknown, residing inside a rectangle in $\mathbb{R}^{2n}$. The A_i–polynomials in (1)–(2) are considered to have underdamped eigenvalues $\rho_{z_1,i}, \rho_{z_2,i}$, which may lay either inside or out of the unit circle in the z–plane. By this assumption, both stable and unstable systems are considered.

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The coefficients $a_{1,i}, a_{2,i}$ of polynomials A_i can be associated with the natural frequencies ω_i and damping ratios ζ_i of their equivalent s–plane eigenvalues, as:

$$\begin{aligned}
\rho_{z,i} &= -\frac{a_{1,i}}{2} \pm j\sqrt{a_{2,i} - \frac{a_{1,i}^2}{4}}, &(3)\\
\omega_i &= \left|\ln\left(\rho_{z,i}\right)\right|, &(4)\\
\zeta_i &= -\cos\left(\arctan\left(\frac{\Im\left\{\ln(\rho_{z,i})\right\}}{\Re\left\{\ln(\rho_{z,i})\right\}}\right)\right), &(5)
\end{aligned}$$

where $\Im$ and $\Re$ denote the imaginary and real part of a complex variable, respectively. It should be noted that $-1 < \zeta_i < 1$, where negative (positive) values of ζ_i are for unstable (stable) systems. The inverse mapping $(\omega_i, \zeta_i) \rightarrow (a_{1,i}, a_{2,i})$ can similarly be computed via the following formulae:

$$\begin{aligned}
\rho_{z,i} &= e^{-\zeta_i \omega_i \pm j\omega_i \sqrt{1-\zeta_i^2}}, &(6)\\
a_{1,i} &= -2\Re\left\{\rho_{z,i}\right\}, &(7)\\
a_{2,i} &= \Re\left\{\rho_{z,i}\right\}^2 + \Im\left\{\rho_{z,i}\right\}^2. &(8)
\end{aligned}$$

The plant (1) is underdamped and stable at the same time when $0 < \zeta_i < 1$ and $0 < \omega_i < \omega_N$, $\forall i = 1, \ldots n$, where ω_N is the underlying Nyquist frequency. On the contrary, if any of the ζ_i is negative, i.e. $\exists i \in \{1, \ldots n\}$: $-1 < \zeta_i < 0$, then the system is unstable. Furthermore, according to Jury's criterion, the condition for (1) to be stable is: $(a_{1,i}, a_{2,i}) \in \Omega = \{(a_{1,i}, a_{2,i}) \in \mathbb{R}^2 \mid (-1 < a_{2,i} < 1) \cap (a_{2,i} > a_{1,i} - 1) \cap (a_{2,i} > -a_{1,i} - 1)\}$, $\forall i$.

For an underdamped plant, the variations of the corresponding natural frequencies ω_i and damping ratios ζ_i are shown in the top and bottom part of Fig. 1, respectively. For a stable plant, it can be seen that the values of ω_i are large for positive values of $a_{1,i}$, while negative values of $a_{1,i}$ correspond to large values of ζ_i. As for the unstable–plant case, the highly nonlinear relationship between the ω_i, ζ_i and $a_{1,i}, a_{2,i}$ is profound due to the large admissible range of the latter.

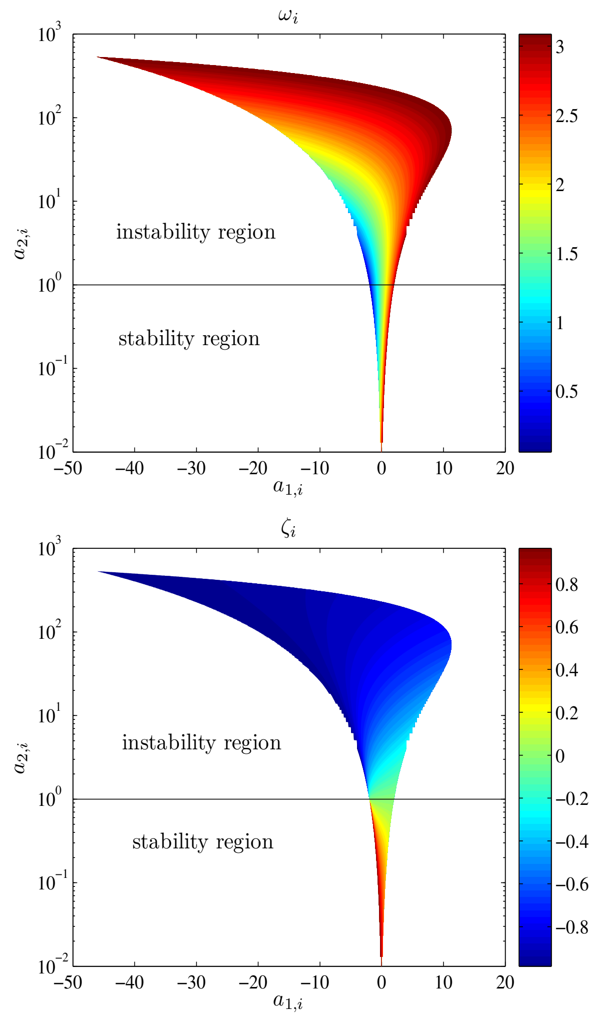

Fig. 1. Variations of ω_i (top), ζ_i (bottom) for an underdamped plant w.r.t. $a_{1,i}, a_{2,i}$. (semi–logarithmic scale)

III. INPUT SHAPING TECHNIQUE FOR DISCRETE SYSTEMS

A. Standard input shaping technique for continuous–time systems

The classical input shaping technique is used in most of the cases where the system to be controlled has an oscillatory response due to lightly–damped eigenvalues. A shaper is a pre–compensator consisted of a number of impulses with which the reference signal is convolved, so that the system reaches the desired state with no (or minimum) oscillation.

In cases where the system under control has multiple modes—like the one defined in (1)—then for each mode i a shaper is designed, based on the estimates of ω_i and ζ_i in order to cancel the oscillating effect of the current mode. The n shapers are then convolved with each other to result in a train of $\prod_{i=1}^{n} N_i$ impulses, where N_i are the number of impulses of each shaper. It should be noted that the number of impulses in each shaper design can be different between each other, depending on the amount of uncertainty in the current mode parameters ω_i, ζ_i. An alternative method for dealing with multi–mode systems is to solve all vibration constraints at the same time, leading in shapers with less number of impulses [18, 19].

A standard shaper of N_i–impulses filters its input according to the following equation:

$$r_{sh,i}(t) = r_i(0) + \sum_{j=1}^{N_i} A_{j,i}\left(r_i(t - t_{j,i}) - r_i(0)\right), \qquad (9)$$

where $r_i(t)$ is the reference input signal to the i–th shaper at time t, N_i is the length of the impulse train, $A_{j,i}$ and $t_{j,i}$ are the amplitudes and time instances of each impulse. The latter can be computed by [15]:

$$\begin{aligned}
A_{j,i} &= \frac{\binom{N_i-1}{j-1} K_i^{j-1}}{\sum_{\ell=0}^{N_i-1} \binom{N_i-1}{\ell} K^{\ell}}, &(10)\\
t_{j,i} &= (j-1)\frac{\pi}{\omega_i\sqrt{1-\zeta_i^2}}, &(11)\\
K_i &= e^{-\zeta_i \pi / \sqrt{1-\zeta_i^2}}, &(12)
\end{aligned}$$

where ω_i and ζ_i are the natural frequency and damping ratio of the mode to be suppressed.

Let us consider an underdamped continuous plant with n modes of vibration expressed in parallel form as:

$$G(s) = \sum_{i=1}^{n} \frac{L_i \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}, \tag{13}$$

where ω_i, ζ_i, L_i are the corresponding natural frequency, damping ratio and dc–gain of each mode i, respectively. Its impulse response is then defined as:

$$g(t) = \sum_{i=1}^{n} \frac{L_i \omega_i}{\sqrt{1-\zeta_i^2}} e^{-\zeta_i \omega_i t} \sin\left(\omega_i \sqrt{1-\zeta_i^2}\, t\right). \tag{14}$$

It is obvious that the time required for the system to reach equilibrium is infinite. If a separate shaper is designed for each mode of the plant according to (10)–(12) and in the sequel the latter are convolved with each other, the resulting shaper's transfer function is given by:

$$S = \prod_{i=1}^{n} \sum_{j=1}^{N_i} A_{j,i} e^{-s t_{j,i}}, \tag{15}$$

where $A_{j,i}, t_{j,i}$ is the amplitude and time instance of the j–th impulse in the i–th shaper, while N_i is the length of each shaper. The resulting shaper in (15) can be rewritten in the form:

$$S = \sum_{j=1}^{\tilde{N}} \tilde{A}_j e^{-s\tilde{t}_j}, \tag{16}$$

where $\tilde{A}_j$, $\tilde{t}_j$ are the amplitudes and time instances of the shaper's impulses ($\tilde{N} = \prod_{i=1}^{n} N_i$ in number) emerged from the convolution of the individual shaping filters. The impulse response of the serial interconnection shaper–plant is then given by the following formulae:

$$\begin{aligned} h(t) = &\sum_{i=1}^{n} \sum_{j=1}^{\tilde{N}} \tilde{A}_j \frac{L_i \omega_i}{\sqrt{1-\zeta_i^2}} e^{-\zeta_i \omega_i (t-\tilde{t}_j)} \\ &\sin\left(\omega_i \sqrt{1-\zeta_i^2}\,(t-\tilde{t}_j)\right), \quad t < \tilde{t}_{\tilde{N}} = \sum_{i=1}^{n} t_{N_i,i}, \end{aligned} \tag{17}$$

Compared to (14), the impulse response (17) of the pre-shaped system is zero after $t = \tilde{t}_{\tilde{N}}$, thus transforming the plant into a finite impulse response filter.

B. Discrete input shaping technique

In cases, though, where the system to be controlled is discrete or its input is given in standard sampling instances, it is profound that the impulses of a standard shaper cannot be applied exactly at the desired time instances. This inability to transform the original plant into an FIR filter, results in an amount of residual vibration in the impulse response of the shaped system, although less in amplitude compared to the unshaped case.

This problem is remedied [20, 21] by applying two impulses instead of one at two consecutive sampling instances, k and $k+1$, such that $kT < t_{j,i} < (k+1)T$, where T is the sampling period. Consequently, if N_i is the number of impulses of a standard shaper, then the discrete version consists of $N_{D,i} = 2N_i - 1$ impulses. After determining the "new" sampling instances, $t^*_{j,i}$, at which the impulses are to be applied, then their amplitudes are computed by solving [20, 21]:

$$\begin{pmatrix} A^*_{1,i} \\ A^*_{2,i} \\ \vdots \\ A^*_{N_{D,i}-1,i} \\ A^*_{N_{D,i},i} \end{pmatrix} = \begin{pmatrix} m_{1,1,i} & \cdots & m_{1,N_{D,i},i} \\ \vdots & \ddots & \vdots \\ m_{N_{D,i}-1,1,i} & \cdots & m_{N_{D,i}-1,N_{D,i},i} \\ 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \tag{18}$$

for $A^*_{j,i}$, where $m_{k,j,i}$ are defined as:

$$m_{k,j,i} = \begin{cases} {t^*_{j,i}}^{\left\lfloor \frac{k+1}{2} \right\rfloor - 1} e^{-\zeta_i \omega_i \left(t^*_{N_D,i} - t^*_{j,i}\right)} \sin\left(t^*_{j,i} \omega_i \sqrt{1-\zeta_i^2}\right), k\text{: odd} \\ {t^*_{j,i}}^{\left\lfloor \frac{k+1}{2} \right\rfloor - 1} e^{-\zeta_i \omega_i \left(t^*_{N_D,i} - t^*_{j,i}\right)} \cos\left(t^*_{j,i} \omega_i \sqrt{1-\zeta_i^2}\right), k\text{: even} \end{cases} \tag{19}$$

Once the amplitudes $A^*_{j,i}$ and the time instances $t^*_{j,i}$ of the impulses are computed for each mode i, the input shaper's transfer function S can be computed as the convolution of the n separate shapers:

$$S = \prod_{i=1}^{n} \sum_{j=1}^{N_{D,i}} A^*_{j,i} z^{-\frac{t^*_{j,i}}{T}}. \tag{20}$$

Preshaping of the plant in (1) with the shaper in (20) leads to a finite–time impulse response of the serially–interconnected system. More specifically, the input–shaped discrete system reaches equilibrium in $\sum_{i=1}^{n} \frac{t^*_{N_{D,i},i}}{T}$ samples.

C. Input shaping technique for unstable LTI plants

Although the classical input shaping technique presented previously was originally derived for stable LTI systems, it can be applied to unstable ones as well. The only difference is in the values of the damping ratio ζ_i, which are negative for the unstable case. From a practical point of view, negative damping means that the enclosing envelope of the system's impulse response does not converge to zero, but goes to infinity instead.

Although input shaping technique described in (9)–(12) can be immediately applied to unstable LTI plants, it hasn't received any attention at all in the related literature. The reason is clear though; if the plant's parameters are not known *exactly* then an open–loop input–shaper compensator is unable to keep the system in equilibrium and the system diverges from stability, since the unstable poles are not completely canceled.

In general, the impulse response of an input–shaped system is consisted of a finite–time impulse response (FIR) and an infinite–time one (IIR), where the cancelation of the latter (IIR) is the goal of the shaper. If the shaper is not tuned correctly (due to the uncertainty embedded in the system modeling) then the residual IIR response is the one responsible for driving the shaped system unstable, since in the unstable–plant case its amplitude is increasing with time.

IV. H_∞ ROBUST CONTROLLER SYNTHESIS

A. Uncertainty model

A discrete input shaper of section III-B is designed for each of the nominal plants G_i^0. However from a practical point of view, there is always some uncertainty in the model of the plant. In this paper the parametric uncertainty is assumed to be embedded within the coefficients $a_{1,i}, a_{2,i}$ of the A_i–polynomials of denominator. Let each of the coefficients $a_{1,i}, a_{2,i}$ be written as:

$$a_{j,i} = a_{j,i}^0 + r_{j,i}\delta_{j,i},\ \ j = 1,2\ ,\ \ i = 1,\ldots n \tag{21}$$

where $a_{j,i}^0$ represents the nominal values and $\delta_{j,i}$ are unknown constants for which $\left\|\delta_{j,i}\right\|_\infty < 1$ holds; $r_{j,i}$ can then be expressed in terms of the bounds for each parameter as:

$$r_{j,i} = a_{j,i}^{\max} - a_{j,i}^0 = a_{j,i}^0 - a_{j,i}^{\min}. \tag{22}$$

Let us denote the serial interconnection of the discrete input shaper with the nominal plant as:

$$L^0\left(z^{-1}\right) = S\left(z^{-1}\right) G^0\left(z^{-1}\right). \tag{23}$$

It should be noted that L^0 is an FIR filter, thus its impulse response reaches equilibrium in finite time (more specifically in $\sum_{i=1}^{n} \frac{t^*_{N_{D,i},i}}{T}$ samples, where $N_{D,i}$ is the number of impulses of the discrete shaper designed for mode i), either the controlled plant is stable or unstable. Substitution of (21) into (1) results in:

$$G_i\left(z^{-1}\right) = \frac{G_i^0\left(z^{-1}\right)}{1 + G_i^0\left(z^{-1}\right)\left(W_{1,i}\left(z^{-1}\right)\delta_{1,i} + W_{2,i}\left(z^{-1}\right)\delta_{2,i}\right)}, \tag{24}$$

where the weighting filters are:

$$W_{j,i}\left(z^{-1}\right) = \frac{r_{j,i} z^{-j}}{B_i\left(z^{-1}\right)},\ \ j = 1,2. \tag{25}$$

A scheme that describes graphically the interconnection of the different subsystems is in Fig. 2.

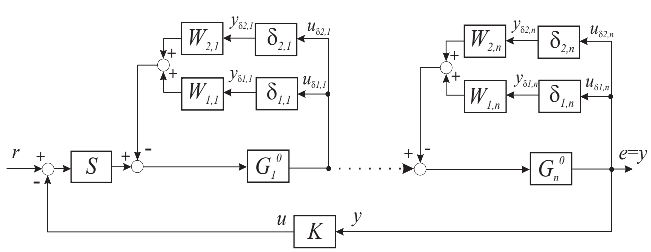

Fig. 2. Interconnection of blocks considering parametric uncertainty in underdamped plant

B. H_∞ controller design

In order to increase the insensitivity of the open–loop input–shaped system w.r.t. the uncertainty in the denominator coefficients, an H_∞ closed–loop controller is to be designed. According to the design procedure, the $\Delta - P - K$ form of the system must be firstly derived, where Δ is the uncertainty of the system in block–diagonal form, P is the interconnection matrix, and K is the controller to be designed, as shown in Fig. 3.

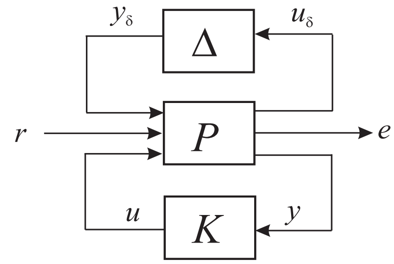

Fig. 3. $\Delta - P - K$ form for robust controller synthesis

For the plant (1) the signals u_δ and y can be expressed in terms of y_δ and u from Fig. 2 as:

$$u_{\delta_{1,i}} = \begin{cases} G_1^0\left(-W_{1,1}y_{\delta_{1,1}} - W_{2,1}y_{\delta_{2,1}} + S\left(r-u\right)\right), & i = 1 \\ G_i^0\left(-W_{1,i}y_{\delta_{1,i}} - W_{2,i}y_{\delta_{2,i}} + u_{\delta_{1,i-1}}\right), & i \geq 2 \end{cases} \tag{26}$$

$$y = u_{\delta_{1,n}} = u_{\delta_{2,n}}, \tag{27}$$

$$e = y - r = u_{\delta_{1,n}} - r. \tag{28}$$

The non–recursive expression of (26) is:

$$u_{\delta_{1,i}} = u_{\delta_{2,i}} = -\sum_{m=1}^{i} W_{1,m}\prod_{k=1}^{i} G_k^0 y_{\delta_{1,m}} - \sum_{m=1}^{i} W_{2,m}\prod_{k=1}^{i} G_k^0 y_{\delta_{2,m}} + S\prod_{k=1}^{i} G_k^0 r - S\prod_{k=1}^{i} G_k^0 u \tag{29}$$

Consequently, the fully interconnected system can be derived by (26)–(28) in a form appropriate for robust controller synthesis. The objective is to find a controller K that minimizes the following cost function:

$$\left\|T_{y_\delta \to u_\delta}\right\|_\infty = \sup_\omega \left\{\sigma_{max}\left(T_{y_\delta \to u_\delta}\left(j\omega\right)\right)\right\} < \gamma. \tag{30}$$

The aforementioned minimization problem is practically transformed into the existence of symmetric solutions R, S of the following system of LMIs [22]:

$$\begin{pmatrix} N_{12} & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} ARA^T - R & ARC_1^T & B_1 \\ C_1RA^T & -\gamma I + C_1RC_1^T & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{pmatrix} \begin{pmatrix} N_{12} & 0 \\ 0 & I \end{pmatrix} < 0,$$

$$\begin{pmatrix} N_{21} & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A^TSA - S & A^TSB_1 & C_1^T \\ B_1^TSA & -\gamma I + B_1^TSB_1 & D_{11}^T \\ C_1 & D_{11} & -\gamma I \end{pmatrix} \begin{pmatrix} N_{21} & 0 \\ 0 & I \end{pmatrix} < 0,$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0,$$

where N_{12} and N_{21} denote bases of the null spaces of $\left(B_2^T, D_{12}^T\right)$ and $\left(C_2, D_{21}\right)$. The matrices $A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22}$ are those of the "packed" state–space realization of the transfer function matrix P, i.e.:

$$P: \begin{cases} \dot{x} = Ax + B_1w + B_2u \\ e = C_1x + D_{11}w + D_{12}u \\ y = C_2x + D_{21}w + D_{22}u \end{cases} \tag{31}$$

where x is the state vector, w is the vector $\left(y_\delta, r\right)^T$ with the exogenous signals to the plant, u and y are the output and input to the controller $K\left(z^{-1}\right)$ respectively, and e are the signal to be kept small.

V. SIMULATION RESULTS

In this section the H_∞ controller described in section IV is designed for an unstable underdamped discrete input–shaped plant. The examined system has a single underdamped mode of vibration which can either be stable or unstable. Let the nominal plant under control be:

$$G^0 = \frac{0.008z^{-1} + 0.004z^{-2}}{1 + 1.1z^{-1} + 0.95z^{-2}}$$

It should be noted that the nominal plant is stable, since $|\rho_z| < 1$, where ρ_z are the complex conjugate roots of the polynomial of the denominator. The corresponding natural frequency and damping factor are $\omega_1 = 2.1705$ and $\zeta_1 = 0.0118$, respectively. The impulse response of the nominal plant is shown in Fig. 4. Let the system parameters be bounded as: $0.8 < a_{1,1} < 1.4$, $0.8 < a_{2,1} < 1.1$. It should be noted that *not all* systems of the form (1) are stable for the given bounds of $a_{1,1}, a_{2,1}$.

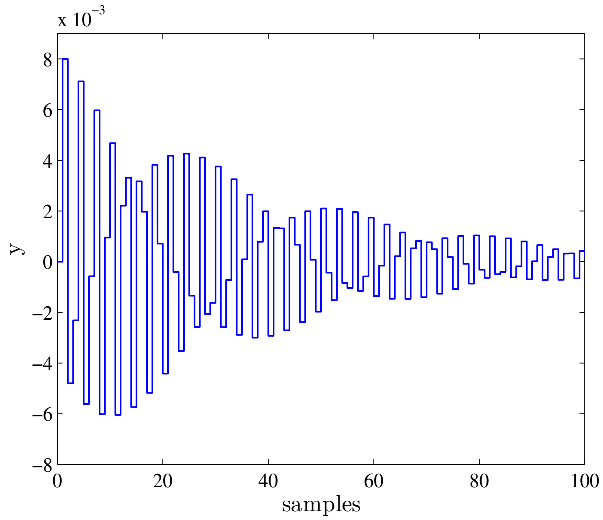

Fig. 4. Impulse response of the nominal plant

In order to eliminate the oscillations in the response of the nominal plant, a discrete input shaper of length $N_1 = 2$ is designed in order to obtain minimum rest–time. The nominal system response when controlled in open–loop with the shaper is shown in Fig. 5 (solid line). As expected, after three samples the oscillation has been eliminated. In the sequel, an H_∞ controller is designed to close the loop around the shaper and the plant, such that all closed–loop systems for any $a_{1,1}, a_{2,1}$ residing inside the uncertainty box are stable, which is derived as:

$$K = \frac{-420 - 825z^{-1} - 798.3z^{-2} - 344.9z^{-3} + 6.394e - 8z^{-4} + 5.112e - 7z^{-5}}{1 + 2.701z^{-1} + 4.214z^{-2} + 3.651z^{-3} + 1.952z^{-4} + 0.4523z^{-5}}$$

The impulse response of the closed–loop system is shown in Fig. 5 (dashed line). Although the system does not reach equilibrium in three samples, it comes at rest after two more samples, although the overshoot is almost double of that in the open–loop case.

Let now the true values for the uncertain parameters be $a_{1,1} = 1.16$, $a_{2,1} = 1.085$, which correspond to $\delta_{1,1} = 0.2$, $\delta_{2,1} = 0.9$. The system's responses when controlled with the shaper and with the H_∞ controller are shown in Fig. 6. The oscillations in the closed–loop case have almost been eliminated after the tenth sample, while closed–loop system stability is guaranteed. On the other hand, in the open–loop case the input–shaped system is unstable since the poles responsible for instability have not been eliminated due to the uncertainty in the system's parameters during the shaper design procedure. It should be noted that even though the nominal plant was considered as a stable one, closing the loop around the shaper guarantees stability of the control system even when the real plant is unstable.

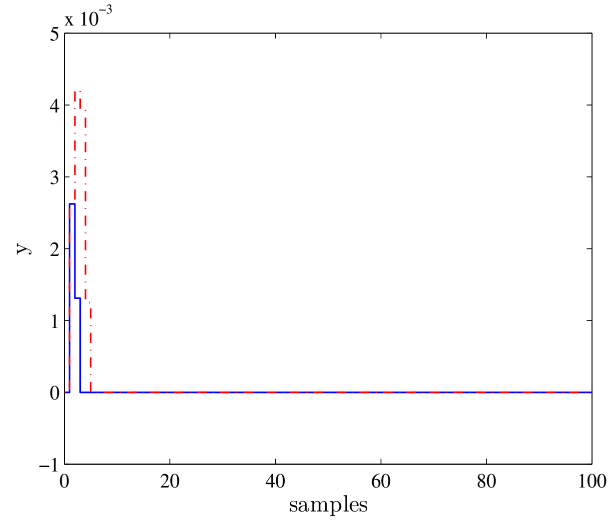

Fig. 5. Impulse response of the nominal input–shaped single–mode plant a) in open–loop (solid) b) with H_∞ closed–loop control (dashed)

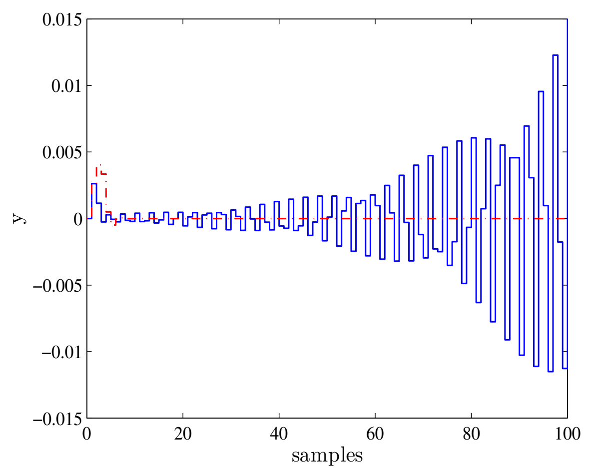

Fig. 6. Impulse response of the perturbed input–shaped single–mode plant a) in open–loop (solid) b) with H_∞ closed–loop control (dashed)

VI. CONCLUSIONS

In this paper a combined H_∞ and input shaping control scheme has been presented for uncertain discrete LTI systems. When the nominal plant is controlled with the discrete version of the input shaping technique, the latter reaches equilibrium in finite time. However, the system response deteriorates (or the latter may be driven unstable) in cases where there is parametric uncertainty in the system model. The control scheme proposed is the design of a robust H_∞ controller to close the loop around both the shaper and the plant. Stability of the closed–loop system is guaranteed for the range of uncertainty for which the H_∞ controller has been designed. Simulation results show the efficacy of this scheme in comparison with the standard open–loop input shaping.

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