# Grasping Control of 3-Joint Dual Finger Robot: Lyapunov Stability Approach 

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#### Abstract

This paper is concerned with the dynamics and control of grasping and regulating motion generated by a 3joint dual finger robot. To manipulate an object, the overall motion of the finger needs to be restricted by the object states. Therefore, we derive the kinematics of dual fingers governed by the object states by using four constraints which are based on the nonslipping assumption between the rigid fingertips and the surface of an object. Then, the control input is derived from Lyapunov stability analysis including the dynamics of the overall system. Further, we propose the solution of the contact forces between fingertips and an object based on physical analysis, which can not be solved mathematically. Finally, computer simulations are presented to verify the effectiveness of the proposed concept and method.


## I. Introduction

Since the beginning of robotics research, the fingered hand robots have been designed to mimic human hand which has the capability of dexterous manipulations and elaborate operations. In the history of development of the fingered hand robots, various hand models with four or five fingers with two or three joints each were reported [1]-[3]. They are so far used only in the open-loop control system which do not consider the relationship between fingers and an object, because there is no way to estimate the forces between them.

In order to overcome the disadvantages of open-loop control system, Arimoto et al. [4]-[6] suggested a pair of robot fingers with hemispherical finger-ends using sensory motor coordination. The main theory of this research is the passivity. The passivity means that the energy variation of the overall system, which is composed of fingers and an object, is caused by the torque generated from a joint motor.[8]. One of the advantages of the using the passivity is that the physical term of finger dynamics can be cancelled. Therefore, it is possible to construct a controller only using the kinematics constraints fingers and object. However, to back up the passivity theorem, some assumptions are needed. The overall system does not allow non-conservative factor such as friction. Even though the effect of these factors can be ignorable, the grasped object can not be controlled when the unknown external force is exerted on it.

In this paper, we propose the method for designing a controller of finger robot which uses the error stability,

[^0]

Fig. 1. 3-joint dual finger robot system
not the passivity. To design a controller, we use the finger dynamics as control input directly. This dynamics consists of two parts briefly. One part is derived from physical properties of the fingers. The other part is derived from terms of contact forces between finger-tips and an grasped objects. We employ the kinematic synthesis for the former part and the inverse dynamics for the latter part to construct a control input as a function of the grasped object posture. Then, we define the Lyapunov candidate function which only consists of the posture of the grasped object in order to guarantee the convergence of the desired posture. To verify the effectiveness of the proposed controller, we employ a 3-joint finger model because it has geometric flexibility in object manipulation such as shifting, rotating and changing contact position simultaneously.

This paper is organized as follows. In Section II, a set of dynamics of the fingers and an object is derived on the basis of Hamilton's principle. In Section III, a control input is designed by using the kinematic synthesis and deriving the contact forces. Simulation results are presented to verify the effectiveness of the proposed method in Section IV.

## II. Dynamics of 3-Joint Dual Finger Robot

For the sake of physical simplicity, we assume that a 3joint dual finger robot shown in Fig. 1 moves on a horizontal plane to ignore the gravitational force. Moreover, we only deal with a solid rectangular object with hard spherical fingertips.

First, let us consider the geometric constraints as follows:

$$
\begin{align*}
Q_{1}= & \left(x-x_{01}\right) \cos \theta-\left(y-y_{01}\right) \sin \theta-r_{1}-\frac{l}{2}=0 \\
Q_{2}= & -\left(x-x_{02}\right) \cos \theta+\left(y-y_{02}\right) \sin \theta \\
& -r_{2}-\frac{l}{2}=0 \\
R_{1}= & Y_{1}-Y_{1}(0)+r_{1}\left(\theta-q_{11}-q_{12}-q_{13}\right)=0 \\
R_{2}= & Y_{2}-Y_{2}(0)+r_{2}\left(-\theta-q_{21}-q_{22}-q_{23}\right)=0 \tag{1}
\end{align*}
$$

where,

$$
\begin{aligned}
x_{01} & =-l_{13} \cos \phi_{13}-l_{12} \cos \phi_{12}-l_{11} \cos \phi_{11} \\
y_{01} & =l_{13} \sin \phi_{13}+l_{12} \sin \phi_{12}+l_{11} \sin \phi_{11} \\
x_{02} & =L+l_{23} \cos \phi_{23}+l_{22} \cos \phi_{22}+l_{21} \cos \phi_{21} \\
y_{02} & =l_{23} \sin \phi_{23}+l_{22} \sin \phi_{22}+l_{21} \sin \phi_{21} \\
\phi_{i 1} & =q_{i 1} \\
\phi_{i 2} & =q_{i 2}+q_{i 3} \\
\phi_{i 3} & =q_{i 1}+q_{i 2}+q_{i 3}, \quad i=1,2
\end{aligned}
$$

Here, all symbols are defined in Fig. 1. $Q_{1}$ and $Q_{2}$ mean the geometric constraints between $O_{c m}$ and $O_{01}$ and between $O_{c m}$ and $O_{02}$, respectively. $x_{0 i}$ and $y_{0 i}, i=1,2$, are the horizontal and vertical components of the distance from the center of the fingertips to the center of mass of an object, respectively. Then, it is reasonable to introduce Lagrange multipliers $f_{1}, f_{2}, \lambda_{1}, \lambda_{2}$ for the corresponding constraint equations, which actually act as the normal and tangential contact forces.

Consider two quantities for the corresponding constraints from (1):

$$
\begin{align*}
Q & =f_{1} Q_{1}+f_{2} Q_{2}=0 \\
R & =\lambda_{1} R_{1}+\lambda_{2} R_{2}=0 \tag{2}
\end{align*}
$$

With these two quantities, the Lagrangian $L$ of the overall system can be defined as

$$
\begin{equation*}
L=K+Q+R \tag{3}
\end{equation*}
$$

where K is the kinetic energy of two fingers and an object, which is defined as

$$
\begin{equation*}
K=\frac{1}{2}\left\{\sum_{i=1}^{2} \dot{\mathbf{q}}_{i}^{T} H_{i} \dot{\mathbf{q}}_{i}+M \dot{x}^{2}+M \dot{y}^{2}+I \dot{\theta}^{2}\right\} \tag{4}
\end{equation*}
$$

where $H_{i}$ denotes the inertia moment of fingers and $\mathbf{q}_{i}=$ $\left[\begin{array}{lll}q_{i 1} & q_{i 2} & q_{i 3}\end{array}\right]^{T}$.

Applying Hamilton's principle to the following equation

$$
\int_{t_{0}}^{t_{1}}\left\{\delta(K+Q+R)+u_{1} \delta q_{1}+u_{2} \delta q_{2}\right\} d t=0
$$

we can obtain the dynamics of the fingers and an object described as follows:

$$
\begin{align*}
& \mathbf{H} \ddot{\mathbf{q}}+\boldsymbol{\Gamma} \dot{\mathbf{q}}+\boldsymbol{\Omega} \mathbf{f}=\mathbf{u}  \tag{5}\\
& \quad \mathbf{M}_{o} \ddot{\mathbf{x}}=\Lambda \mathbf{f} \tag{6}
\end{align*}
$$

where,

$$
\begin{aligned}
& \mathbf{H}=\operatorname{diag}\left[H_{1}, H_{2}\right], \quad \boldsymbol{\Gamma}=\left[\Gamma_{1}^{T}, \Gamma_{2}^{T}\right], \\
& \boldsymbol{\Omega}=\left[\begin{array}{c}
\left.-J_{01}^{T}\left[\begin{array}{c}
-\cos \theta \\
\sin \theta
\end{array}\right], \mathbf{0}_{3 \times 1},-J_{01}^{T}\left[\begin{array}{c}
\sin \theta \\
\cos \theta
\end{array}\right]+r_{1}\left[\begin{array}{l}
1 \\
1 \\
1
\end{array}\right], \mathbf{0}_{3 \times 1}\right] \\
\mathbf{0}_{3 \times 1}, J_{02}^{T}\left[\begin{array}{c}
-\cos \theta \\
\sin \theta
\end{array}\right], \mathbf{0}_{3 \times 1},-J_{02}^{T}\left[\begin{array}{c}
\sin \theta \\
\cos \theta
\end{array}\right]+r_{2}\left[\begin{array}{l}
1 \\
1 \\
1
\end{array}\right]
\end{array}\right], \\
& \mathbf{M}_{o}=\left[\begin{array}{ccc}
M & 0 & 0 \\
0 & M & 0 \\
0 & 0 & I
\end{array}\right], \\
& \boldsymbol{\Lambda}=\left[\begin{array}{cccc}
\cos \theta & -\cos \theta & -\sin \theta & -\sin \theta \\
-\sin \theta & \sin \theta & -\cos \theta & -\cos \theta \\
Y_{1} & -Y_{2} & -\frac{l}{2} & \frac{l}{2}
\end{array}\right], \\
& \mathbf{f}=\left[\begin{array}{llll}
f_{1} & f_{2} & \lambda_{1} & \lambda_{2}
\end{array}\right]^{T}, \quad \mathbf{q}=\left[\mathbf{q}_{1}^{T}, \mathbf{q}_{2}^{T}\right]^{T} .
\end{aligned}
$$

Here, $\Gamma_{1}$ and $\Gamma_{2}$ stand for the coefficient of $\dot{\mathbf{q}}$ including coriolis, centrifugal forces and differential functions of inertia moment. M and I are the mass and the inertia moment of an object, respectively. $f_{i}$ and $\lambda_{i}$ stand for the normal and tangential contact forces, which are exerted on an object for secure grasp and dexterous movements, respectively. $J_{0 i}$, $i=1,2$, denotes the Jacobean matrices of $\left(x_{0 i}, y_{0 i}\right)^{T}$ with respect to $\mathbf{q}_{i}$ and can be formulated as follows:

$$
\begin{aligned}
& J_{01}^{T}=\left[\begin{array}{ll}
J_{011} & J_{012}
\end{array}\right], \\
& J_{02}^{T}=\left[\begin{array}{ll}
J_{021} & J_{022}
\end{array}\right],
\end{aligned}
$$

where,

$$
\begin{aligned}
& J_{011}=\left[\begin{array}{c}
l_{13} \sin \phi_{3}+l_{12} \sin \phi_{2}+l_{11} \sin \phi_{11} \\
l_{13} \sin \phi_{13}+l_{12} \sin \phi_{12} \\
l_{13} \sin \phi_{13}
\end{array}\right] \\
& J_{012}=\left[\begin{array}{c}
l_{13} \cos \phi_{13}+l_{12} \cos \phi_{12}+l_{11} \cos \phi_{11} \\
l_{13} \cos \phi_{13}+l_{12} \cos \phi_{12} \\
l_{13} \cos \phi_{13}
\end{array}\right] \\
& J_{021}=\left[\begin{array}{c}
-l_{23} \sin \phi_{23}-l_{22} \sin \phi_{22}+l_{21} \sin \phi_{21} \\
-l_{23} \sin \phi_{23}-l_{22} \sin \phi_{22} \\
-l_{23} \sin \phi_{23}
\end{array}\right] \\
& J_{022}=\left[\begin{array}{c}
l_{23} \cos \phi_{23}+l_{22} \cos \phi_{22}+l_{21} \cos \phi_{21} \\
l_{23} \cos \phi_{23}+l_{22} \cos \phi_{22} \\
l_{23} \cos \phi_{23}
\end{array}\right]
\end{aligned}
$$

## III. Design of Control Input

## A. Kinematic Synthesis

The kinematics of each finger can be obtained from $\mathbf{x}=$ $[x, y, \theta]^{T}$ and $\mathbf{Y}=\left[Y_{1}, Y_{2}\right]^{T}$ by transforming constraints in (1). The transformed equations are represented as follows:

$$
\begin{gathered}
l_{13} \cos \phi_{13}+l_{12} \cos \phi_{12}+l_{11} \cos \phi_{11} \\
=-x+\left(\frac{l}{2}+r_{1}\right) \cos \theta-Y_{1} \sin \theta \\
l_{13} \sin \phi_{13}+l_{12} \sin \phi_{12}+l_{11} \sin \phi_{11}
\end{gathered}
$$



Fig. 2. Diagram of overall dynamic system with controller

$$
\begin{align*}
& =y+\left(\frac{l}{2}+r_{1}\right) \sin \theta+Y_{1} \cos \theta \\
& \phi_{13}=\frac{Y_{1}-Y_{1}(0)}{r_{1}}+\theta \\
& l_{23} \cos \phi_{23}+l_{22} \cos \phi_{22}+l_{21} \cos \phi_{21} \\
& \quad=x+\left(\frac{l}{2}+r_{2}\right) \cos \theta+Y_{2} \sin \theta-L \\
& l_{23} \sin \phi_{23}+l_{22} \sin \phi_{22}+l_{21} \sin \phi_{21} \\
& \quad=y-\left(\frac{l}{2}+r_{2}\right) \sin \theta+Y_{2} \cos \theta \\
& \phi_{23}=\frac{Y_{2}-Y_{2}(0)}{r_{2}}-\theta \tag{7}
\end{align*}
$$

Numerically, we can find a solution of $\mathbf{q}_{1}$ and $\mathbf{q}_{2}$ by solving a set of nonlinear equations in (7). However, we need the differential forms to derive the control input of the overall dynamic system. The differential form of the kinematics of the fingers can be derived as follows:

$$
\begin{equation*}
A \ddot{\Phi}=B \ddot{\mathbf{z}}+C \dot{\Phi}+D \tag{8}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \Phi=\left[\begin{array}{llllll}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{21} & \phi_{22} & \phi_{23}
\end{array}\right]^{T}, \\
& \mathbf{z}=\left[\begin{array}{ll}
\mathbf{x}^{T} & \mathbf{Y}^{T}
\end{array}\right]^{T}, \\
& A=\operatorname{diag}\left[A_{1}, A_{2}\right], \quad B=\left[B_{1}^{T}, B_{2}^{T}\right]^{T}, \\
& C=\operatorname{diag}\left[C_{1}, C_{2}\right], \quad D=\left[D_{1}^{T}, D_{2}^{T}\right]^{T}, \\
& A_{i}=\left[\begin{array}{ccc}
l_{i 1} \cos \phi_{i 1} & l_{i 2} \cos \phi_{i 2} & l_{i 3} \cos \phi_{i 3} \\
l_{i 1} \sin \phi_{i 1} & l_{i 2} \sin \phi_{i 2} & l_{i 3} \sin \phi_{i 3} \\
0 & 0 & r_{i}
\end{array}\right], \\
& B_{1}=\left[\begin{array}{ccccc}
0 & 1 & \left(\frac{l}{2}+r_{1}\right) \cos \theta-Y_{1} \sin \theta & \cos \theta & 0 \\
1 & 0 & \left(\frac{l}{2}+r_{1}\right) \sin \theta-Y_{1} \cos \theta & \sin \theta & 0 \\
0 & 0 & r_{1} & 1 & 0
\end{array}\right], \\
& B_{2}=\left[\begin{array}{ccccc}
0 & 1 & -\left(\frac{l}{2}+r_{2}\right) \cos \theta-Y_{2} \sin \theta & 0 & \cos \theta \\
-1 & 0 & \left(\frac{l}{2}+r_{2}\right) \sin \theta-Y_{2} \cos \theta & 0 & -\sin \theta \\
0 & 0 & -r_{2} & 0 & 1
\end{array}\right],
\end{aligned}
$$

$\left.C_{i}=\left[\begin{array}{ccc}l_{i 1} \sin \phi_{i 1} & \dot{\phi}_{i 1} & l_{i 2} \sin \phi_{i 2} \dot{\phi}_{i 2} \\ -l_{i 1} \cos \phi_{i 1} \dot{\phi}_{i 1} & -l_{i 3} \sin \phi_{i 3} \dot{\phi}_{i 3} \\ 0 & 0 & \phi_{i 2} \dot{\phi}_{i 2}\end{array}\right]-l_{i 3} \cos \phi_{i 3} \dot{\phi}_{i 3}\right]$,
$D_{i}=\left[\begin{array}{c}-2 \dot{Y}_{i} \sin \theta \dot{\theta}-\dot{\theta}^{2}\left(\left(\frac{l}{2}+r_{i}\right) \sin \theta+Y_{i} \cos \theta\right) \\ (-1)^{i+1} 2 \dot{Y}_{i} \cos \theta \dot{\theta}+\dot{\theta}^{2} \times \\ \left(\left(\frac{l}{2}+r_{i}\right) \cos \theta+(-1)^{i} Y_{i} \sin \theta\right) \\ 0\end{array}\right]$.
Using $\mathbf{q}$, (8) can be also written as follows:

$$
\begin{equation*}
\ddot{\mathbf{q}}=T^{-1} A^{-1}(B \ddot{\mathbf{z}}+C T \dot{\mathbf{q}}+D) \tag{9}
\end{equation*}
$$

where,

$$
T=\operatorname{diag}\left[T_{i n}, T_{i n}\right], \quad T_{i n}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
1 & 1 & 0 \\
1 & 1 & 1
\end{array}\right]
$$

The above equation means that joint angles can be governed by the posture of an object and the contact points. Therefore, all the kinematic positions are a function of desired posture of the object and the contact points.

Assumption 1: Invertibility of $A$ is guaranteed when the determinant of $A_{i}$ is nonzero as follows:

$$
\begin{aligned}
\operatorname{det}\left(A_{i}\right)= & r_{i}\left\{l_{i 1} \cos q_{i 1} l_{i 2} \sin \left(q_{i 1}+q_{i 2}\right)\right. \\
& \left.-l_{i 1} \sin q_{i 1} l_{i 2} \cos \left(q_{i 1}+q_{i 2}\right)\right\} \\
= & r_{i} l_{i 1} l_{i 2} \sin q_{i 2} \neq 0, \quad i=1,2
\end{aligned}
$$

Therefore, we assume that $q_{12} \neq 0$ and $q_{22} \neq 0$ for invertibility of $A$.

## B. Contact Forces

To use the finger dynamics as control input, the contact forces between finger tips and object surface should be calculated. The contact forces are derived from (6) inversely. Since matrix $\boldsymbol{\Lambda}$, however, is not rectangular, we develop the contact force condition via physical insight. We can find this condition from the physical meaning of an object motion. To grasp an object securely, the desired normal

TABLE I
NORMAL CONTACT FORCES

| Condition | $\ddot{x} \cos \theta-\ddot{y} \sin \theta>0$ | $\ddot{x} \cos \theta-\ddot{y} \sin \theta<0$ |
| :---: | :---: | :---: |
| Normal | $f_{1}=f_{d}+\Delta f$ | $f_{2}=f_{d}$ |
| Forces | $f_{1}=f_{d}$ | $f_{2}=f_{d}-\Delta f$ |

force $f_{d}$ should be exerted continuously. Only the additional force $\Delta f=M \times(\ddot{x} \cos \theta-\ddot{y} \sin \theta)$ is added to accelerate and decelerate an object. The condition for $f_{1}, f_{2}$ can be summarized as shown in Table I.

With the condition listed in Table I and (6), finally we can calculate the contact forces as follows:

$$
\begin{equation*}
\mathbf{f}=\boldsymbol{\Pi} \ddot{\mathbf{z}}+\boldsymbol{\Xi} f_{d} \tag{10}
\end{equation*}
$$

where,

$$
\begin{gathered}
\boldsymbol{\Pi}=\left[\begin{array}{ll}
\boldsymbol{\Pi}_{i n} & \mathbf{0}_{3 \times 1}
\end{array}\right] \\
\boldsymbol{\Pi}_{i n}=\left[\begin{array}{lll}
\Pi_{1} & \Pi_{2} & \Pi_{3} \\
\Pi_{4}
\end{array}\right]^{T}, \\
\Pi_{1}=\left[\begin{array}{c}
\frac{M}{2}(\cos \theta)\{1+\operatorname{sgn}(\Delta f)\} \\
-\frac{M}{2}(\sin \theta)\{1+\operatorname{sgn}(\Delta f)\} \\
0
\end{array}\right] \\
\Pi_{2}=\left[\begin{array}{c}
\frac{M}{2}(\cos \theta)\{-1+\operatorname{sgn}(\Delta f)\} \\
-\frac{M}{2}(\sin \theta)\{1-\operatorname{sgn}(\Delta f)\} \\
0
\end{array}\right], \\
\Pi_{3}=\left[\begin{array}{c}
\frac{M}{l}\left(\frac{Y_{1}+Y_{2}}{2}+\frac{Y_{1}-Y_{2}}{2} \operatorname{sgn}(\Delta f)\right) \cos \theta-\frac{M}{2} \sin \theta \\
-\frac{M}{l}\left(\frac{Y_{1}+Y_{2}}{2}+\frac{Y_{1}-Y_{2}}{2} \operatorname{sgn}(\Delta f)\right) \sin \theta-\frac{M}{2} \cos \theta \\
-\frac{I}{l}
\end{array}\right] \\
\Pi_{4}=\left[\begin{array}{c}
-\frac{M}{l}\left(\frac{Y_{1}+Y_{2}}{2}+\frac{Y_{1}-Y_{2}}{2} \operatorname{sgn}(\Delta f)\right) \cos \theta-\frac{M}{2} \sin \theta \\
\frac{M}{l}\left(\frac{Y_{1}+Y_{2}}{2}+\frac{Y_{1}-Y_{2}}{2} \operatorname{sgn}(\Delta f)\right) \sin \theta-\frac{M}{2} \cos \theta \\
\frac{I}{l}
\end{array}\right] .
\end{gathered}
$$

(10) is the function of the object accelerations and the desired normal force. That means it can be controlled by generating the posture of a pinched object.

## C. Design of Control Input

The objective of the proposed controller is to manipulate an object using the dynamics of fingers. Therefore, The control input should consist of the states of an object, ultimately.

Substituting (9) and (10) into (5), we can obtain

$$
\begin{equation*}
\mathbf{u}=\mathbf{H}^{\prime} \ddot{\mathbf{z}}+\Psi(\mathbf{q}, \dot{\mathbf{q}})+\boldsymbol{\Omega} \boldsymbol{\Xi} f_{d} \tag{11}
\end{equation*}
$$

where,

$$
\begin{aligned}
\mathbf{H}^{\prime} & =H T^{-1} A^{-1} B+\boldsymbol{\Omega} \boldsymbol{\Pi} \\
\Psi(\mathbf{q}, \dot{\mathbf{q}}) & =\left\{H T^{-1} A^{-1} C T+\Gamma\right\} \dot{\mathbf{q}}
\end{aligned}
$$

For regulating the posture of an object, it should be guaranteed in the states z to converge to the desired states $z_{d}$.

Theorem 1: Assume that the control input is formulated as follows:

$$
\begin{equation*}
\mathbf{u}=\Psi(\mathbf{q}, \dot{\mathbf{q}})+\boldsymbol{\Omega} \boldsymbol{\Xi} f_{d}-\mathbf{H}^{\prime}\left\{P_{1}\left(\dot{\mathbf{z}}-\dot{\mathbf{z}}_{d}\right)+P_{2}\left(\mathbf{z}-\mathbf{z}_{d}\right)\right\} \tag{12}
\end{equation*}
$$

where, $P_{1}, P_{2} \in \mathbb{R}^{5 \times 5}$ are the strictly positive diagonal matrices. Then, the states $\mathbf{z}$ are guaranteed to converge to the desired states $\mathbf{z}_{\mathbf{d}}$.

Proof: Let us consider the following Lyapunov candidate function:

$$
\begin{equation*}
V=\frac{1}{2} \mathbf{s}^{T} \mathbf{s}>0 \tag{13}
\end{equation*}
$$

where,

$$
\begin{aligned}
s & =\dot{\tilde{\mathbf{z}}}+P_{\mathbf{z}} \tilde{\mathbf{z}} \\
\tilde{\mathbf{z}} & =\mathbf{z}-\mathbf{z}_{d} .
\end{aligned}
$$

Differentiating (13) and substituting (11), we can obtain

$$
\begin{align*}
\dot{V} & =s^{T} \dot{s} \\
& =s^{T}\left\{\mathbf{H}^{\prime}\left(\mathbf{u}-\Psi(\mathbf{q}, \dot{\mathbf{q}})-\boldsymbol{\Omega} \boldsymbol{\Xi} f_{d}\right)+P_{z}\left(\mathbf{z}-\mathbf{z}_{d}\right)\right\} . \tag{14}
\end{align*}
$$

Then, substituting (12) into (14) yields

$$
\begin{equation*}
\dot{V}=-s^{T} s \leq 0 \tag{15}
\end{equation*}
$$

Therefore, Lyapunov function guarantees the asymptotic stability of states $\mathbf{s}$. Since $\mathbf{s}$ converges to zero, $\tilde{\mathbf{z}}$ should converge to zero. Therefore, the posture of an object and the contact positions moves toward desired positions.

## IV. SIMULATION RESULTS

We carry out computer simulations in Matlab. The physical parameters are given in Table II. The damping gains are tuned as 10 . The initial and final postures are given as $\left[x_{0}\right.$, $\left.y_{0}, \theta_{0}, Y_{10}, Y_{20}\right]=[0.125,0.3488,0,0,0]$ and $\left[x_{f}, y_{f}, \theta_{f}\right.$, $\left.Y_{1 f}, Y_{2 f}\right]=[0.2,0.38,0.2,-0.005]$, respectively.
From Figs. 3 and 4, we can confirm that the proposed control system performs the secure grasp and manipulation such as shifting and changing the contact position simultaneously. Fig. 6 shows that the transient responses of the center of mass and the rolling contact position converge to the desired value. From the result of Fig. 7, we can confirm that

TABLE II
Physical Parameters

| $m_{11}=m_{21}$ | link mass | $0.04[\mathrm{~kg}]$ |
| :---: | :---: | :---: |
| $m_{12}=m_{22}$ | link mass | $0.025[\mathrm{~kg}]$ |
| $m_{13}=m_{23}$ | link mass | $0.035[\mathrm{~kg}]$ |
| $l_{11}=l_{21}$ | link length | $0.2[\mathrm{~m}]$ |
| $l_{12}=l_{22}$ | link length | $0.2[\mathrm{~m}]$ |
| $l_{13}=l_{23}$ | link length | $0.15[\mathrm{~m}]$ |
| $I_{11}=I_{21}$ | inertia moment | $1.5 \times 10^{-5}\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]$ |
| $I_{12}=I_{22}$ | inertia moment | $1.2 \times 10^{-5}\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]$ |
| $I_{13}=I_{23}$ | inertia moment | $1.1 \times 10^{-5}\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]$ |
| $M$ | object mass | $0.2[\mathrm{~kg}]$ |
| $I$ | object inertia moment | $5 \times 10^{-4}\left[\mathrm{~kg} \cdot \mathrm{~m}^{2}\right]$ |
| $f_{d}$ | internal force | $0.3[\mathrm{~N}]$ |



Fig. 3. Initial posture


Fig. 4. Final posture


Fig. 5. Transient responses for the angles of the fingers 1 and 2
the normal contact forces $f_{1}$ and $f_{2}$, which accelerate and decelerate an object, respectively, are induced in order and


Fig. 6. Transient responses for the center of mass and rolling contact position


Fig. 7. Normal and tangential contact forces by fingertips


Fig. 8. Transient responses for the first angle of the fingers 1 and 2
converge to the desired force $f_{d}$ within a second eventually. We can also confirm that the tangential contact forces $\lambda_{1}$ and $\lambda_{2}$ are induced to shift an object toward y -axis direction and eventually converge to zero. Thus, the simulation results demonstrate the effectiveness of the controller proposed in this paper. Furthermore, we show how the responses of the finger angles and the contact forces can be affected by the


Fig. 9. Normal contact force by fingertips
changes of the damping gains. Fig. 8 shows that the speed of convergence of finger angles is proportional to the damping gains. In addition, Fig. 9 shows that the contact forces are also proportional to the damping gains within smaller time.

## V. Conclusion

This paper has dealt with a 3-joint dual finger robot for grasping and regulating the posture and position of an object. We derive and analyze the dynamics of a setup of a 3-joint dual finger robot with spherical fingertips pinching a rigid object. In order to calculate the kinematics of fingers, we use four geometric constraint based on the assumption of nonslipping condition between the fingertips and an object. To design the control input, we propose the contact forces for manipulating an object, which are derived via physical insight. The computer simulation results verify the effectiveness of our proposed method. Furthermore, we can confirm the advantage of the proposed control system such that the states of the fingers and an object do not fluctuate in a transient response.

## References

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