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Adaptive Robust Tracking Control for a Class of Distributed Systems with Faulty Actuators and Interconnections

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Abstract— **In this paper, direct adaptive control schemes are developed to solve the robust tracking problem of a class of distributed large scale systems with actuator faults, faulty and perturbed interconnection links, and external disturbances. The adaptation laws are proposed to update controller parameters on-line when all eventual faults, the upper bounds of the perturbations and disturbances are assumed to be unknown. Then a class of distributed state feedback controllers is constructed to automatically compensate the fault, perturbation and disturbance effects based on the information from adaptive schemes. The proposed tracking controller can ensure that the resulting closed-loop large-scale system is stable and the tracking error decreases asymptotically to zero. The proposed adaptive design technique is finally evaluated in the light of a simulation example.**

I. INTRODUCTION

A lot of practical control systems can be considered as large-scale systems composed with a large number of spatially interconnected units, such as the control of vehicular platoons [20], cross-directional control in the chemical industry [21]. And many approaches have been developed to synthesize some types of distributed controllers to make the dynamic large scale system well-posed, stable, and contractive (see, e.g., $[1]-[9]$, and the references therein). Recently, some issues which always exist in interconnections are put forward by some researchers [1], $[2]-[5]$ and $[10]-[11]$, such as single attenuations, time delays, bandwidth limitations (bit rate limitations) and perturbations. In [1], the problem of signal attenuations in interconnection channels is considered, and the necessary and sufficient conditions for well-posedness, stability, and contractiveness are obtained by using integral quadratic constraints (IQCs) method. Recursive information flow [3] is proposed to compensate for the effect of bandwidth limitations. In [2], [4], distributed controllers are constructed by using some special effective approaches for stability and performance of the closed-loop interconnected systems with the problem of time-delays in channels. For the problem of perturbations, transmission noise in channels between the controllers is presented in [5] and the transmission signal-to-noise power is kept limited using an LMI method. However, to the best of our knowledge, the problem of asymptotic tracking for distributed control systems with signal attenuations and perturbations in interconnections has not yet been investigated.

Recently, fault-tolerant control (FTC) system design, which can make the systems operate in safety and with proper performances whenever components are healthy or faulted, has received significant attention (see e.g., [6], [8], $[14]-[19]$). Some of above works adopt adaptive methods to deal with the problem of tracking control in the case of actuator faults on systems, such as $[14]-[18]$. However, as we know that the disturbances play an important role in systems, some of above works, such as $[14]-[17]$, have not consider the disturbances within the systems, and the proposed methods may not be suitable for the FTC systems if there exist disturbances. Moreover, $[18]-[19]$ consider the disturbances under some special conditions, such as $\lim_{t\to\infty} z(t)=0$ ($z(t)$ is disturbance) [18] and constant disturbance [19]. Therefore, the capability of disturbance rejection for the above FTC systems seems weak. On the other hand, a direct adaptive method proposed in [15] can compensate for the time-varying parameterizable stuck-actuator failures, but for the unparameterizable failures, approximations of the stuck-actuator failures must be employed and the closed-loop system can be guaranteed to be stable but asymptotic tracking cannot be ensured [16]. Furthermore, [17] considers the unparameterizable failures in the system, but the requirement of knowledge of upper bounds of failures is needed and asymptotic tracking also cannot be ensured. In this paper, the new proposed robust adaptive schemes can solve the problem of FTC with more general actuator failures than above works $[14]-[19]$ and make sure the system asymptotically track the states of model reference system under the influence of actuator unparameterizable time-varying failures and disturbances.

This paper considers the direct adaptive method to solve the robust asymptotic tracking control problem for a class of distributed control systems with actuator and interconnection failures, perturbations in interconnection channels, and disturbances on subsystems. The adaptive compensation design approach will be used for a general actuator fault model, which covers the cases of normal operation, loss of effectiveness, outage, and stuck. General faults of signal attenuations of interconnections are also considered. Each control effectiveness, upper bounds of perturbations and external disturbances are assumed to be unknown. A direct adaptive method is proposed to solve the problem

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for developing some distributed state feedback controllers. For this purpose, we first propose some adaptation laws to update the controller parameters. Then, the distributed controllers are constructed using the updated values of these estimations. Based on the Lyapunov stability theory, the adaptive closed-loop large-scale system can be guaranteed to be stable and each subsystem can asymptotically track the corresponding state of model reference system. The corresponding distributed fault-tolerant controller design via a different adaptive method is presented in [8].

II. Preliminaries and Problem Statement

We first introduce our notation. Notation $\mathrm{cat}_{k=1}^{n} x_k$ denotes the signal or vector $(x_1, x_2, \ldots, x_n)$ formed by concatenating x_k. This is also usually denoted $\mathrm{cat}_k x_k$ for brevity.

In this paper, we consider a large-scale system G composed of N interconnected linear time-invariant continuous time subsystems G_i, $i = 1, 2, \ldots, N$. Each subsystem is captured the following state-space equation:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \end{bmatrix} = \begin{bmatrix} A^i_{TT} & A^i_{TS} & B^i_{Td} & B^i_{Tu} \\ A^i_{ST} & A^i_{SS} & B^i_{Sd} & B^i_{Su} \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i(t) \\ u_i(t) \end{bmatrix} \tag{1}$$

where $x_i(t) \in R^{n_i}$ is the state, $u_i(t) \in R^{m_i}$ is the control input, $y_i(t) \in R^{l_i}$ is the measured output, $d_i \in R^{p_i}$ is the external disturbance, and $v_i := \mathrm{cat}_j(v_{ij})$, $v_{ij} \in R^{q_{ji}}$ and $w_i := \mathrm{cat}_j(w_{ij})$, $w_{ij} \in R^{q_{ij}}$, $j = 1, 2, \ldots, N$ is the interconnection input to each subsystem and the interconnection output from each subsystem, respectively. All system matrices are known real constant matrices with appropriate dimensions.

In the normal case, once the relationships between the inputs and outputs at each subsystem have been defined, the distributed system can be described by closing all loops by imposing the constraints of interconnection with the interconnection condition such that

$$v_{ij}(t) = w_{ji}(t). \tag{2}$$

We assume every subsystem is controllable and the states of each subsystem are available at every instant. Moreover, every state has its transmission channel interconnected with other subsystems, and the controller and plant use identical transmission channels. Then, a state feedback controller with same interconnection structure for this system has subsystems K_i given by

$$\begin{bmatrix} u_i(t) \\ w_i^K(t) \end{bmatrix} = \begin{bmatrix} K_{i11} & K_{i12} \\ K_{i21} & K_{i22} \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i^K(t) \end{bmatrix} \tag{3}$$

where $v_i^K(t) := \mathrm{cat}_j(v_{ij}^K)$, $w_i^K(t) := \mathrm{cat}_j(w_{ij}^K)$ and $v_{ij}^K(t)$, $w_{ji}^K(t) \in R^{q_{ji}^K}$ also have interconnection condition with

$$v_{ij}^K(t) = w_{ji}^K(t) \tag{4}$$

in the normal case. Then, the closed-loop system can be illustrated for example in Fig.1.

In this paper, we formulate the faults including actuator outage, loss of effectiveness and stuck on subsystem G_i and

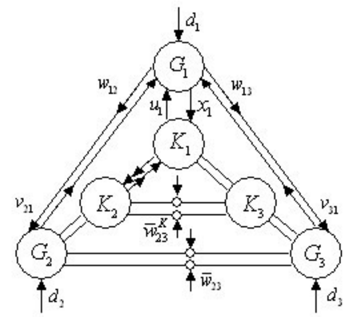

Fig. 1. Example of interconnected closed-loop system with N = 3 subsystems.

signal attenuations of interconnections between communicating subsystems. Let $u_{if}^{hF}(t)$ and $v_{ij}^{hF}(t)$ represent the signals from the fth actuator in G_i and jth communicating subsystem that have failed in the hth faulty mode, respectively. Then we denote the fault model as follows:

$$u_{if}^{hF}(t) = (1 - \rho_{if}^h(t)) u_{if}(t) + \tau_{if}^h u_{sif}(t),\ v_{ij}^{hF}(t) = \sigma_{ji}^h(t) v_{ij}(t) \tag{5}$$

where $i, j = 1, 2, \ldots, N$, $f = 1, 2, \ldots, m_i$, $h = 1, 2, \ldots, L$, $\rho_{if}^h(t)$ and $\sigma_{ji}^h(t)$ are unknown actuator efficiency factor and interconnected factor, the index h denotes the hth faulty mode and L is the total faulty modes. For every faulty mode, $\underline{\rho}_{if}^h$ and $\bar{\rho}_{if}^h$ represent the lower and upper bounds of $\rho_{if}^h(t)$, respectively. Similarly, $\underline{\sigma}_{ji}^h$ and $\bar{\sigma}_{ji}^h$ represent the lower and upper bounds of $\sigma_{ji}^h(t)$ respectively. $u_{sif}(t)$ is an unparametrizable bounded time-varying stuck-actuator fault [17] in the fth actuator of ith subsystem G_i. Note the practical case, we have $0 \le \underline{\rho}_{if}^h \le \rho_{if}^h(t) \le \bar{\rho}_{if}^h$, $0 \le \underline{\sigma}_{ji}^h \le \sigma_{ji}^h(t) \le \bar{\sigma}_{ji}^h$ and τ_{if}^h is unknown constant defined as:

$$\tau_{if}^h = \begin{cases} 0, & \rho_{if}^h < 1 \\ 0 \text{ or } 1, & \rho_{if}^h = 1. \end{cases}$$

Then, Table 1 can be given to illustrate the actuator fault model.

Denote

$$\begin{aligned} u_i^{hF}(t) &= [u_{i1}^{hF}(t), u_{i2}^{hF}(t), \cdots, u_{im_i}^{hF}(t)]^T = (I - \rho_i^h(t)) u_i(t), \\ v_{ij}^{hF}(t) &= [v_{ij1}^{hF}(t), v_{ij2}^{hF}(t), \cdots, v_{ijq_{ji}}^{hF}(t)]^T = \sigma_{ji}^h(t) v_{ij}(t) \end{aligned} \tag{6}$$

where $\rho_i^h(t) = \mathrm{diag}_f[\rho_{if}^h(t)]$, $\rho_{if}^h(t) \in [\underline{\rho}_{if}^h, \bar{\rho}_{if}^h]$, $\sigma_{ji}^h(t) = \mathrm{diag}_k[\sigma_{jik}^h(t)]$, $\sigma_{jik}^h \in [\underline{\sigma}_{jik}^h, \bar{\sigma}_{jik}^h]$, $i, j = 1, 2, \ldots, N$, $h = 1, 2, \ldots, L$, $f = 1, 2, \ldots, m_i$ and $k = 1, 2, \ldots, q_{ji}$. Then, the sets of operators with above structures are denoted by

$$\begin{aligned} \Delta_{\rho_i^h} &= \{\rho_i^h(t) : \rho_i^h(t) = \mathrm{diag}_f[\rho_{if}^h(t)], \rho_{if}^h(t) \in [\underline{\rho}_{if}^h, \bar{\rho}_{if}^h]\}, \\ \Delta_{\sigma_{ji}^h} &= \{\sigma_{ji}^h(t) : \sigma_{ji}^h(t) = \mathrm{diag}_k[\sigma_{jik}^h(t)], \sigma_{jik}^h(t) \in [\underline{\sigma}_{jik}^h, \bar{\sigma}_{jik}^h]\}. \end{aligned} \tag{7}$$

For convenience in the following sections, for all possible faulty modes L, the following uniform actuator and interconnection fault model is exploited:

$$\begin{aligned} u_i^F(t) &= (I - \rho_i(t)) u_i(t) + \tau_i u_{si}(t), \quad \rho_i \in \{\rho_i^1 \cdots \rho_i^L\} \\ v_{ij}^F(t) &= \sigma_{ji}(t) v_{ij}(t), \quad \sigma_{ji} \in \{\sigma_{ji}^1 \cdots \sigma_{ji}^L\}. \end{aligned} \tag{8}$$

TABLE I

ACTUATOR FAULT MODEL

Fault model	$\underline{\rho}_{if}^h$	$\bar{\rho}_{if}^h$	τ_{if}^h
Normal	0	0	0
Outage	1	1	0
Loss of effectiveness	>0	<1	0
Stuck	1	1	1

We make the following assumption on the subsystem matrices:

Assumption 1. Consider (1) and (3), the subsystems are interconnected only through their states, that means,

$$A_{SS}^i = 0, \quad B_{Su}^i = 0, \quad B_{Sd}^i = 0, \quad K_{i22} = 0. \tag{9}$$

Here, we let $\bar{w}_{ji}(t) \in R^{q_{ji}}$ denote perturbation which combined by unknown time-varying parameter variation, noise, and nonlinearity in the transmission channel between ith and jth subsystems.

Then, based on the above description, the equation (2) can be represented by

$$v_{ij}(t) = \sigma_{ji}(t) w_{ji}(t) + \bar{w}_{ji}(t) \tag{10}$$

for all $i, j = 1 \ldots N$, and the dynamics with faulty and perturbed interconnection links (1) can be rewritten by

$$\begin{aligned} \dot{x}_i(t) = A_{TT}^i x_i(t) + \sum_{j=1}^N A_{TS}^{ij} \sigma_{ji}(t) A_{ST}^{ji} x_j(t) + \sum_{j=1}^N A_{TS}^{ij} \bar{w}_{ji}(t) \\ + B_{Tu}^i \rho_i(t) u_i(t) + B_{Tu}^i \tau_i u_{si}(t) + B_{Td}^i d_i(t). \end{aligned} \tag{11}$$

Since we assume the controller and plant use identical transmission channels, then we have

$$v_{ij}^K(t) = \sigma_{ji}(t) w_{ji}^K(t) + \bar{w}_{ji}^K(t), \quad i, j = 1 \ldots N \tag{12}$$

where $w_{ji}^K(t) \in R^{q_{ji}^K}$ respects the perturbation which influences on the controller. Then, in terms of (3) and Assumption 1, the controller form can be described as:

$$\begin{aligned} u_i(t) = & \hat{K}_{i1}(t) x_i(t) \\ & + \sum_{j=1}^N K_{i2} \sigma_{ji}(t) \hat{K}_{i3}(t) x_j(t) + \sum_{j=1}^N K_{i2} \sigma_{ji}(t) \bar{w}_{ji}^K(t) + K_{i4}(t) \end{aligned} \tag{13}$$

where $\sigma_{ji}(t) \in \Delta_{\sigma_{ji}^h}$; $\hat{K}_{i1}(t)$, $\hat{K}_{i3}(t)$ are the estimate of $K_{i1}(t)$ and $K_{i3}(t)$, respectively; K_{i2} is an appropriate dimensions matrix chosen by the system designer; $K_{i4}(t)$ is given by a function. All the controller parameters will be designed in section 3 in detail.

For achieving the fault-tolerant asymptotic tracking control objective, the following assumptions in design are assumed to be valid.

Assumption 2. All pairs $[A_{TT}^i, B_{Tu}^i (I - \rho_i(t))]$ are uniformly completely controllable for any actuator failure mode $\rho_i \in \{\rho_i^1 \ldots \rho_i^L\}$ under consideration. That is, there exists constant matrix $K_{i1} \in R^{m_i \times n_i}$ such that, for an asymptotically stable matrix A_{mi},

$$A_{TT}^i + B_{Tu}^i (I - \rho_i(t)) K_{i1} = A_{mi}. \tag{14}$$

Assumption 3. There exists a constant matrix $K_{i3} \in R^{q_{ij} \times n_i}$ such that

$$A_{TS}^{ij} \sigma_{ji} A_{ST}^{ji} + B_{Tu}^i K_{i2} \sigma_{ji} K_{i3} = A_{mij}, \quad i, j = 1, 2, \ldots, N \tag{15}$$

for any $\sigma_{ji} \in \Delta_{\sigma_{ji}^h}$.

Assumption 4. Subsystem control input matrix B_{Tu}^i is known and other subsystem matrices are unknown.

Now, the reference model for each individual subsystem G_i, as introduced in [12], is adopted here

$$\dot{x}_{mi}(t) = A_{mi} x_{mi}(t) + B_{mi} r_i(t) + \sum_{j=1}^N A_{mij} x_{mj}(t) \tag{16}$$

where $i, j = 1, 2, \ldots, N$, $x_{mi}(t) \in R^{n_i}$ and $x_{mj}(t) \in R^{n_j}$ are the state variables, $r_i(t) \in R^{m_i}$ is the bounded and piecewise continuous reference input. A_{mi} and B_{mi} are the known constant matrices with appropriate dimensions, and A_{mi} is stable. For the sake of designing robust fault-tolerant tracking controller, we make the following standard assumption on the reference model:

Assumption 5. Define $\zeta_i = \lambda_{\max}(\sum_{j=1}^N \varepsilon_i^{-1} A_{mji}^T A_{mji})$, $\varepsilon_i > 0$, $\xi_i > 0$, the following matrix

$$H_i = \begin{bmatrix} A_{mi} & NI \\ -\varepsilon_i^{-1}(\zeta_i + \xi_i) I & -A_{mi}^T \end{bmatrix}$$

has no eigenvalues on the imaginary axis.

Now, an assumption which is quite natural and common in the fault-tolerant controller design and robust control literature introduced as follows:

Then, consider the large scale system described by (11) with actuator faults (5), interconnection faults and perturbations (10), and external disturbances. The design problem under consideration is to find a direct adaptive state feedback controller (13) such that

1). During normal operation, the closed-loop system is stable, and the state vector $x_i(t)$ tracks the given reference state vector $x_{mi}(t)$ without steady-state error, that is $\lim_{t \to \infty} e_i(t) = 0$,

$$e_i(t) = x_i(t) - x_{mi}(t), \quad i = 1, 2, \ldots, N. \tag{17}$$

2). In the event of actuator and interconnection faults, the closed-loop system is still stable, and the state vector $x_i(t)$ tracks the given reference state vector $x_{mi}(t)$ without steady-state error.

III. DISTRIBUTED ADAPTIVE ROBUST TRACKING CONTROL SYSTEMS DESIGN

Consider a large scale system G_i described by (11) and controller model given by (13), the controller gain $\hat{K}_{i1}(t) = [\hat{K}_{i1,1}(t), \hat{K}_{i1,2}(t), \ldots, \hat{K}_{i1,m_i}(t)]^T \in R^{m_i \times n_i}$ updated by the following adaptive law: $i = 1, 2, \ldots, N, f = 1, 2, \ldots, m_i$

$$\frac{d\hat{K}_{i1,f}(t)}{dt} = -\Gamma_{i1,f} x_i e_i^T P b_{Tu,f}^i \tag{18}$$

where $\Gamma_{i1,f}$ is any positive constant, $\hat{K}_{i1,f}(t_0)$ is finite, and $b_{Tu,f}^i$ is the fth column of B_{Tu}^i; K_{i2} is an appropriate dimensions matrix chosen by the system designer; $\hat{K}_{i3}(t) =$

$[\hat{K}_{i3,1}(t),\hat{K}_{i3,2}(t),\ldots,\hat{K}_{i3,q_{ji}}(t)]^T \in R^{q_{ji}\times n_i}$ updated according to the adaptive law: $i=1,2,\ldots,N, k=1,2,\ldots,q_{ji}$

$$\frac{d\hat{K}_{i3,k}(t)}{dt} = -\Gamma_{i3,k} x_j e_i^T P B_{Tu}^i k_{i2,k} \tag{19}$$

where Γ_{3k} is any positive constant, $\hat{K}_{i3,k}(t_0)$ is finite, $k_{i2,k}$ is the kth column of K_{i2}; $K_{i4}(t)$ is given by the following function:

$$K_{i4}(t) = \frac{-(e_i^T P_i B_{Tu}^i)^T \beta_i \parallel e_i^T P_i \parallel \hat{k}_{i5}(t)}{\parallel e_i^T P_i B_{Tu}^i \parallel^2 \alpha_i}, \quad i=1,2,\ldots,N \tag{20}$$

where α_i, β_i are suitable positive constants which satisfied:

$$\parallel e_i^T P_i B_{Tu}^i \parallel^2 \alpha_i \leq \parallel e_i^T P_i B_{Tu}^i \sqrt{\underline{\rho}_i^h} \parallel^2 \beta_i \tag{21}$$

for any $\underline{\rho}_i^h = \mathrm{diag}_f[\underline{\rho}_{if}^h] \in \Delta_{\rho_i^h}$, $i=1,2,\ldots,N$, $f=1,2,\ldots,m_i$, $h=1,2,\ldots,L$; and $\hat{k}_{i5}(t)\in R$ is updated by the following adaptive law:

$$\frac{d\hat{k}_{i5}(t)}{dt} = \gamma_i \parallel e_i^T P_i \parallel \quad i=1,2,\ldots,N \tag{22}$$

where γ_i is any positive constant, $\hat{k}_{i5}(t_0)$ is finite, and from (22), we can see $\hat{k}_{i5}(t)\geq 0$ if $\hat{k}_{i5}(t_0)\geq 0$.

On the other hand, letting

$$\begin{aligned}\tilde{K}_{i1,f}(t) &= \hat{K}_{i1,f}(t) - K_{i1,f},\\ \tilde{K}_{i3,k}(t) &= \hat{K}_{i3,k}(t) - K_{i3,k},\\ \tilde{k}_{i5}(t) &= \hat{k}_{i5}(t) - k_{i5}.\end{aligned} \tag{23}$$

Due to $K_{i1,f}$, $K_{i3,k}$, and k_{i5} are unknown constants, we can write the following error system

$$\begin{aligned}\frac{d\tilde{K}_{i1,f}(t)}{dt} &= -\Gamma_{i1,f} x_i e_i^T P b_{Tu,f}^i,\\ \frac{d\tilde{K}_{i3,k}(t)}{dt} &= -\Gamma_{i3,k} x_j e_i^T P B_{Tu}^i k_{i2,k},\\ \frac{d\tilde{k}_{i5}(t)}{dt} &= \gamma_i \parallel e_i^T P_i \parallel .\end{aligned} \tag{24}$$

where $i=1,2,\ldots,N, f=1,2,\ldots,m_i, k=1,2,\ldots,q_{ji}$.

Then, following above description, the large scale closed-loop system error model can be written by

$$\begin{aligned}\dot{e}_i(t) = {}& A_{mi}e_i(t) + \sum_{j=1}^{N} A_{mij}e_j(t) + B_{Tu}^i \tau_i u_{si}(t) + B_{Td}^i d_i(t)\\ & - B_{mi}r_i(t) - \sum_{j=1}^{N} B_{Tu}^i \rho_i(t) K_{i2}\sigma_{ji}(t)\hat{K}_{i3}(t)x_j(t)\\ & + B_{Tu}^i \rho_i(t)\sum_{j=1}^{N} K_{i2}\sigma_{ji}(t)\bar{w}_{ji}^K(t) + B_{Tu}^i K_{i4}(t)\\ & + \sum_{j=1}^{N} A_{ST}^{ij}\bar{w}_{ji}(t) + (A_{TT}^i + B_{Tu}^i\rho_i(t)\tilde{K}_{i1}(t))x_i(t)\\ & + \sum_{j=1}^{N} B_{Tu}^i K_{i2}\sigma_{ji}(t)\tilde{K}_{i3}(t)x_j(t).\end{aligned} \tag{25}$$

Before giving our main result, the following Lemmas are introduced firstly:

Lemma 1. For appropriate dimensions matrices X, Y, and any $\varepsilon>0$, the inequality

$$X^TY + Y^TX \leq \varepsilon X^TX + \varepsilon^{-1}Y^TY \tag{26}$$

holds true.

Lemma 2. ([12], [22]) Consider the following Algebraic Riccati Equation:

$$A^TP + PA + PRP + Q = 0. \tag{27}$$

If $R=R^T\geq 0$, $Q=Q^T>0$, A is Hurwitz, and the associated Hamiltonian matrix

$$H = \begin{bmatrix} A & R \\ -Q & -A^T \end{bmatrix}$$

has no eigenvalues on the imaginary axis, then there exists a $P=P^T>0$, which is the solution of (27).

In the following, by $(e_i,\tilde{K}_{i1},\tilde{K}_{i3},\tilde{k}_{i5})(t)$ we denote a solution of the closed-loop system and the error system. Then, the following theorem can be obtained which shows globally boundedness of the solutions of the adaptive closed-loop system described by (25) and (24).

Theorem 1. Consider the adaptive closed-loop system described by (25) and (24) under Assumptions 1-5. The closed-loop large scale system is uniformly bounded and the tracking error $e(t)$ converges asymptotically to zero as the time t goes to infinity for any $\rho(t)\in\Delta_{\rho_i^h}$ and $\sigma(t)\in\Delta_{\sigma_{ji}^h}$, if there exist a symmetric matrix $P>0$, and $\hat{K}_{i1,f}$, $\hat{K}_{i3,k}$, $\hat{k}_{i5}$ determined according to the adaptive laws (18), (19) and (22), and control gain function K_{i4} given by (20).

Proof: Due to the space limitations, we omit the proof. ■

Thus, for the large-scale error system (25) with actuator failures, signal attenuations and perturbations in interconnection channels, and external disturbances, from adaptive laws (18), (19), (22), and control gain function (20), we have the information of $\hat{K}_{i1}$, $\hat{K}_{i3}$, K_{i4}, $i=1,2,\ldots,N$, respectively. Then, we can obtain the distributed adaptive controllers (13), by which the solutions of the resulting adaptive closed-loop large-scale system can be guaranteed to be stable, and the state of each subsystem is uniformly asymptotically tracking the corresponding state of model-reference system.

IV. Numerical Example

A large-scale dynamical system, with faults on actuators and interconnections, perturbations in interconnection channels, and disturbances, is composed of two dynamical subsystems. The system matrices described by

$$A_{TT}^1 = \begin{bmatrix} -3 & -1 \\ 0 & 2 \end{bmatrix}, A_{TT}^2 = \begin{bmatrix} -3 & 0 \\ 1 & 2 \end{bmatrix},$$
$$A_{TS}^{21} = \begin{bmatrix} 2 & -2 \\ 0.5 & -1 \end{bmatrix}, A_{TS}^{12} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix},$$
$$A_{ST}^{21} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}, A_{ST}^{12} = \begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix},$$
$$B_{Td}^1 = \begin{bmatrix} 1 & -0.5 \\ -0.1 & 1 \end{bmatrix}, B_{Td}^2 = \begin{bmatrix} 1 & 0.5 \\ -0.5 & 1 \end{bmatrix},$$
$$B_{Tu}^1 = \begin{bmatrix} 0.1 & 0.5 & -1 \\ -1 & 0.3 & 1 \end{bmatrix}, B_{Tu}^2 = \begin{bmatrix} 0.2 & 1 & -1.5 \\ -1 & 1 & 0.5 \end{bmatrix}.$$

We let C_{ij}^s denote the sth interconnection links with signals transmit from the ith subsystem to the jth subsystem. Then, we assume each of the four interconnection links may lose its effectiveness and consider the following two possible faulty

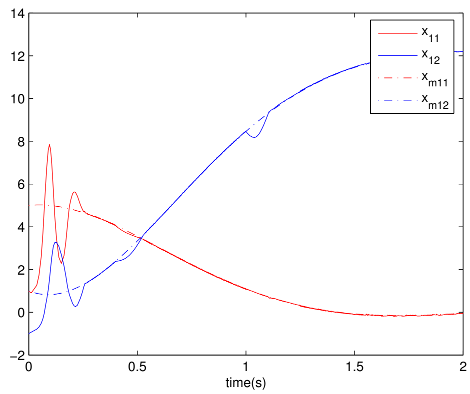

Fig. 2. Response curves of $x_{1f}(t)$ and $x_{m1f}(t), f = 1,2$ for the first subsystem with distributed adaptive tracking controllers.

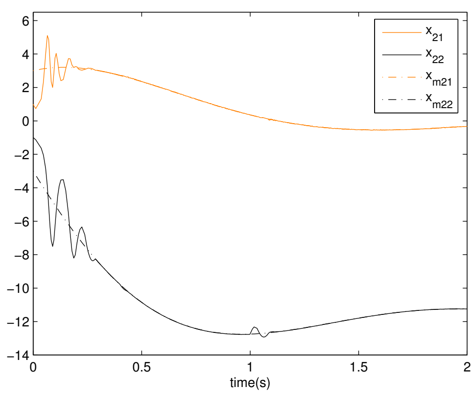

Fig. 3. Response curves of $x_{2f}(t)$ and $x_{m2f}(t), f = 1,2$ for the second subsystem with distributed adaptive tracking controllers..

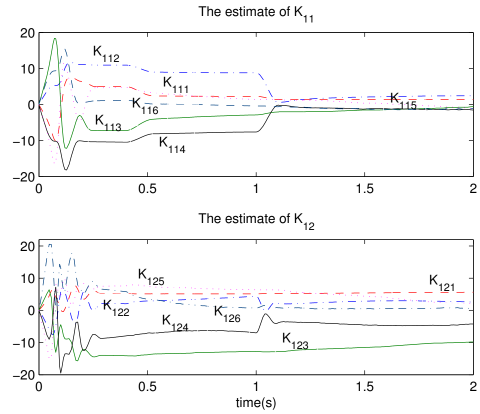

Fig. 4. Response curves of the estimates of controller parameters $K_{i1}, i = 1,2$.

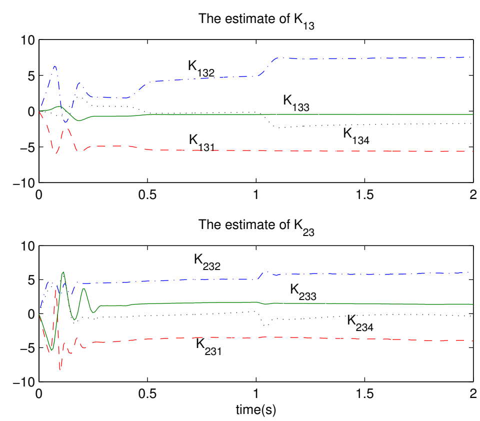

Fig. 5. Response curves of the estimates of controller parameters $K_{i3}, i = 1,2$.

modes:
Normal mode 1: Both of the two subsystems' actuators and interconnection links are normal, described by,

$$\rho_{if}^1(t) = 0, \quad \sigma_{ijk}^1(t) = 1, \quad i,j = 1,2, f = 1,2,3, k = 1,2.$$

Faulty mode 2: The first actuators of subsystems G_1 and G_2 are outage, and other actuators may be normal or loss of effectiveness, that is,

$$\rho_{11}^2(t) = \rho_{21}^2(t) = 1, \quad 0 \leq \rho_{if}^2(t) < 1, i = 1,2, f = 2,3.$$

On the other hand, interconnection links C_{12}^1 and C_{21}^1 are completely disconnected, the other links may be normal or attenuations, that is,

$$\sigma_{11}^2(t) = \sigma_{21}^2(t) = 0, \quad 0 < \sigma_{12}^2(t) \leq 1, 0 < \sigma_{22}^2(t) \leq 1.$$

To verify the effectiveness of the proposed adaptive method, the simulations are given with the following parameters and initial conditions:

$\Gamma_{11,f} = 10I_2$, $\Gamma_{21,f} = 10I_2$, $\Gamma_{13,k} = 2I_2$, $\Gamma_{23,k} = 5I_2$, $\gamma_i = 1$, $\alpha_i = 1$, $\beta_i = 10$, $x_i(0) = [1,-1]^T$, $x_{m1}(0) = [5,1]^T$, $x_{m2}(0) = [3,-3]^T$, $\hat{K}_{i1,f}(0) = 0$, $\hat{K}_{i3,k}(0) = 0$, $k_{i5}(0) = 0$, $K_{i2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $\varepsilon_i = 1, i = 1,2,\ f = 1,2,3,\ k = 1,2$.

Here, the reference model matrices [11] are denoted by

$$A_{m1} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B_{m1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$A_{m2} = \begin{bmatrix} 0 & 1 \\ -3 & -1 \end{bmatrix}, B_{m2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and $r_1 = 2sin(t)$, $r_2 = cos(t)$.

Then, the following faulty case is considered in the simulations, that is, before 0.4 second, the interconnected systems operate in normal case, and the external disturbances $d_1(t) = [-5,\ 5 + sin(2t)]^T$ and $d_2(t) = [-5,\ 5sin(2t)]^T$ enter into the subsystems at the beginning $(t \geq 0)$, respectively. At 0.4 second, some faults on actuators and transmission channels have occur, described by $\rho_1 = \mathrm{diag}[1, 0.5t, 0]$, $\rho_2 = \mathrm{diag}[1, 0.4t, 0]$, $\sigma_{12} = \mathrm{diag}[0,1]$ and $\sigma_{21} = \mathrm{diag}[0,1]$, and at

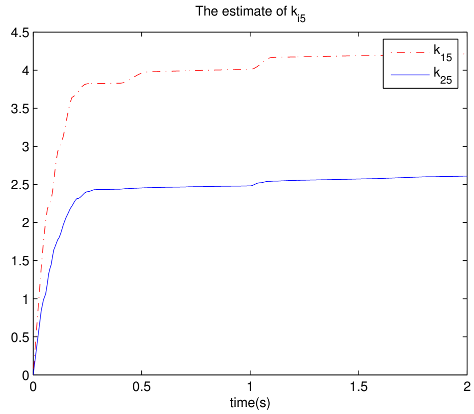

Fig. 6. Response curves of the estimates of controller parameters $K_{i5}, i = 1,2$.

the same time, the perturbations in interconnection links $\bar{w}_{12}(t) = [5,\ 8+cos(2t)]^T$ and $\bar{w}_{21}(t) = [-5,\ 5+2sin(5t)]^T$ enter into the subsystems G_1 and G_2, and $\bar{w}^K_{12}(t) = [-5,\ 5+2sin(2t)]^T$ and $\bar{w}^K_{21}(t) = [-5,\ 5cos(2t)]^T$ enter into the controllers, respectively. At 1 second, the first actuators of subsystem G_1 and G_2 have stuck at $u_{s11}(t) = 30+5sin(t)$ and $u_{s21}(t) = -30-3sin(3t)+5cos(4t)$, respectively, and the second actuators of two subsystem loss of effectiveness 50%.

Fig.2 and Fig.3 are the model reference state curves and the subsystem G_1 and G_2 state curves of with adaptive state feedback controller in above-mentioned cases, respectively. It can be easy see that the states of subsystems can asymptotically track the corresponding states of model reference systems in the presence of faults on actuators and interconnections, perturbations in interconnection channels, and external disturbances. Fig.4-Fig.6 are the response curves of the estimations of controller parameters $\hat{K}_{i1}$, $\hat{K}_{i3}$ and $\hat{k}_{i5}$, $i = 1,2$, respectively. It is easy see the estimations can converge and all signals are uniformly bounded.

V. Conclusions

This paper has presented a direct adaptive design method for solving the robust asymptotic tracking control problem for a class of large scale systems with actuator failures, faulty and perturbed interconnection links, and disturbances. For the sake of automatically compensating for the effects, the distributed state feedback controllers are constructed with the adaptive schemes, which are based on the updated adaptation laws to estimate the unknown controller parameters on-line. On the basis of Lyapunov stability theory, it has shown that the resulting adaptive closed-loop large-scale system can be guaranteed to be stable and each subsystem can asymptotically track the corresponding state of the model-reference system. A numerical example has been given to illustrate the effectiveness of the proposed method.

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