2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Stable multiple model adaptive control of nonlinear multivariable discrete-time systems

Yue Fu, Tianyou Chai and Hong Wang

Abstract— **In this paper, to further relax the restriction on the higher order nonlinearity in [7], a stable multiple model adaptive control (SMMAC) method is developed. First a new robust adaptive controller is designed, which can guarantee the stability of the closed-loop system. Then to improve the system performance, the SMMAC method is presented by switching between the robust adaptive controller and a conventional neural network (NN) adaptive controller. Theory analysis and simulation results are presented to show the effectiveness of the proposed method.**

I. INTRODUCTION

Nonlinear adaptive control of continuous-time systems has been intensively studied in the past several decades [1-3]. It is only recently that, along with the introduction of neural networks (NNs), issues have related to nonlinear adaptive control of discrete-time systems [4-6]. In practical applications, it is desirable to have a systematic method of ensuring stability and performance properties of the overall neural network (NN) adaptive system. However, just as what is described in [7], due to the complexity of the structure of an NN and the nonlinear dependence of its map on the parameter values, stability analysis of the resulting adaptive systems has always been very difficult and quite often intractable.

Although stability and convergence results for adaptive control using NNs have been presented [8-12], some problems remain unsolved, and most of them suffer from one or more of the following drawbacks [7]: (i) The NNs used are linearly parameterized. Even when the structure allows for nonlinear parameters, they are kept fixed. (ii) The system to be controlled is of a special structure, e.g., an affine structure. Such results cannot be directly applied to a system which is nonlinearly dependent on the current input variables. (iii) The stability result is obtained based on the assumption that the estimate of an NN has to be close enough to the true nonlinearity. In practice, the closeness is hard to decide, since it is well known that parameters usually do not converge to their true values even after extensive off-line training.

To overcome the problems mentioned above, a stable multiple model adaptive control method is proposed in [7], which is composed of a linear robust adaptive controller, an NN based nonlinear adaptive controller and a switching mechanism, and can lead to the result that all the signals in the closed-loop systems are globally bounded. In [7], the method is designed for a single-input-single-output system, and the assumption made on the nonlinear system is that it can be modeled by the sum of a linear part and a globally bounded nonlinear part. The problem becomes extremely challenging when the nonlinearity is not globally bounded. Two directions of research in this respect is indicated in [13]: one is to establish some global result for the closed-loop system by imposing more structural constraints on the plant, as is done in the integrator backstepping methods for continuous time systems. The other is to establish some local results using continuity arguments: if the initial conditions and the amplitude of the desired trajectory are small, then the system will evolve in a neighborhood of the equilibrium that satisfies the bound on the nonlinearity. However, whether there may exist a strategy that does not rely on any more structural constraints, and can establish some global result? The answer is positive. By introducing a delay difference operator, [14] proposes a globally stable multiple model NN adaptive control method for the same structure multi-input-multi-output systems and relaxes the assumption of global boundedness on the higher order nonlinear term.

In [14], the assumption is that the difference of the higher order nonlinearity is globally bounded. To further relax the restriction on the higher order nonlinearity, in this paper, we first design a new robust adaptive controller, which can guarantee the global stability of the closed-loop system, then to improve the system performance, we present the stable multiple model adaptive control (SMMAC) method by switching between the robust adaptive controller and a conventional NN adaptive controller.

The rest of the paper is organized as follows: In Section 2, the system under consideration is described, and the control problem is stated. In Section 3, the SMMAC method composed of a linear robust adaptive controller, a nonlinear NN adaptive controller and a switching mechanism is proposed. And the stability and tracking performance of the closed-loop system is analyzed. Section 4 provide simulation results showing the effectiveness of the proposed control scheme, and finally some conclusions are drawn in Section 5.

This work is partially supported by the National Fundamental Research Program of China (No. 2009CB320601), the State Key Program of National Natural Science of China (60534010), the Funds for Creative Research Groups of China (60821063), and the Program for Chang-jiang Scholars and Innovative Research Team in University (IRT0421).

Y. Fu and T. Chai are with the Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang, Liaoning Province, 110004, China; and the Research Center of Automation, Northeastern University, Shenyang, Liaoning Province, 110004, China. Email: fuyue@mail.neu.edu.cn; tychai@mail.neu.edu.cn

H. Wang is with the Control Systems Center, School of Electrical and Electronic Engineering, The University of Manchester , M60 1QD, Manchester, The United Kingdom. Email: hong.wang@manchester.ac.uk

978-1-4244-4524-0/09/$25.00 ©2009 AACC

II. PROBLEM FORMULATION

It is assumed that the discrete-time multivariable system under control can be expressed as the following model.

$$A(z^{-1})y(t+1) = B(z^{-1})u(t) + v[y(t), \cdots, y(t-n_a+1), u(t), \cdots, u(t-n_b)] \quad (1)$$

where $u(t), y(t) \in \mathscr{R}^n$ are the system input and output vectors respectively; $v[\cdot] \in \mathscr{R}^n$ is an unknown continuous vector-valued nonlinear function; $A(z^{-1})$ and $B(z^{-1})$ are two $n \times n$ matrix polynomials in the backward shift operator z^{-1}, i.e.,

$$\begin{aligned} A(z^{-1}) &= I + A_1 z^{-1} + \cdots + A_{n_a} z^{-n_a}, \\ B(z^{-1}) &= B_0 + B_1 z^{-1} + \cdots + B_{n_b} z^{-n_b} \end{aligned} \quad (2)$$

with I being an identity matrix; $A_i, i = 1, \cdots, n_a; B_j, j = 0, \cdots, n_b$ being unknown constant matrices and n_a, n_b being known structural orders. To make the design procedure easier to follow, define

$$x(t) = [y(t), \cdots, y(t-n_a+1), u(t), \cdots, u(t-n_b)] \quad (3)$$

Denoting the n-dimension reference input vector as $w(t)$, the output tracking error can be expressed as

$$e_c(t) = y(t) - w(t) \quad (4)$$

Assumption 1 [14]. (i) The linear parameter matrices $A_i, i = 1, \cdots, n_a; B_j, j = 0, \cdots, n_b$ lie in a compact region $\sum$, and B_0 is nonsingular; (ii) The system has a globally uniformly asymptotically stable zero dynamics.

Assumption 2. The higher order nonlinearity satisfies

$$\|v[x(t), u(t)]\| \leq \alpha_0 + \alpha_1 \|y(t)\| + \cdots + \alpha_{n_a} \|y(t-n_a+1)\| \quad (5)$$

where $\alpha_i|_{i=0,\cdots,n_a} > 0$ and is known.

Assumption 3. For the nonlinear equations

$$B_0 \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} + \begin{pmatrix} f_1(u_1, u_2, \cdots, u_n, X_0) \\ f_2(u_1, u_2, \cdots, u_n, X_0) \\ \vdots \\ f_n(u_1, u_2, \cdots, u_n, X_0) \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} \quad (6)$$

where $u_1, u_2, \cdots, u_n$ are unknown variables; $X_0 \in \mathscr{R}^m$ is an arbitrary given vector; $f_i|_{i=1,\cdots,n} : \mathscr{R}^n \times \mathscr{R}^m \rightarrow \mathscr{R}$ are continuous bounded nonlinear functions and $B_0 \in \mathscr{R}^{n \times n}$ is a nonsingular matrix, there exist $u_1^*, u_2^*, \cdots, u_n^*$ which satisfy Eq. (6) for arbitrary ideal constants $r_1, r_2, \cdots, r_n$.

Remark 1. In general, the right hand side of Ineq (5) should include all the variables of the function $v[x(t), u(t)]$, i.e., the condition,

$$\begin{aligned} \|v[x(t), u(t)]\| \leq{}& \alpha_0 + \alpha_1 \|y(t)\| + \cdots + \\ & \alpha_{n_a} \|y(t-n_a+1)\| + \beta_0 \|u(t)\| \\ & + \cdots + \beta_{n_b} \|u(t-n_b)\| \end{aligned}$$

with $\beta_j|_{j=0,\cdots,n_b} > 0$ and being also known, should be used. However, in this paper, in the right hand side of Ineq (5), the variables $u(t), u(t-1), , u(t-n_b)$ of $v[x(t), u(t)]$ are not included. In practice, if $u(t)$ in $v[x(t), u(t)]$ is constrained by a bounded function, or if $u(t)$ in Eq. (1) is saturated, then Ineq (5) can be commonly satisfied. Comparing with [14], where the difference of the nonlinearity is assumed to be globally bounded, it is obvious that Assumption 2 is still quite mild. The aim of the design is to determine a control signal so that the input and output signals of the closed-loop system remain bounded, whilst the output tracking error is made as small as possible. Similar to [7] and [14], the structure of the control system is composed of a linear model, a nonlinear model, a robust adaptive controller, an NN adaptive controller and a switching mechanism.

III. LINEAR ROBUST ADAPTIVE CONTROL

For preliminary, we choose a diagonal matrix polynomial

$$P(z^{-1}) = P_0 + P_1 z^{-1} + \cdots + P_{n_p} z^{-n_p} \quad (7)$$

with $n_p \leq n_a$, and then introduce the following equation.

$$P(z^{-1}) = A(z^{-1}) + z^{-1} G(z^{-1}) \quad (8)$$

where $G(z^{-1})$ with the order $n_a - 1$ is a matrix polynomial and uniquely determined by Eq. (8). Combining Eq. (1) and Eq. (8), we can obtain

$$\begin{aligned} P(z^{-1})y(t+1) ={}& G(z^{-1})y(t) \\ & + B(z^{-1})u(t) + v[x(t), u(t)] \end{aligned} \quad (9)$$

Denote $G(z^{-1}) = G_0 + G_1 z^{-1} + \cdots + G_{n_a-1} z^{-n_a+1}; \Theta = [G_0, \cdots, G_{n_a-1}, B_0, \cdots, B_{n_b}]^{\mathrm{T}}; X(t) = [y(t)^{\mathrm{T}}, \cdots, y(t-n_a+1)^{\mathrm{T}}, u(t)^{\mathrm{T}}, \cdots, u(t-n_b)^{\mathrm{T}}]^{\mathrm{T}}; \phi(t+1) = P(z^{-1})y(t+1)$, then Eq. (9) can be rewritten as

$$\phi(t+1) = \Theta^{\mathrm{T}} X(t) + v[x(t), u(t)] \quad (10)$$

First, the linear estimate model of Eq. (10) is defined as

$$\hat{\phi}(t+1) = \hat{\Theta}(t)^{\mathrm{T}} X(t) \quad (11)$$

where $\hat{\Theta}(t) = [\hat{G}_0(t), \cdots, \hat{G}_{n_a-1}(t), \hat{B}_0(t), \hat{B}_1(t), \cdots, \hat{B}_{n_b}(t)]^{\mathrm{T}}$ is an estimate of Θ at time instant t. The linear model error is

$$\begin{aligned} e(t) &= \phi(t) - \hat{\phi}(t) \\ &:= \phi(t) - \hat{\Theta}(t-1)^{\mathrm{T}} X(t-1) \\ &:= [\Theta - \hat{\Theta}(t-1)]^{\mathrm{T}} X(t-1) \\ &\quad + v[x(t-1), u(t-1)] \end{aligned} \quad (12)$$

Denote

$$\begin{aligned} M(t) &= \alpha_0 + \alpha_1 \|y(t)\| + \cdots + \alpha_{n_a} \|y(t-n_a+1)\| \\ &= \alpha_0 + (\alpha_1 z^{-1} + \cdots + \alpha_{n_a} z^{-n_a}) \|y(t+1)\| \end{aligned} \quad (13)$$

and introduce the relative deadzone

$$\lambda(t) = \begin{cases} 1 & \text{if } \|e(t)\| > 2M(t-1) \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

In this paper, the following identification algorithm is used to update $\hat{\Theta}(t)$.

$$\hat{\Theta}'(t) = \hat{\Theta}(t-1) + \frac{\lambda(t) X(t-1) e(t)^{\mathrm{T}}}{1 + X(t-1)^{\mathrm{T}} X(t-1)} \quad (15)$$

$$\hat{\Theta}(t) = \text{proj}\{\hat{\Theta}'(t)\} \tag{16}$$

where $\hat{\Theta}'(t) = [\hat{G}_0(t), \cdots, \hat{G}_{n_a-1}(t), \hat{B}'_0(t), \hat{B}_1(t), \cdots, \hat{B}_{n_b}(t)]^{\mathrm{T}}$; proj is a projection operator, satisfying

$$\text{proj}\{\hat{\Theta}'(t)\} = \begin{cases} \hat{\Theta}'(t); & \hat{B}'_0(t) \text{ is nonsingular} \\ [\cdots, \hat{B}'_0(t-1), \cdots]; & \text{otherwise} \end{cases} \tag{17}$$

The linear robust adaptive controller is then obtained as

$$\hat{\Theta}(t)^{\mathrm{T}} X(t) = P(z^{-1}) w(t+1) \tag{18}$$

Lemma 1. If the recursive least square algorithm described by Eqs. (11)-(17) is used to update the estimate $\hat{\Theta}(t)$, we have the following results.

(i) $\hat{\Theta}(t)$ is bounded for all $t \geq 0$;

$$(ii) \;\; \lim_{N\to\infty} \sum_{t=1}^{N} \frac{\lambda(t)[\|e(t)\| - 2M(t-1)]^2}{2(1 + \|X(t-1)\|^2)} < \infty;$$

$$(iii) \;\; \lim_{t\to\infty} \frac{\lambda(t)[\|e(t)\| - 2M(t-1)]^2}{2(1 + \|X(t-1)\|^2)} \to 0.$$

Proof. Define $\tilde{\Theta}(t) = \hat{\Theta}(t) - \Theta$, then from Eqs. (15)-(17), using Eqs. (12), (14) and adopting the similar approach as what is described in [8], it follows that

$$\begin{aligned} \|\tilde{\Theta}(t)\|^2 &\leq \|\tilde{\Theta}(t-1)\|^2 - \frac{\lambda(t)[\|e(t)\|^2 - 4M(t-1)^2]}{2(1 + \|X(t-1)\|^2)} \\ &\leq \|\tilde{\Theta}(t-1)\|^2 - \frac{\lambda(t)[\|e(t)\| - 2M(t-1)]^2}{2(1 + \|X(t-1)\|^2)} \end{aligned} \tag{19}$$

Since $\lambda(t) = 1$ for $\|e(t)\| > 2M(t-1)$ and is 0 otherwise, $\{\|\tilde{\Theta}(t)\|^2\}$ is a nonincreasing sequence. Consenquently, results $(i) - (iii)$ are valid.

Theorem 1. For the system (1) with the adaptive algorithm (11)-(17) and the robust adaptive controller (18), if the stable matrix polynomial $P(z^{-1})$ is chosen such that

$$\begin{aligned} S(z^{-1}) = \|P_0\| &- (\|P_1\| + 2\alpha_1) z^{-1} \\ &- \cdots - (\|P_{n_p}\| + 2\alpha_{n_p}) z^{-n_p} \\ &- 2\alpha_{n_p+1} z^{-n_p-1} - \cdots - 2\alpha_{n_a} z^{-n_a} \end{aligned} \tag{20}$$

is also stable, then we have

$$(iv) \;\; \lim_{t\to\infty} \lambda(t)[\|e(t)\| - 2M(t-1)]^2 \to \infty, \text{i.e.,}$$

$$\lim_{t\to\infty} \sup[\|e(t)\| - 2M(t-1)] \leq 0;$$

(v) the input and output signals in the closed-loop system are bounded.

Proof. Denote the generalized tracking error

$$\bar{e}(t) = P(z^{-1}) y(t) - P(z^{-1}) w(t) \tag{21}$$

then from Eqs. (12), (18), and the certainty equivalence principle, we have

$$\begin{aligned} e(t) &= \phi(t) - \hat{\phi}(t) \\ &= P(z^{-1}) y(t) - \hat{\Theta}(t)^{\mathrm{T}} X(t) \\ &= P(z^{-1}) y(t) - P(z^{-1}) w(t) \\ &= \bar{e}(t) \end{aligned} \tag{22}$$

Consequently,

$$\begin{aligned} &\|P(z^{-1})y(t)\| \leq \|e(t)\| + \|P(z^{-1})w(t)\| \\ &\Rightarrow \|P(z^{-1})y(t)\| - 2M(t-1) \\ &\leq \|e(t)\| - 2M(t-1) + \|P(z^{-1})w(t)\| \\ &\Rightarrow \|P(z^{-1})y(t)\| - 2\alpha_0 - 2(\alpha_1 z^{-1} + \cdots + \alpha_{n_a} z^{-n_a})\|y(t)\| \\ &\leq \|e(t)\| - 2M(t-1) + \|P(z^{-1})w(t)\| \\ &\Rightarrow [\|P_0\| - (\|P_1\| + 2\alpha_1)z^{-1} - \cdots - (\|P_{n_p} + 2\alpha_{n_p})z^{-n_p} \\ &\quad -2\alpha_{n_p+1} z^{-n_p-1} - \cdots - 2\alpha_{n_a} z^{-n_a}]\|y(t)\| \\ &\leq \|e(t)\| - 2M(t-1) + \|P(z^{-1})w(t)\| + 2\alpha_0 \end{aligned}$$

i.e.,

$$\begin{aligned} S(z^{-1})\|y(t)\| \leq \|e(t)\| &- 2M(t-1) \\ &+ \|P(z^{-1})w(t)\| + 2\alpha_0 \end{aligned} \tag{23}$$

Since $S(z^{-1})$ is stable, and $\|P(z^{-1})w(t)\|$ and $2\alpha_0$ are bounded, there exist constants C_1, C_2 such that

$$\|y(t)\| \leq C_1 + C_2 \max_{0\leq\tau\leq t} [\|e(\tau)\| - 2M(\tau - 1)] \tag{24}$$

By Assumption 2 and the fact $X(t) = [y(t)^{\mathrm{T}}, \cdots, u(t)^{\mathrm{T}}, \cdots]^{\mathrm{T}}$, there exist constants C_3, C_4 such that

$$\|X(t)\| \leq C_3 + C_4 \max_{0\leq\tau\leq t} [\|e(\tau)\| - 2M(\tau - 1)] \tag{25}$$

From Eq. (25) and (iii) in Lemma 1, and using the key technique lemma in [15], we can obtain the results (iv) and (v).

IV. Stable multiple model adaptive control

A. Linear model and robust adaptive controller

The linear estimate model of Eq. (10) is defined as

$$\hat{\phi}_1(t+1) = \hat{\Theta}_1(t)^{\mathrm{T}} X(t) \tag{26}$$

where $\hat{\Theta}_1(t) = [\hat{G}_0(t), \cdots, \hat{G}_{n_a-1}(t), \hat{B}_0(t), \hat{B}_1(t), \cdots, \hat{B}_{n_b}(t)]^{\mathrm{T}}$ is an estimate of Θ at time instant t, and is updated as

$$\hat{\Theta}'_1(t) = \hat{\Theta}_1(t-1) + \frac{\lambda_1(t) X(t-1) e_1(t)^{\mathrm{T}}}{1 + X(t-1)^{\mathrm{T}} X(t-1)} \tag{27}$$

$$\hat{\Theta}_1(t) = \text{proj}\{\hat{\Theta}'_1(t)\} \tag{28}$$

$$\lambda_1(t) = \begin{cases} 1 & \text{if } \|e_1(t)\| > 2M(t-1) \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

$$\begin{aligned} e_1(t) &= \phi(t) - \hat{\phi}_1(t) \\ &:= \phi(t) - \hat{\Theta}_1(t-1)^{\mathrm{T}} X(t-1) \\ &:= [\Theta - \hat{\Theta}_1(t-1)]^{\mathrm{T}} X(t-1) \\ &\quad + v[x(t-1), u(t-1)] \end{aligned} \tag{30}$$

where $\hat{\Theta}'_1(t) = [\hat{G}_0(t), \cdots, \hat{G}_{n_a-1}(t), \hat{B}'_0(t), \hat{B}_1(t), \cdots, \hat{B}_{n_b}(t)]^{\mathrm{T}}$; proj is a projection operator, satisfying

$$\text{proj}\{\hat{\Theta}'_1(t)\} = \begin{cases} \hat{\Theta}'_1(t); & \hat{B}'_0(t) \text{ is nonsingular} \\ [\cdots, \hat{B}'_0(t-1), \cdots]; & \text{otherwise} \end{cases} \tag{31}$$

The linear robust adaptive controller $u_1(t)$ considered in this paper can be computed by

$$\hat{\Theta}_1(t)^{\mathrm{T}} X_1(t) = P(z^{-1}) w(t+1) \tag{32}$$

where $X_1(t) = [y(t)^{\mathrm{T}}, \cdots, y(t-n_a+1)^{\mathrm{T}}, u_1(t)^{\mathrm{T}}, u(t-1)\mathrm{T}, \cdots, u(t-n_b)\mathrm{T}]^{\mathrm{T}}$.

B. Nonlinear model and NN adaptive controller

The nonlinear estimate model of Eq. (10) is defined as

$$\hat{\phi}_2(t+1) = \hat{\Theta}_2(t)^{\mathrm{T}} X(t) + \hat{v}^*[x(t), u(t)] \tag{33}$$

where $\hat{\Theta}_2(t) = [\cdots, \hat{B}_{2,0}(t), \cdots]^{\mathrm{T}}$ is an another estimate of Θ at time instant t; $\hat{v}^*[x(t), u(t)]$ is an NN estimate of $v^*[x(t), u(t)]$ at time instant t with $\hat{v}^*[x(t), u(t)] = \phi(t+1) - \hat{\Theta}_2(t)^{\mathrm{T}} X(t)$, i.e.,

$$\hat{v}^*[x(t), u(t)] = NN[\hat{W}(t), X(t)] \tag{34}$$

where $NN[\cdot]$ represents the structure of the adopted NN; $X(t)$ is the input vector; $\hat{W}(t)$ is the estimate of the ideal weight matrix W^*. The nonlinear model error is

$$\begin{aligned} e_2(t) &= \phi(t) - \hat{\phi}_2(t) \\ &:= \phi(t) - \hat{\Theta}_2(t-1)^{\mathrm{T}} X(t-1) \\ &\quad - \hat{v}^*[x(t-1), u(t-1)] \end{aligned} \tag{35}$$

Similar to [7] and [14], no restriction is made on how the parameters $\hat{\Theta}_2(t)$ and $\hat{W}(t)$ are updated except that they always lie in-side some predefined compact region Ω, and $\hat{B}_{2,0}(t)$ is nonsingular, i.e.,

$$\hat{\Theta}_2(t), \hat{W}(t) \in \Omega, \text{ and } \hat{B}_{2,0}(t) \text{ is nonsingular } \ \forall t \tag{36}$$

The NN adaptive controller $u_2(t)$ considered in this paper can be computed by

$$\hat{\Theta}_2(t)^{\mathrm{T}} X_2(t) + \hat{v}^*[x(t), u_2(t)] = P(z^{-1}) w(t+1) \tag{37}$$

where $X_2(t) = [y(t)^{\mathrm{T}}, \cdots, u_2(t)^{\mathrm{T}}, u(t-1)^{\mathrm{T}}, \cdots]^{\mathrm{T}}$.

Remark 2. From Assumption 3 and Eq. (36), there exists an exact $u_2(t)$ which exactly satisfies Eq. (37). However, it may be noticed that a connotative $u_2(t)$ inside $\hat{v}^*[x(t), u_2(t)]$ will lead to difficult nonlinear calculation of the control output $u_2(t)$. For this reason, we expand $\hat{v}^*[x(t), u_2(t)]$ at its current state $X(t) = [y(t)^{\mathrm{T}}, \cdots, y(t-n_a+1)^{\mathrm{T}}, u(t-1)^{\mathrm{T}}, u(t-1)^{\mathrm{T}}, \cdots, u(t-n_b)^{\mathrm{T}}]^{\mathrm{T}}$. With the higher order term omitted, it holds that

$$\begin{aligned} \hat{v}^*[x(t), u_2(t)] &\approx \hat{v}^*[x(t), u(t-1)] \\ &+ \frac{\partial \hat{v}^*[x(t), u_2(t)]}{\partial u_2(t)}\Big|_{u_2(t)=u(t-1)} \cdot [u_2(t) - u(t-1)] \end{aligned} \tag{38}$$

It is necessary to note that the above equation is used in the control input calculation and it does not affect the accuracy of the controller $u_2(t)$, and it only takes $u_2(t)$ out of the nonlinear function and makes the calculation easier.

C. Switching mechanism

In this section, the problem of SMMAC by switching between the linear robust adaptive controller (32) and the nonlinear NN adaptive controller (37) is considered. Adopt the similar switching rule as described in [14], i.e.,

$$\begin{aligned} J_j(t) &= \sum_{l=1}^{t} \frac{\lambda_j(l)[\|e_j(l)\| - 2M(t-1)]^2}{1 + \|X(l-1)\|^2} \\ &\quad + c \sum_{l=t-N+1}^{t} (1 - \lambda_j(l)) \|e_j(l)\|^2 \end{aligned} \tag{39}$$

$$\lambda_j(t) = \begin{cases} 1, & \text{if } \ \|e_j(t)\| > 2M(t-1) \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

where N is an integer and $c \geq 0$ is a predefined constant.

By comparing $J_1(t)$ and $J_2(t)$, the controller C_* corresponding to the smaller J_* is chosen to control the system.

Theorem 2. For the system (1) with the adaptive algorithm (25)-(39), if the stable polynomial matrix $P(z^{-1})$ is chosen such that Eq. (20) is stable, then we have

(vi) the input and output signals in the closed-loop switching system are bounded.

Moreover, for any predefined arbitrary small positive number ϵ, there exists an instant T, such that, if the nonlinear controller $u_2(t)$ is chosen, the generalized tracking error of the system satisfies

(vii) $\|\bar{e}(t)\| < \epsilon.$ for $t > T$.

Especially, for the above case, if the reference inputs are step signals, at steady state, the tracking error of the system satisfies

$(viii)$ $\|e_c(t)\| < \epsilon / P(1)$.

Proof. From By Eqs. (32), (37), and the certainty equivalence principle, at every instant t,

$$\bar{e}(t) = e_1(t) \ \text{ or } \ e_2(t) \tag{41}$$

One the other hand, at every instant t, the model error of the closed-loop switching system

$$e(t) = e_1(t) \ \text{ or } \ e_2(t) \tag{42}$$

Therefore, using the same line as Theorem 1, we have

$$\begin{aligned} S(z^{-1})\|y(t)\| &\leq \|e(t)\| - 2M(t-1) \\ &\quad + \|P(z^{-1}) w(t)\| + 2\alpha_0 \end{aligned} \tag{43}$$

Consequently, there exist positive constants D_3, D_4 such that

$$\|X(t)\| \leq D_3 + D_4 \max_{0 \leq \tau \leq t} [\|e(\tau)\| - 2M(\tau-1)] \tag{44}$$

By Eq. (40), the second term in Eq. (39) is always bounded, so $J_1(t)$ is bounded by employing (ii) in Lemm1. For $J_2(t)$, there can be two cases:

(I) $J_2(t)$ is bounded. By the switching rule (39), it follows that

$$\lim_{t \to \infty} \frac{\lambda_2(t)[\|e_2(t)\| - 2M(t-1)]^2}{2(1 + \|X(t-1)\|^2)} = 0$$

Therefore the model error of the system, $e(t) = e_1(t)$ or $e_2(t)$, satisfies

$$\lim_{t \to \infty} \frac{\lambda(t)[\|e(t)\| - 2M(t-1)]^2}{2(1 + \|X(t-1)\|^2)} = 0 \tag{45}$$

where

$$\lambda(t) = \begin{cases} 1 & \text{if } \ \|e(t)\| > 2M(t-1) \\ 0 & \text{otherwise.} \end{cases}$$

(II) $J_2(t)$ is unbounded. Since $J_1(t)$ is bounded, there exists an instant t_0 such that $J_1(t) \leq J_2(t)$, $\forall t \geq t_0$. Therefore when $t \geq t_0 + 1$, by the switching mechanism, the model $e(t) = e_1(t)$ and also satisfies Eq. (45).

From Eqs. (44) and (45), and using the key technique lemma in [15], we can obtain the results (vi).

From the boundedness of the input and output signals, and Eq. (45), there exists an instant T and the bound Δ, such that when $t > T$, the model error $e(t)$ of the closed-loop switching system satisfies $\|e(t)\| < \Delta$. For the nonlinear model error, from Eq. (35), we have

$$\begin{aligned} e_2(t) &= \phi(t) - \hat{\phi}_2(t) \\ &= \phi(t) - \hat{\Theta}_2(t-1)^{\mathrm{T}} X(t-1) \\ &\quad - \hat{v}^*[x(t-1), u(t-1)] \\ &= \phi(t) - \{\phi(t) - v^*[x(t-1), u(t-1)]\} \\ &\quad - \hat{v}^*[x(t-1), u(t-1)] \\ &= v^*[\cdot] - \hat{v}^*[\cdot] \end{aligned} \tag{46}$$

By properly choosing the structure and parameters of an NN, for a predefined arbitrary small positive number ϵ, when $t > T$, $\|e_2(t)\| = \|v^*[\cdot] - \hat{v}^*[\cdot]\| \leq \epsilon$ can be achieved. Thus, from Eq. (41), if the nonlinear controller $u_2(t)$ is used, the generalized tracking error of the system will equal $e_2(t)$, and satisfies $\|\bar{e}(t)\| = \|e_2(t)\| \leq \epsilon$; Consequently, the result (vii) is obtained. From Eqs. (4) and (21), when the reference inputs are step signals, at steady state, $e_c(t) = \bar{e}(t)/\|P(1)\|$, then the result $(viii)$ is easlily obtained.

V. Simulation Results

This section presents an example and its simulation results to illustrate the proposed algorithm. Consider the following double-input-double-output discrete-time nonlinear dynamic system.

$$\begin{pmatrix} 1 - 0.6z^{-1} - 1.5z^{-2} & -1.2z^{-1} - 0.3z^{-2} \\ -2.4z^{-1} + 0.2z^{-2} & 1 - 0.1z^{-1} - 1.8z^{-2} \end{pmatrix} y(t+1)$$
$$= \begin{pmatrix} 1.1 & 0.8 \\ 0.32z^{-1} & 1.25 + 0.1z^{-1} \end{pmatrix} u(t) + v[x(t), u(t)]$$

where $y(t) = [y_1(t), y_2(t)]^{\mathrm{T}}, u(t) = [u_1(t), u_2(t)]^{\mathrm{T}}, x(t) = [y(t)^{\mathrm{T}}, y(t-1)^{\mathrm{T}}, u(t-1)^{\mathrm{T}}]^{\mathrm{T}}$, $n_a = 2, n_b = 1$. The nonlinear vector function $v[x(t), u(t)]$ is described by

$$v[x(t), u(t)] = \begin{pmatrix} 0.5\sin(t)[2\sin[u(t)] + y_1(t) + y_2(t) \\ 0.5\sin(t)[2\sin[u(t)] + \sin(t)y_1(t) + \sin(t)y_2(t) \end{pmatrix}$$

It is easy to know that $u(t)$ in the above $v[x(t), u(t)]$ is constrained by the bounded function $\sin[u(t)]$. Consequently, $\|v[x(t), u(t)]\| \leq 2 + 2\|y(t)\|$. Hence $\alpha_0 = 2, \alpha_1 = 2$. Choosing $n_p = 1$, and $S(z^{-1}) = 10 - 3z^{-1}$, then to satisfy

$$\begin{aligned} \|P_0\| - \|P_1\|z^{-1} &= 10 - 3z^{-1} + \alpha_1 z^{-1} \\ &= 10 - z^{-1} \end{aligned}$$

we choose

$$P(z^{-1}) = \begin{pmatrix} 5\sqrt{2} - \frac{\sqrt{2}}{2}z^{-1} & 0 \\ 0 & 5\sqrt{2} - \frac{\sqrt{2}}{2}z^{-1} \end{pmatrix}$$

Reference trajectories, same to [14], $w_1(t) = 1.5(\sin 2\pi t/10 + \sin 2\pi t/25)$ and $w_2 = w_1$ are chosen to be followed.

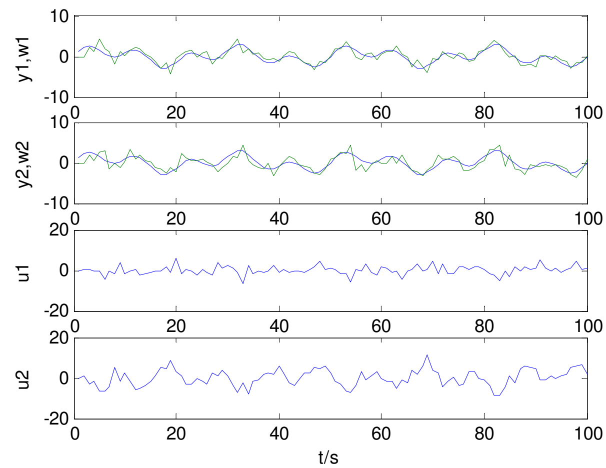

Fig.1 Performance when the robust adaptive controller is used

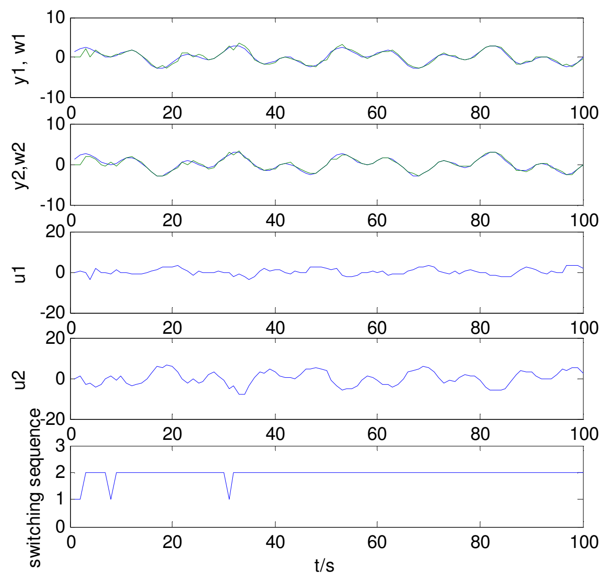

Fig.2 Performance and switching sequence when the SMMAC method is used

Fig. 1 shows the performance when only the linear ro-bust adaptive controller is used. Obviously, the input and output signals in the closed-loop system are bounded. However, the tracking performance is relatively bad.

Since there is no special requirement for the NN adopted in this paper, an NN with single hidden layer is chosen and the back-propagation algorithm with adaptive learning rate in batch mode is used. In order to determine the optimum number of hidden nodes, same to [14], a cross-validation procedure is used, which starts by moving bottom-up [16]. As a result, when the number of the hidden nodes is larger than 22, the cross validation error can not be reduced comparing with the case of 22. So the number 22 is chosen as the optimal number of hidden nodes. The learning rate is 0.1.

Fig. 2 illustrates the performance when the SMMAC method proposed in this paper is used. The parameters in Eq. (33) are chosen to be $c = 1.5$ and $N = 2$. Obviously, the good tracking performance of the output signals and the small amplitude of the input signals are all achieved.

VI. CONCLUSION

This paper proposes a SMMAC approach for a class of discrete-time nonlinear multi-input-multi-output dynamic systems. Comparing with [14], the condition that the difference of the higher order nonlinear term of a system is globally bounded is relaxed. Theory analysis and simulation results show the effectiveness of the proposed method.

REFERENCES

[1] F. C. Chen, and C. C. Liu, "Adaptively controlling nonlin-ear continuous-time systems using multilayer neural networks," *IEEE Transactions on Automatic Control*, vol.39, no.6, pp.1306-1310, 1994.

[2] S. Shin, and T. Kitamori, "Continuous-time model reference adaptive control for an unknown nonlinear system," *International Journal of Control*, vol.49, no.2, pp.513-520, 1989.

[3] X. S. Wang, C. Y. Su, and H. Hong, "Robust adaptive control of a class of nonlinear systems with unknown dead-zone," *Automatica*, vol.40, no.3, pp.407-413, 2004.

[4] J. B. D. Cabrera, and K. S. Narendra, "Issues in the application of neural networks for tracking based on inverse control," *IEEE Transactions on Automatic Control*, vol.44, no.7, pp.2007-2027, 1999.

[5] S. S. G, G. Y. Li, J. Zhang, and T. H. Lee, "Direct adaptive control for a class of MIMO nonlinear systems using neural networks," *IEEE Transactions on Automatic Control*, vol.49,no.10, pp.2001-2004, 2004.

[6] Q. M. Zhu, and L. Z. Guo, "Stable Adaptive Neurocontrol for Nonlinear Discrete-Time Systems," *IEEE Transactions on Neural Networks*, vol.15, no.3, pp.653-662, 2004.

[7] L. J. Chen, and K. S. Narendra, "Nonlinear adaptive control using neural networks and multiple models," *Automatica*, vol.37, no.7, pp.1245-1255, 2001.

[8] F. C. Chen, "Back-propagation neural networks for nonlinear self-tuning adaptive control," *IEEE Control System Magazine*, vol.10, no.3, pp.44-48, 1990.

[9] F. C. Chen, "Adaptive control of a class of nonlinear discrete-time system using neural networks," *IEEE Transactions Automatic Control*, vol.40, no.5, pp.791-801, 1995.

[10] F. C. Chen, andd H. Khalili, "Adaptive control of nonlinear systems using neural networks," *International Journal of Control*, vol.55, no.6, pp.1299-1317, 1992.

[11] S. S. G, T. H. Lee, G. Y. Li, and J. Zhang, "Adaptive NN control for a class of discrete-time non-linear systems," *International Journal of Control*, vol.76, no.4, pp.334-354, 2003.

[12] Q. M. Zhu, Z. Ma, and K. Warwick, "Neural network enhanced generalized minimum variance self-tuning controller for nonlinear discrete-time systems," *IEE Proceedings-Control Theory and Application*, vol.146, no.4, pp.319-326, 1999.

[13] L. J. Chen, "Nonlinear adaptive control of descrete-time systems using neural networks and multiple models," Ph.D. thesis, Yale University, 2001.

[14] Y. Fu, and T. Y. Chai, "Nonlinear multivariable adaptive control using multiple models and neural networks. *Automatica*, vol.43, no.6, pp.1101-1110, 2007.

[15] G. C. Goodwin, P. J. Ramadge, and P. E. Caines, "Discrete-time multivariable adaptive control," *IEEE Transactions on Automatic Control*, vol.25, no.3, pp.449-456, 1980.

[16] U. Anders, and O. Korn, "Model selection in neural networks," *Neural Network*, vol.12, no.2, pp.309-323, 1999.