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Fine and simplified dynamic modelling of complex hydraulic systems

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Abstract— **This paper deals with the dynamic modelling of a complex electro-hydraulic system. Modelling is based on physical laws and the system knowledge. The main idea is to obtain a simple and reliable model that can be used for controller synthesis and implementation by using the architecture of Embedded Model Control. Simplifications are made by taking as basis the ideas of singular perturbation. A subsequent identification procedure is made in order to acquire some important parameters, required for carrying out a simulation.**

I. INTRODUCTION

Model building is a constant issue in control engineering since it is needed for many different tasks, such as analysis, control design, training, etc. The accuracy level relies on the type of application it is intended for.

The paper is concerned with fine and simplified modelling of hydraulic systems in view of control design and testing. The adopted approach follows the framework of the Embedded Model Control given in [1]. In this approach the control law is designed and implemented around a discrete-time simplified dynamics of the plant to be controlled, enhanced with the uncertainty dynamics to be rejected to guarantee performance. The problem of model simplification is well known in the control literature, where the main goal is to look for efficient algorithms capable of reducing the dynamics within a specified bound of the modelling error between the complex and the simplified model under the same class of commands. Different techniques have been proposed: singular perturbations, singular values, etc., all of them addressing linear dynamics.

Here a rather different approach is followed from two different standpoints. The so called 'true' model, here called 'fine model', which for complex hydraulic systems is usually assembled from standard component models and commercial packages like AMESIM, is studied for each component to enhance the dynamics within a frequency bandwidth, which is sufficiently larger than the target control bandwidth. Under this approach a simplified model version is obtained, which is referred as 'neglected dynamics' in [1] also called unstructured uncertainties in control literature. This is not an easy task, since component data from manufacturers do not care for dynamics but very often only for static performance, especially when the system includes hydraulic compensation devices designed and arranged to ensure them. On the other hand, such models do not need to be very accurate, say to know exact time or frequency responses, especially when their dynamics are expected to lie beyond the target control bandwidth. Usually, as it will be shown, a triple of dynamics have to be accounted for, namely, mass dynamics of the moving parts of valves, distributors, pumps, usually of the second order, pressure dynamics (continuity equation) of the lumped hydraulic capacities of each device; hydraulic links, of accumulators, usually first order dynamics, and finally electromechanical dynamics of driving solenoids when appropriate. The pressure dynamics is usually interconnected to mass dynamics through the flow permitted by the moving parts, where pressure enters in a nonlinear way if flow becomes turbulent. Solenoid dynamics is usually the fastest one, being regulated by appropriate electronics. The result is a set of nonlinear state equations with their working conditions and limits, where state variables are position, velocity, pressure, current; commands are solenoid driving voltage; measurements are pressures, flows, currents; disturbances are leakages, flows, discharge pressures, frictions, etc. An equilibrium point is then looked for, not for dynamic linearization but to fix the intermediate point of the different variables, for instance zero-hydraulic positions, and their working limits.

Simplification method looks very similar to singular perturbation method [2][3] and may be facilitated to certain extent by hydraulic systems to be designed as the interconnection of well defined lumped dynamics. As a baseline when cut-off frequencies (or eigenvalues) of a set of equations are outside a pre-specified bandwidth, equations are reduced to be static by zeroing the derivative term on the LHS of the equations. From a hydro-mechanical standpoint it corresponds to assume negligible fluid volume variation $\dot{p}_i V_i / \beta_i$ [m^3/s] due to pressure rate V_i / β_i, where V_i is the

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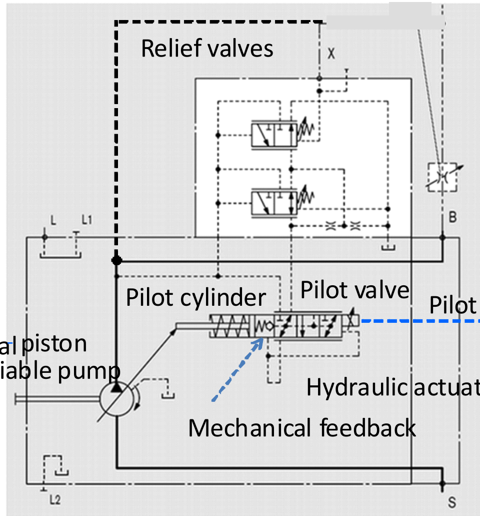

Fig. 1. Essential hydraulic circuit.

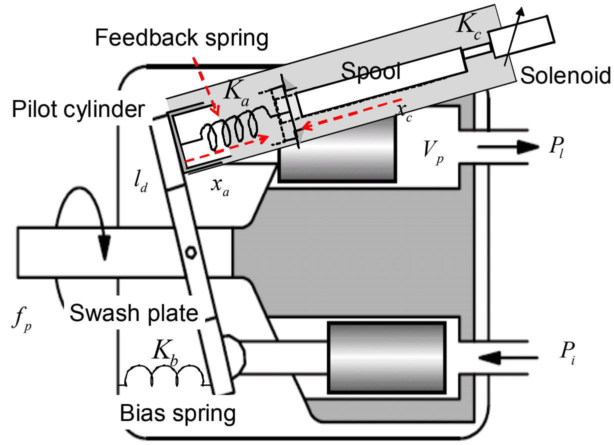

Fig. 2. Basic axial pump diagram.

capacitance volume, $1/\beta_i$ the fluid compressibility; to assume small inertia forces $m_k \ddot{x}_k$ of the moving parts under the balance of hydraulic, visco-elastic and solenoid forces or to assume a small current error rate $d\left(I_{ref} - I\right)/dt$. This method assumes weak interconnection between different dynamics, which may not occur and therefore must be carefully examined by frequency or modal analysis of the overall equations or part of them.

The paper is organized as follows, section 2 is devoted to system description and problem statement; section 3 boards the development of a fine model for the system under study. Sections 4 and 5 deal with the simplification of the complex obtained model, as well as the transfer function analysis; section 6 considers an identification process of some parameters. Finally some conclusions are presented.

II. PLANT DESCRIPTION

A. *The hydraulic circuit*

The plant under study is a subset of a more complex hydraulic circuit supplying loads to off-highway vehicles. The essential elements of the network shown in Fig. 1 are

1) an axial-piston variable pump driven by an electro-hydraulic actuator,
2) a set of relief valves to limit pressure in the circuit,
3) the hydraulic load, composed by several parallel loads, part of them to be continuously supplied (priority loads) and part of them on demand,
4) a load-sensing control system, not represented in Fig. 1, fed by load measurements and in charge of regulating the pump flow according to load demand.

The paper will restrict to the sole pump and the electro-hydraulic actuator, while the load is simplified to be a passive pressure drop device as in Fig. 1 (port B).

B. *The pump*

The pump is an axial-piston variable type (see Fig. 2), the flow being varied by tilting a swash plate against a bias spring with stiffness K_b. The plate tilting and the subsequent flow regulation may be obtained in two ways:

1) active regulation: the plate tilt is measured by a suitable sensor and fed to a control loop commanding the electro-hydraulic actuators [4][5];
2) mechanical feedback: the hydraulic actuator stroke x_a (the pilot cylinder in Fig. 2) is servoed to the spool position x_c of a proportional valve by means of an elastic link (the feedback spring in Fig. 2) having stiffness K_a.

The second (passive) solution is more compact and robust as the sensor becomes useless and only one pilot cylinder may be employed. Careful design ensures the pump flow Q_p to be proportional to the current i_s of the pilot solenoid, under steady state conditions, less some hysteresis due to friction.

The goal of the paper is to derive the state equations of the pump and the actuator (fine model) and then to simplify them (design LTI model) within the bandwidth required by the load-sensing control.

A complete derivation of the open-loop pump dynamics, starting from the dynamics of each pump piston, can be found in the literature [6][7][8]. As shall be proved, pump dynamics is not the core of the target model, due to mechanical feedback which replaces the pump closed-loop and therefore simplifies current to flow dynamics.

III. FINE MODEL

A. *Mechanical equivalence*

The mechanical chain from solenoid to plate can be represented as in Fig. 3 by masses and springs. Hydraulics enters through the pressure forces.

Modelling assumptions are:

1) electrodynamics of the solenoid is neglected, under assumption of a suitable current regulator, with bandwidth better than 1 kHz,
2) each body is considered as a second order mass-spring system with position and rate as state variables;
3) rigidly connected elements as plate and actuator body will be reduced to a single body;
4) equations are given in a variational way with respect to a well specific equilibrium condition corresponding to zero spool position, namely $x_c = 0$, and the central plate position, defined by the zero actuator stroke, i.e. $x_a = 0$. note the resulting equations are not small variation equations, but large variations, holding in the whole range of the state variables.

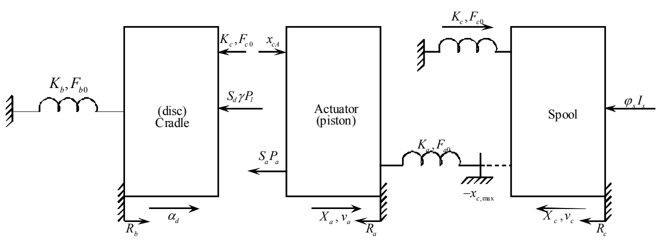

Fig. 3. Mechanical equivalent

At the end a state equation system of the sixth order is obtained, driven by the solenoid current i_s and the pump discharge pressure P_l, in the following referred to as line pressure.

B. Spool equation

According to sign conventions in Fig. 3 and the above guidelines, the second order spool equation is given by

$$m_c \ddot{x}_c(t) = -A_c(x_c, \dot{x}_c) - (K_c + K_a)x_c - K_a x_a + \varphi_s i_s$$
$$x_c(0) = x_{c0}, \quad -x_{c,\max} \le x_c \le x_{c,\max}, \quad -I_{\max} \le i_s \le I_{\max} \tag{1}$$

where x_c is the spool displacement, m_c the spool mass, $K_c << K_a$ the stiffness of the bias spring defining the spool rest position $-x_{c,\max}$. Finally, A_c accounts for friction and fluid forces, whereas φ_s is the solenoid electro-mechanical constant. The symmetric current range defined by $I_{\max}$ is the flow regulating range, corresponding to the actual current range

$$0 < I_{\min} \le I_s = I_{\min} + i_s + I_{\max} \le I_{\min} + 2I_{\max}. \tag{2}$$

The current region below $I_{\min}$ is used to move the spool in the regulation range, an issue not treated here.

C. Actuator equations

As the pilot cylinder body is assumed to be rigidly connected to the swash plate through a spherical joint, only the hydraulic equation is considered here. Denote the current hydraulic volume with V_a and the active area with S_a; neglecting leakages, the continuity equation is written in terms of the total actuator internal pressure P_a as follows

$$\dot{P}_a(t) = \frac{\beta}{V_a}\left[-Q_a(t) + S_a \dot{x}_a(t)\right], \; P_a \ge 0, \tag{3}$$

where β is the compressibility coefficient and Q_a is the input/output flow, which is assumed as positive when going out [5]. Equation (3) must be completed with the flow equations

$$\begin{aligned} &Q_a(t) = Q_{a,in}(t) = \mu_a x_c \sqrt{P_l - P_a}, \; x_c < 0, P_l - P_a \ge 0 \\ &Q_a(t) = 0, \; x_c = 0 \\ &Q_a(t) = Q_{a,out}(t) = \mu_a x_c \sqrt{P_a}, \; x_c > 0, P_a \ge 0 \\ &\mu_a = h_a \sqrt{2/\rho}, \end{aligned} \tag{4}$$

where ρ is the fluid density and h_a is the equivalent height of the orifice, assumed constant at any x_c. Note the flow Q_a is assumed to be proportional to the spool displacement x_c which may be approximate at the extremes of the spool stroke. Note further the flow is zero for $x_c = 0$.

D. Plate dynamics

As was already said, a simplified equation of the plate dynamics is reported, which is however coherent with the simplified equations reported in the literature [8]. By assuming actuator and plate rigidly connected, a single equation may be written, and directly in the actuator stroke x_a, which is proportional to plate tilt α_d through the kinematic link

$$x_a = \alpha_d l_d, \tag{5}$$

with l_d being the actuator arm. Then, the force balance on the actuator and plate ensemble reads

$$\begin{aligned} &m_a \ddot{x}_a(t) = -A_a(x_a, \dot{x}_a) - K_b x_a - K_a(x_a + x_c) - p_a S_a - p_l S_d \\ &-x_{a,\max} \le x_a \le x_{a,\max} \end{aligned} \tag{6}$$

where A_a accounts for friction and fluid forces, m_a is the equivalent mass of the ensemble and $S_d << S_a$ is the total active area of the line pressure in the pump pistons, generating a force over the swash plate. Equation (6) being a variation equation in x_a, has been written in terms of the pressure variations

$$p_a = P_a - \bar{P}_a, \; p_l = P_l - \bar{P}_l \tag{7},$$

where the fixed pressures $\bar{P}_a, \bar{P}_l$ are defined by the bias spring preload.

E. Pump output dynamics

The swash plate tilting allows to express the discharge fluid volume as proportional to α_d and then to x_a. The rotation of the tilted pistons allows expressing flow as proportional to shaft angular frequency $f_p = \omega_p (2\pi)^{-1}$ in Hertz units, ω_p being the shaft angular rate. At the end, the pump flow equation can be written as follows,

$$Q_p(t) = V_p f_p \left(\frac{1}{2} + \frac{x_a(t)}{x_{a,\max}}\right) = \bar{Q}_p + q_p(t) \tag{8}$$

i.e. as the sum of the mean flow $\bar{Q}_p$ and its variation $q_p(t)$.

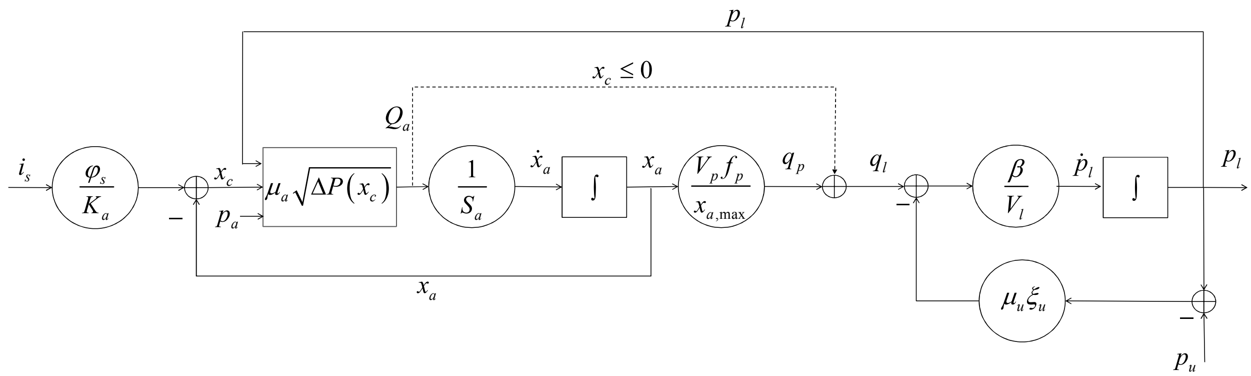

Fig. 4. Block scheme of the simplified model.

F. Load dynamics

Load dynamics may be very complex; here it is assumed a single fluid volume V_l at the line pressure P_l supplied by the pump flow, and discharged by a variable flow depending on P_l. Actually, the output flow must be complemented with the actuator input flow $Q_{a,in}$ only existing when $x_c \le 0$. The corresponding equation holds

$$\begin{aligned} \dot{P}_l &= \frac{\beta}{V_l}\left(Q_p + Q_a - \mu_u \xi_u \left(P_l - P_u\right)^{1/\gamma_u}\right),\ x_c < 0 \\ \dot{P}_l &= \frac{\beta}{V_l}\left(Q_p - \mu_u \xi_u \left(P_l - P_u\right)^{1/\gamma_u}\right),\ x_c \ge 0, \end{aligned} \tag{9}$$

where the output flow may be driven by the load pressure $P_u \le P_l$ and/or the aperture degree $0 \le \xi_u \le 1$, and $1 \le \gamma_u \le 2$.

IV. SIMPLIFIED MODEL

The methodology followed here takes advantage of the so called *singular perturbation method* [2][3][9], that averages state variables, inputs and outputs over a given time period T. To this end, small parameters, denoted as $\varepsilon = \tau/T$, pre-multiplying the n-th derivative (for example, $n=1$ for the case of continuity equations and $n=2$ in case of mass-spring equations), make them so small to be approximated to zero, except along small time intervals comparable with τ, playing the role of a time constant. Moreover, $\tau_c = \sqrt{m_c / K_c} \le 5$ ms holds for the spool equation, $\tau_a = V_a / \left(\beta \times \partial Q_a / \partial P_a\right) \le 0.1$ ms for the actuator hydraulics and $\tau_b = \sqrt{m_a / K_b} \le 5$ ms for the plate equation. Roughly speaking, if the load-sensing control averages over times longer than $T = 50$ ms, the above dynamics can be neglected, with an error that can be evaluated by the perturbation method.

A. Spool dynamics

Singular perturbation applied to equation (1) and $K_c << K_a$ provides

$$x_c = \frac{\varphi_s i_s}{K_a} - x_a, \tag{10}$$

showing the spool position playing the role of the control error between the current command and the actuator displacement; since $x_c = 0$ at steady state condition, from equation (10) x_a is proportional to i_s and pump flow variation q_p in equation (8) can be rewritten as

$$q_p(t) = V_p f_p \frac{i_s}{I_{\max}}, \tag{11}$$

which is the standard form of the linear pump characteristics.

B. Actuator and plate equation

Singular perturbation applied to equations (3) and (6) provides

$$\begin{aligned} \dot{x}_a &= Q_a / S_a \\ p_a &= -\frac{K_b x_a + \varphi_s i_s + p_l S_d}{S_a} \simeq -\frac{K_b x_a + \varphi_s i_s}{S_a}, \end{aligned} \tag{12}$$

where second equation relates the actuator pressure to command and actuator displacement.

C. Load equation

Assuming for simplicity $\gamma_u = 1$ in (9), and q_p from (8), the load equation holds

$$\begin{aligned} \dot{p}_l &= \frac{\beta}{V_l}\left(b_p x_a + S_a \dot{x}_a - \mu_u \xi_u \left(p_l - p_u\right)\right),\ x_c < 0 \\ \dot{p}_l &= \frac{\beta}{V_l}\left(b_p x_a - \mu_u \xi_u \left(p_l - p_u\right)\right),\ x_c \ge 0 \end{aligned} \tag{13}$$

where $b_p = V_p f_p / x_{a,\max}$ and $\bar{P}_l$ defines the equilibrium for a constant $\bar{P}_u$.

The block diagram for this simplified version of model is depicted in Fig. 4.

V. TRANSFER FUNCTION ANALYSIS

As a next step, a transfer function synthesis is done. As can be seen from equations (10)-(13), the simplified model remains non-linear for variations of P_a and P_l, more specifically the signal Q_a is a non-linear function of the difference between the line and actuator pressure, whom in turn depends on the variation of spool position x_c. Once again an analysis for each condition of x_c (positive or negative) must be done.

The linearization procedure is based on fixating the variations of signals with respect to their nominal values. i.e., taking the functions $\sqrt{P_a}$ or $\sqrt{P_l - P_a}$ as a fixed parameter.

A. Positive error $x_c > 0$

This situation corresponds to an increase of pump output flow, it means, there is no feedback flow to the pilot valve.

From (10)-(13) and replacing the pilot valve flow (4), the linear system is given by

$$\begin{bmatrix} \dot{x}_a \\ \dot{p}_l \end{bmatrix} = \begin{bmatrix} -\dfrac{\mu_a\sqrt{\overline{P}_a}}{S_a} & 0 \\ \dfrac{\beta b_p}{V_l} & -\dfrac{\beta\mu_u\xi_u}{V_l} \end{bmatrix} \begin{bmatrix} x_a \\ p_l \end{bmatrix} + \begin{bmatrix} \dfrac{\mu_a\sqrt{\overline{P}_a}\varphi_s}{S_a K_a} & 0 \\ 0 & \dfrac{\beta\mu_u\xi_u}{V_l} \end{bmatrix} \begin{bmatrix} i_s \\ p_u \end{bmatrix}$$

$$\begin{bmatrix} p_l \\ q_l \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ b_p & 0 \end{bmatrix} \begin{bmatrix} x_a \\ p_l \end{bmatrix} \tag{14}$$

where q_l is the line flow (see Fig. 4).

B. Negative error $x_c < 0$

This situation corresponds to a drop in the output pump flow. Additionally there is a flow request from the pilot valve, in order to reach a minimum pressure level to reduce the plate inclination. By following same guidelines as for $x_c > 0$, the linear state space is given by

$$\begin{bmatrix} \dot{x}_a \\ \dot{p}_l \end{bmatrix} = \begin{bmatrix} -\dfrac{\mu_a\sqrt{\Delta\overline{P}}}{S_a} & 0 \\ \dfrac{\beta_p}{V_l}\left(b_p - \mu_a\sqrt{\Delta\overline{P}}\right) & -\dfrac{\beta\mu_u\xi_u}{V_l} \end{bmatrix} \begin{bmatrix} x_a \\ p_l \end{bmatrix} + $$

$$+\begin{bmatrix} \dfrac{\mu_a\sqrt{\Delta\overline{P}}\varphi_s}{S_a K_a} & 0 \\ \dfrac{\beta\mu_a\sqrt{\Delta\overline{P}}\varphi_s}{V_l K_a} & \dfrac{\beta\mu_u\xi_u}{V_l} \end{bmatrix} \begin{bmatrix} i_s \\ p_u \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} p_l \\ q_l \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ b_p - \mu_a\sqrt{\Delta\overline{P}} & 0 \end{bmatrix} \begin{bmatrix} x_a \\ p_l \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{\mu_a\sqrt{\Delta\overline{P}}\varphi_s}{K_a} & 0 \end{bmatrix} \begin{bmatrix} i_s \\ p_u \end{bmatrix},$$

where $\Delta\overline{P} = \overline{P}_l - \overline{P}_a$.

The matrix transfer function of systems (14) and (15) is

$$G(s) = \begin{bmatrix} \dfrac{\eta\beta f_p\left(\overline{P}_l,\overline{P}_a\right) f_z\left(s,x_c\right)}{\left(sS_a + \mu_a f_p\left(\overline{P}_l,\overline{P}_a\right)\right)\left(sV_l + \beta\mu_u\xi_u\right)} & \dfrac{\beta\mu_u\xi_u}{sV_l + \beta\mu_u\xi_u} \\ \dfrac{\eta f_p\left(\overline{P}_l,\overline{P}_a\right) f_z\left(s,x_c\right)}{\left(sS_a + \mu_a f_p\left(\overline{P}_l,\overline{P}_a\right)\right)} & 0 \end{bmatrix}, \tag{16}$$

where s stands for the Laplace variable, $\eta = \mu_a\varphi_s / K_a$ and the functions

$$f_z\left(s,x_c\right) = \begin{cases} b_p, & x_c > 0 \\ b_p + sS_a, & x_c < 0 \end{cases} \tag{17}$$

$$f_p\left(\overline{P}_l,\overline{P}_a\right) = \begin{cases} \sqrt{\overline{P}_a}, & x_c > 0 \\ \sqrt{\overline{P}_l - \overline{P}_a}, & x_c < 0. \end{cases} \tag{18}$$

It is worth noting that transfer function given by (16) is valid for the non linear system as well; more exactly this approximation is valid in the case of small variations of terms p_a and p_l.

Function (17) reveals the existence of a zero when there is a flow drop in the pump. A further analysis to this fact reflects a time constant of the zero around $\tau_z \approx 10$ ms, which can be considered negligible with respect to predominant dynamics within the system.

In order to complete the modelling process, some parameters must be estimated. To this aim, an identification process is effectuated, regarding, for a first instance, only the path from i_s to q_l, that is to say the element 2,1 of matrix (16). The proposed model obeys to a first order delayed system, i.e.,

$$G_{id}(s) = G_{21}(s)e^{-st_d} = \frac{G_0}{1+\tau s}e^{-st_d}, \tag{19}$$

with t_d, τ the time delay and time constant respectively and G_0 corresponds to the DC gain. Note that the time delay t_d (not present in model (16)) is added just to better fit the acquired data and it is due not only to the neglected dynamics but mainly to signal transmission delays and sensor delays of the experimental setting. The estimation procedure is based on a non linear least square optimisation process.

As a first step in the identification, it is necessary to acquire the seed for the optimisation process. This initial information is obtained from the data of tests made over the plant. This identification process is effectuated only for the growing flow zone, i.e., $x_c > 0$.

The cost function to be minimized is given by

$$J = \sum_{i=1}^{N}\left(q_{l,m} - q_{l,i}\right)^2, \tag{20}$$

where terms $q_{l,m}, q_{l,i}$ are the measured and estimated flow respectively. The estimated flow is generated in the search process by evaluating the estimated parameters into (19). It should be clarified that the data set is finite and discrete, however, it is being evaluated in the continuous time transfer function (19) in order to generate the required sequence $q_{l,i}$. The used optimisation algorithm is called *Nelder-Mead* simplex search method and it is codified in Matlab under the `fminsearch()` function name. From the identification process, applied to nine different data sets of acquired signals, parameters value and uncertainty are obtained:

$$G_0 = 100 \pm 0.7, \quad \tau = 100 \pm 2 \text{ ms}, \quad t_d = 55 \pm 5.5 \text{ ms}.$$

VI. SIMULATION RESULTS

In order to show the effectiveness of the obtained models, some simulations were carried on and compared with the acquired data.

Fig. 5 shows simulation results of the optimization process of Section V. , for one data set, and of the simplified model described in the block scheme of Fig. 4 of Section IV. Dash dotted and solid lines correspond to measured current input and measured flow output respectively. Dashed line represents the simulated flow output obtained at the end of the optimization procedure, while dotted line represents the simulated flow output obtained from block scheme of Fig. 4. Note that all these signals are normalized, for the sake of confidentiality, and that the measured signals are filtered in order to eliminate the dither contribution.

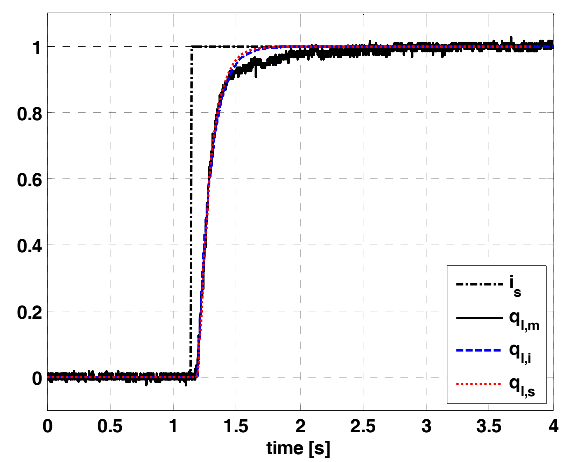

Fig. 5. Pump responses, measured and identified.

Moreover, Fig. 6 presents the final estimation error $e_e(t) = q_{l,m}(t) - q_{l,i}(t)$ obtained.

As can be seen, the distance between the responses is small enough as to be accepted the parameters. The variations observed in the plot, are mainly due to quantization effects within the measured signal.

VII. CONCLUSIONS

In this note, a modelling process of a complex hydraulic system has been shown. The system under study is highly complex and non-linear, however as a first step just the basic feeding component has been analyzed, with a subsequent simplification of the obtained model.

A key issue in this modelling process relies in the load attached to the pump, since this load may affect in a large extent the general system behaviour. Even though in the real system the load is composed of a set of mechanic-hydraulic elements such as non-return valves or load distributors, most of them with a local compensation, in the procedure shown here, for the sake of simplicity this load system has been assumed to be a pressure drop. The called fine model has been obtained from the physical system behaviour, together with the analysis of preloaded equilibriums. Once this fine model was presented, a simplified version was obtained, turning into a local linearized system model, that anyway shows a time behaviour strictly fitting actual acquired data.

At the end of the paper a simple identification process was presented, with the purpose of acquire some important parameters required to complete the system modelling.

Future work will be devoted to the analysis and modelling of the load system, as well as the corresponding simulation of the obtained model.

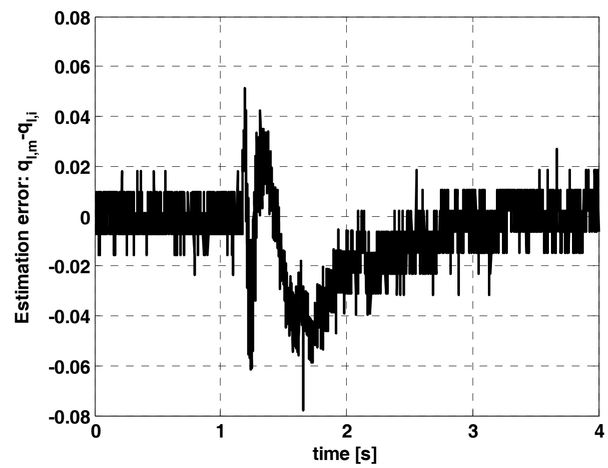

Fig. 6. Estimation error

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