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New H_∞ Controller Design Method for Networked Control Systems with Quantized State Feedback

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Abstract— **The problems of H_∞ stability and stabilization for networked control systems (NCSs) with state and control signal quantizations are considered, and a new sector bound approach is proposed. Unlike the previous works, the transform of system models to uncertain systems is not needed, which results in that the proposed controller design method is simpler. Even for the NCSs with only state quantizations, the newly obtained results are less conservative than the existing ones. Meanwhile, they are also less complex since fewer decision variables are involved. Illustrative examples are given to show the effectiveness and less conservatism of the proposed methods.**

I. INTRODUCTION

The signal transmission delay and the data packet dropout phenomenon in NCSs have been investigated by many researchers [1], [2], [3], [4], [5], [6], [7]. In [3], necessary and sufficient LMI conditions for the synthesis of the H_∞ optimal controller for a discrete-time jump system were presented. In [8], [9], continuous-time NCSs with both time delay and packet dropout were modeled as linear delay systems, such that the results of linear systems with fast delay (for example, [10]) can be directly applied to the analysis and synthesis problems of NCSs.

Another important issue in a communication channel is the quantization effect, which has significant impact on the performance of NCSs [11], [12]. As we can see, quantization problems have been investigated by many researchers for both linear systems [13], [14] and nonlinear systems [15]. In [14], Elia and Mitter indicated that the coarsest, or least dense, quantizer is logarithmic, and they considered the quadratic stabilization for SISO systems by using quantized state feedback. In [16], Fu and Xie extended the method of [14] to MIMO systems and generalized their results to the problems of performance control.

To notice that most of the aforementioned results only consider one or two aspects of the communication issues (time delay, packet dropout and quantization), while few papers address the analysis or synthesis problems with simultaneous consideration of the three important communication issues. More recently, [17], [18] addressed the analysis or synthesis problems of such NCSs. Yue et al. [17] considered the guaranteed cost control of continuous systems over networks with state and input quantizations, based on the sector bound condition of quantizer given in [16] and the transformation of original nonlinear systems to linear systems with uncertainty was needed. Since these exist uncertainties on both sides of the controller gain matrix K, so the process of the transformation was complicated and the presented LMI-based conditions in [17] were difficult to check. Gao et al. [18] combined the transformation method similar to [17] with a technique of two successive delay components, investigated problems of H_∞ stability and stabilization for continuous-time NCSs with only quantized state feedback, and the presented LMI-based conditions were also complex and conservative.

In this paper, the problems of H_∞ stability and stabilization for NCSs with both sensor-to-controller and controller-to-actuator quantizations are considered. A set of new sector bound conditions of quantizers is developed, and a new method is proposed to deal with H_∞ stabilization of NCSs. Unlike the previous works, the transformation of system models to uncertain systems is not needed, then the proposed controller design process is simpler than the existing ones. Even there exist only state quantizations, the newly obtained results are less conservative than the existing ones. Furthermore, the new H_∞ stability criteria and H_∞ controller design methods are simpler since fewer decision variables are involved.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a typical NCS shown in Fig. 1, where the physical plant is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E\omega(t), \\ z(t) = Cx(t) + Du(t) + F\omega(t). \end{cases} \tag{1}$$

Here $x(t) \in R^n$ is the system state vector, $u(t) \in R^p$ is the control input; $\omega(t) \in R^l$ is the disturbance input which belongs to $L_2[0, \infty)$; $z(t) \in R^q$ is the output; A, B, C, D, E and F are system matrices with appropriate dimensions. The pair (A, B) is assumed to be stabilizable.

It is assumed that the sensor is clock-driven, while the controller and zero-order hold (ZOH) are event-driven. The sampling period is denoted by h, the state variable $x(t)$ is assumed to be measurable, and the measurements of $x(t)$ are firstly quantized via a quantizer, and then transmitted with a single packet. Similarly, the control signals generated by the controller are quantized before they are transmitted. In

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addition, if one packet sampled at the sensor node reaches the destination later than its successors, then it will be dropped and the latest one will be used.

Denote the instant the actuator receives the kth control signal as t_k, and this control signal is based on the state of plant at instant $i_k h$, thus $\{i_1,\ i_2,\ i_3,\ \cdots\} \subset Z^+$ and $i_k < i_{k+1}\ \forall\ k \in \{1,\ 2,\ \cdots\}$. So, the control signal is given by

$$u(t) = g(Kf(x(i_k h))), \quad t \in [i_k h + \tau_k,\ i_{k+1}h + \tau_{k+1}), \quad (2)$$

where $\tau_k = t_k - i_k h$ $(k = 1,\ 2,\ \cdots)$ represents the transmission delay of the data packet sampled at instant $i_k h$ from the sensor to the actuator, K is the state feedback gain matrix, quantizers $f(\cdot) = \begin{bmatrix} f_1(\cdot) & f_2(\cdot) & \cdots & f_n(\cdot) \end{bmatrix}^T$ and $g(\cdot) = \begin{bmatrix} g_1(\cdot) & g_2(\cdot) & \cdots & g_p(\cdot) \end{bmatrix}^T$ are assumed to be symmetric, that is, $f_j(-v) = -f_j(v)$ $(j = 1,\ 2,\ \cdots,\ n)$ and $g_m(-v) = -g_m(v)$ $(m = 1,\ 2,\ \cdots,\ p)$. Similar to [17], [18] and [19], the quantizers considered in this paper are logarithmic static and time-invariant.

As pointed out in [8], [9], under the assumption:

$$(i_{k+1} - i_k)h + \tau_{k+1} \le \eta, \quad k = 1, 2, \cdots, \quad (3)$$

$$\tau \le \tau_k, \quad k = 1, 2, \cdots, \quad (4)$$

where η and τ are constants satisfying $0 \le \tau < \eta$, then system (1) can be rewritten as follows:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bg(Kf(x(i_k h))) + E\omega(t), \\ z(t) &= Cx(t) + Dg(Kf(x(i_k h))) + F\omega(t), \end{cases} \quad (5)$$

where $t \in [i_k h + \tau_k,\ i_{k+1}h + \tau_{k+1})$ and $k = 1,\ 2,\ \cdots$.

Obviously, system (5) is nonlinear and it contains system (1) with only quantized state as a special case.

According to [16], for each $f_j(\cdot)$, the set of quantized levels is described by

$$\mathscr{U}_j = \{\pm u_i^{(j)},\ u_i^{(j)} = \rho_j^i u_0^{(j)},\ i = \pm 1,\ \pm 2,\ \cdots\} \cup \{\pm u_0^{(j)}\} \cup \{0\},\ 0 < \rho_j < 1,\ u_0^{(j)} > 0. \quad (6)$$

The associated quantizer $f_j(\cdot)$ are defined as

$$f_j(v) = \begin{cases} u_i^{(j)} & \text{if } \frac{1}{1+\sigma_j}u_i^{(j)} < v \le \frac{1}{1-\sigma_j}u_i^{(j)},\ v > 0 \\ 0 & \text{if } v = 0, \\ -f_j(-v) & \text{if } v < 0, \end{cases}$$

where $\sigma_j = \frac{1-\rho_j}{1+\rho_j}$. According to [16], ρ_j is also called the quantization density of quantizer $f_j(\cdot)$.

Similarly, the quantizer $g_j(\cdot)$ $(j \in \{1,\ 2,\ \cdots,\ p\})$ are of quantization densities ρ_j, respectively, and denote $\pi_j = \frac{1-\rho_j}{1+\rho_j}$.

For given logarithmic quantizer f_j, a sector bound condition was provided in [16] as follows:

$$f_j(v) = (1 + \Lambda_j)v, \quad (7)$$

where $\Lambda_j \in [-\sigma_j, \sigma_j]$. And then, the systems via quantized linear state feedback can be transferred to linear systems with norm-bounded uncertainty. The same transformations were employed in [17], [18] and [19]. However, by using such technique, it is difficult to design controller for the NCSs with state and control quantizations since these exist uncertainties on both sides of controller gain matrix K.

In this paper, we also study the problems of H_∞ stability and stabilization for system (5). New sector bound conditions of quantizers are developed, and based on these conditions, a new approach to networked-based control is proposed. The model transformation of system is not needed, and simple H_∞ stability and stabilization criteria are presented. Compared with the existing ones, the newly obtained results are less conservative and less complex.

III. Main results

In this section, new sector bound-based H_∞ stability condition and H_∞ stabilization method will be given. For convenience, we denote

$$\Delta = diag\{\sigma_1,\ \sigma_2,\ \cdots,\ \sigma_n\}, \quad \Delta_0 = I - \Delta, \quad \Delta_1 = I + \Delta,$$
$$\Pi = diag\{\pi_1,\ \pi_2,\ \cdots,\ \pi_p\}, \quad \Pi_0 = I - \Pi, \quad \Pi_1 = I + \Pi.$$

A. New sector bound conditions of quantizers

From the definition of logarithmic quantizers, new sector bound conditions can be derived.

Lemma 1. The logarithmic static quantizers $f_j(\cdot)$ $(j = 1,\ 2,\ \cdots,\ n)$ and $g_m(\cdot)$ $(m = 1,\ 2,\ \cdots,\ p)$ are of quantization densities ρ_j and ρ_m, respectively, then for any diagonal matrices $S > 0,\ H > 0$, the following inequalities are true:

$$\Big(f(x(t)) - \Delta_0 x(t)\Big)^T S\Big(f(x(t)) - \Delta_1 x(t)\Big) \le 0, \quad (8)$$

$$\Big(g(Kf(\cdot)) - \Pi_0 Kf(\cdot)\Big)^T H\Big(g(Kf(\cdot)) - \Pi_1 Kf(\cdot)\Big) \le 0. \quad (9)$$

Proof: Trivial. ∎

Remark 1. Lemma 1 presents new sector bound conditions of quantizers. Unlike the existing works (for example, [17], [18] and [19]), the difficulty of stability analysis and controller design for NCSs with both state and control quantizations can be overcome effectively by using these conditions, and LMI-based stability and stabilization results will be derived in the sequel.

B. H_∞ performance analysis

Based on Lemma 1, we can give an H_∞ performance analysis result as follows.

Theorem 1. Given the controller gain matrix K and positive scalars $\tau,\ \eta$ $(0 < \tau < \eta)$, the closed-loop system (5) is asymptotically stable with an H_∞ disturbance attention level γ if there exist matrices $P > 0,\ R \ge 0,\ Q \ge 0,\ Z_i > 0$ $(i = 1,\ 2,\ 3)$, and diagonal matrices $S > 0,\ H > 0$ satisfying

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \mathscr{A}Z \\ * & -I & 0 \\ * & * & -Z \end{bmatrix} < 0, \quad (10)$$

where

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & 0 & \Gamma_{16} & 0 & 0 & PB & PE \\ * & \Gamma_{12} & \frac{Z_2}{\eta-\tau} & \frac{Z_2}{\eta-\tau} & 2S & 0 & 0 \\ * & * & \Gamma_{13} & \frac{Z_3}{\eta-\tau} & 0 & 0 & 0 \\ * & * & * & \Gamma_{14} & 0 & 0 & 0 \\ * & * & * & * & \Gamma_{15} & 2K^T H & 0 \\ * & * & * & * & * & -2H & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$
\begin{aligned}
\Gamma_{11} &= PA + A^T P + Q + R - \frac{1}{\tau}(Z_1 + Z_3), \\
\Gamma_{12} &= -\frac{2}{\eta - \tau} Z_2 - 2\Delta_1 S \Delta_0, \\
\Gamma_{13} &= -Q - \frac{1}{\tau}(Z_1 + Z_3) - \frac{1}{\eta - \tau}(Z_2 + Z_3), \\
\Gamma_{14} &= -R - \frac{1}{\eta - \tau}(Z_2 + Z_3), \\
\Gamma_{15} &= -2S - 2K^T \Pi_0 H \Pi_1 K, \\
\Gamma_{16} &= \frac{1}{\tau}(Z_1 + Z_3), \\
\Gamma_2 &= \begin{bmatrix} C & 0 & 0 & 0 & 0 & D & F \end{bmatrix}^T, \\
\mathscr{A} &= \begin{bmatrix} A & 0 & 0 & 0 & 0 & B & E \end{bmatrix}^T, \\
Z &= \tau Z_1 + (\eta - \tau) Z_2 + \eta Z_3.
\end{aligned} \tag{11}
$$

Proof: Choose a Lyapunov-Krasovskii functional candidate as follows:

$$
\begin{aligned}
V(t) =& x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds + \int_{t-\eta}^{t-\tau} x^T(s)Rx(s)ds \\
&+ \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta \\
&+ \int_{-\eta}^{-\tau}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta \\
&+ \int_{-\eta}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_3\dot{x}(s)dsd\theta.
\end{aligned} \tag{12}
$$

Calculating the time derivative of $V(t)$ along the solution of (5) for $\forall t \in [i_k h + \tau_k,\ i_{k+1}h + \tau_{k+1})$, it yields

$$
\begin{aligned}
\dot{V}(t) =& 2x^T(t)P\dot{x}(t) + x^T(t)(Q+R)x(t) - x^T(t-\tau)Qx(t-\tau) \\
&- x^T(t-\eta)Rx(t-\eta) + \dot{x}^T(t)Z\dot{x}(t) \\
&- \int_{t-\tau}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{i_k h}^{t-\tau} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
&- \int_{t-\eta}^{i_k h} \dot{x}^T(s)Z_2\dot{x}(s)ds.
\end{aligned} \tag{13}
$$

From the Jensen's integral inequality [20], and Lemma 1, it gets that

$$
\begin{aligned}
&\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\
&\le \zeta^T(t)[\Gamma_1 + \mathscr{A}Z\mathscr{A}^T + \Gamma_2\Gamma_2^T]\zeta(t),
\end{aligned} \tag{14}
$$

where $\zeta(t) = [x^T(t) \quad x^T(i_k h) \quad x^T(t-\tau) \quad x^T(t-\eta) \quad f^T(x(i_k h)) \quad g^T(Kf(x(i_k h))) \quad \omega^T(t)]^T$. Since (10) is equivalent to $\Gamma_1 + \mathscr{A}Z\mathscr{A}^T + \Gamma_2\Gamma_2^T < 0$, so the following inequality holds for all nonzero $\omega(t) \in L_2[0,\ \infty)$:

$$
z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(x_t) < 0. \tag{15}
$$

Under zero initial condition, we have $V(0) = 0$ and $V(T) \ge 0$ for any $T > 0$. Integrating both sides of (15) yields $||z||_2 < \gamma||\omega||_2$ for all nonzero $\omega(t) \in L_2[0,\ \infty)$, and the proof is completed. ∎

Remark 2. By taking new sector bound conditions of quantizers into account and using the Jensen's integral inequality, Theorem 1 presents a new sufficient condition of H_∞ stability for system (1) with both quantized state and control signals. Compared with [9], [10] and [18], the newly proposed condition is more simple and more applicable. Meanwhile, it is also less conservative than that in [18], which will be proved in the next section. Therefore, Theorem 1 is more effective.

C. H_∞ controller design

Note that $-2\Delta_1 S\Delta_0 + 2S = 2\Delta S\Delta$, $-2\Pi_1 H\Pi_0 + 2H = 2\Pi H\Pi$, and by the Schur complement, one can get the following lemma from Theorem 1.

Lemma 2. Inequality $\Gamma < 0$ in (10) is equivalent to

$$
\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_5 \\ * & -I & 0 & 0 \\ * & * & \Phi_4 & 0 \\ * & * & * & \Phi_6 \end{bmatrix} < 0, \tag{16}
$$

where

$$
\Phi_1 = \begin{bmatrix}
\Phi_{11} & PBK & \frac{Z_1+Z_3}{\tau} & 0 & PBK & PB & PE \\
* & \Phi_{12} & \frac{Z_2}{\eta-\tau} & \frac{Z_2}{\eta-\tau} & 0 & 0 & 0 \\
* & * & \Phi_{13} & \frac{Z_3}{\eta-\tau} & 0 & 0 & 0 \\
* & * & * & \Phi_{14} & 0 & 0 & 0 \\
* & * & * & * & -2S & 0 & 0 \\
* & * & * & * & * & -2H & 0 \\
* & * & * & * & * & * & -\gamma^2 I
\end{bmatrix},
$$

$$
\begin{aligned}
\Phi_{11} &= PA + A^T P + Q + R - \frac{1}{\tau}(Z_1 + Z_3), \\
\Phi_{12} &= -\frac{2}{\eta - \tau} Z_2, \\
\Phi_{13} &= -Q - \frac{1}{\tau}(Z_1 + Z_3) - \frac{1}{\eta - \tau}(Z_2 + Z_3), \\
\Phi_{14} &= -R - \frac{1}{\eta - \tau}(Z_2 + Z_3), \\
\Phi_2 &= \begin{bmatrix} C & DK & 0 & 0 & DK & D & F \end{bmatrix}^T, \\
\Phi_3 &= \begin{bmatrix} \mathscr{B} & \mathscr{B} & \mathscr{B} \end{bmatrix} \times \Phi_4, \\
\mathscr{B} &= \begin{bmatrix} A & BK & 0 & 0 & BK & B & E \end{bmatrix}^T, \\
\Phi_4 &= diag\{-\tau Z_1,\ -(\eta - \tau)Z_2,\ -\eta Z_3\}, \\
\Phi_5 &= \begin{bmatrix} 0 & 2\Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\Pi K & 0 & 0 & 2\Pi K & 0 & 0 \end{bmatrix}^T, \\
\Phi_6 &= diag\{-2S^{-1},\ -2H^{-1}\}.
\end{aligned}
$$

Denoting

$$
\begin{aligned}
&\bar{P} = P^{-1}, \quad \bar{R} = \bar{P}R\bar{P}, \quad \bar{Q} = \bar{P}Q\bar{P}, \\
&Y = K\bar{P}, \quad \bar{S} = \bar{P}S\bar{P}, \quad \bar{Z}_i = \bar{P}Z_i\bar{P}\ (i = 1,\ 2,\ 3),
\end{aligned}
$$

and noticing that $Z_i^{-1} = \bar{P}\bar{Z}_i^{-1}\bar{P}$ $(i = 1,\ 2,\ 3)$ and $S^{-1} = \bar{P}\bar{S}^{-1}\bar{P}$, the following inequalities are true for any positive scalars θ_i $(i = 1,\ 2,\ \cdots,\ 5)$:

$$
-Z_i^{-1} \le \theta_1^2 \bar{Z}_i - 2\theta_1 \bar{P} \quad (i = 1,\ 2,\ 3), \tag{17}
$$

$$
-S^{-1} \le \theta_4^2 \bar{S} - 2\theta_4 \bar{P}, \tag{18}
$$

$$
-H^{-1} \le \theta_5^2 H - 2\theta_5 I. \tag{19}
$$

Then, pre- and post-multiplying $diag\{P^{-1},\ P^{-1},\ P^{-1},\ P^{-1},\ P^{-1},\ I,\ I,\ I,\ Z_1^{-1},\ Z_2^{-1},\ Z_3^{-1},\ I,\ I\}$ and its transpose on both sides of Φ in (16), the following theorem can be obtained.

Theorem 2. Given positive scalars $\tau,\ \eta\ (0<\tau<\eta)$ and $\theta_i>0\ (i=1,\ 2,\ 3)$, the closed-loop system (5) is asymptotically stable with an H_∞ disturbance attention level γ if there exist matrices $\bar{P}>0,\ \bar{R}\geq 0,\ \bar{Q}\geq 0,\ \bar{Z}_i>0\ (i=1,\ 2,\ 3)$, and diagonal matrices $\bar{S}>0,\ H>0$ satisfying

$$\bar{\Phi}=\begin{bmatrix} \bar{\Phi}_1 & \bar{\Phi}_2 & \bar{\Phi}_3 & \bar{\Phi}_5 \\ * & -I & 0 & 0 \\ * & * & \bar{\Phi}_4 & 0 \\ * & * & * & \bar{\Phi}_6 \end{bmatrix}<0, \tag{20}$$

where

$$\bar{\Phi}_1=\begin{bmatrix} \bar{\Phi}_{11} & BY & \frac{\bar{Z}_1+\bar{Z}_3}{\tau} & 0 & BY & B & E \\ * & -\frac{2\bar{Z}_2}{\eta-\tau} & \frac{\bar{Z}_2}{\eta-\tau} & \frac{\bar{Z}_2}{\eta-\tau} & 0 & 0 & 0 \\ * & * & \bar{\Phi}_{13} & \frac{\bar{Z}_3}{\eta-\tau} & 0 & 0 & 0 \\ * & * & * & \bar{\Phi}_{14} & 0 & 0 & 0 \\ * & * & * & * & -2\bar{S} & 0 & 0 \\ * & * & * & * & * & -2H & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{\Phi}_{11}=A\bar{P}+\bar{P}A^T+\bar{Q}+\bar{R}-\frac{1}{\tau}(\bar{Z}_1+\bar{Z}_3),$$

$$\bar{\Phi}_{13}=-\bar{Q}-\frac{1}{\tau}(\bar{Z}_1+\bar{Z}_3)-\frac{1}{\eta-\tau}(\bar{Z}_2+\bar{Z}_3),$$

$$\bar{\Phi}_{14}=-\bar{R}-\frac{1}{\eta-\tau}(\bar{Z}_2+\bar{Z}_3),$$

$$\bar{\Phi}_2=\begin{bmatrix} C\bar{P} & DY & 0 & 0 & DY & D & F \end{bmatrix}^T,$$

$$\bar{\Phi}_3=\begin{bmatrix} \tau\mathscr{C} & (\eta-\tau)\mathscr{C} & \eta\mathscr{C} \end{bmatrix},$$

$$\mathscr{C}=\begin{bmatrix} A\bar{P} & BY & 0 & 0 & BY & B & E \end{bmatrix}^T,$$

$$\bar{\Phi}_4=diag\{\tau(\theta_1^2\bar{Z}_1-2\theta_1\bar{P}),\ (\eta-\tau)(\theta_2^2\bar{Z}_2-2\theta_2\bar{P}),\ \eta(\theta_3^2\bar{Z}_3-2\theta_3\bar{P})\},$$

$$\bar{\Phi}_5=\begin{bmatrix} 0 & 2\Delta\bar{P} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\Pi Y & 0 & 0 & 2\Pi Y & 0 & 0 \end{bmatrix}^T,$$

$$\bar{\Phi}_6=diag\{2\theta_4^2\bar{S}-4\theta_4\bar{P},\ 2\theta_5^2 H-4\theta_5 I\}.$$

Moreover, a desired H_∞ controller gain matrix is given by

$$K=Y\bar{P}^{-1}. \tag{21}$$

Remark 3. Theorem 2 presents an LMI-based condition for the existence of desired H_∞ state-feedback controllers. Compared with the corresponding results in [18] and [17], Theorem 2 contains less decision variables, and the structure of matrix $\bar{\Phi}$ is also simpler. It is obvious that Theorem 2 is also more applicable and less conservative than the corresponding one in [18].

Remark 4. In the process of deriving Theorem 2 from Lemma 3, since $\bar{Z}_i$ and $\bar{Z}_i^{-1}$ $(i=1,\ 2,\ 3)$, $\bar{S}$ and $\bar{S}^{-1}$, H and H^{-1} exist simultaneously, so a non-convex problem yields. Though the cone complementarity linearization (CCL) method given in [21] is commonly used to solve such problem, it is well known that the iterative process is very complex. To reduce the calculational complexity, the inequalities $-\bar{P}\bar{Z}_i^{-1}\bar{P}\leq\bar{Z}_i-2\bar{P}$ $(i=1,\ 2,\ 3)$ were employed in [18]. In order to reduce conservatism, we introduce tuning parameters $\theta_i>0$ $(i=1,\ 2,\ \cdots,\ 5)$ satisfying (17)-(19).

IV. Comparison with the existing results

In this section, we will prove that Theorem 1 is less conservative than the corresponding one in [18]. For convenience, the H_∞ stability result for system (5) with only quantized state (that is, $g(v)=v$ and $\Pi=0$) in [18] is listed as the following lemma.

Lemma 3. [18] Given the controller gain matrix K and a positive constant γ, the closed-loop system (5) with $g(v)=v$ is asymptotically stable with an H_∞ disturbance attention level γ if there exist matrices $P>0,\ Q\geq 0,\ R\geq 0,\ Z_i>0\ (i=1,\ 2),\ M>0,\ S,\ T,\ U,\ V$, and a diagonal matrix $W>0$ satisfying

$$\Theta=\begin{bmatrix} \Theta_1+\Theta_2+\Theta_2^T & \Theta_3 & \Theta_5^T & \Theta_7^T & \Theta_8 \\ * & \Theta_4 & 0 & 0 & 0 \\ * & * & \Theta_6 & 0 & \Theta_9 \\ * & * & * & -I & DK \\ * & * & * & * & -W \end{bmatrix}<0, \tag{22}$$

where

$$\Theta_1=\begin{bmatrix} PA+A^TP+Q+R & 0 & PBK & 0 & PE \\ * & -Q & 0 & 0 & 0 \\ * & * & \Delta^2 W & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Theta_2=\begin{bmatrix} S+V & T-S & U-T & -U-V & 0 \end{bmatrix},$$

$$\Theta_3=\begin{bmatrix} S & T & U & V \end{bmatrix},$$

$$\Theta_4=diag\{-\tau^{-1}Z_1,\ -(\eta-\tau)^{-1}Z_2,\ -\eta^{-1}Z_2,\ -\eta^{-1}M\},$$

$$\Theta_5=\begin{bmatrix} Z_1A & 0 & Z_1BK & 0 & Z_1E \\ Z_2A & 0 & Z_2BK & 0 & Z_2E \\ MA & 0 & MBK & 0 & ME \end{bmatrix},$$

$$\Theta_6=diag\{-\tau^{-1}Z_1,\ -(\eta-\tau)^{-1}Z_2,\ -\eta^{-1}M\},$$

$$\Theta_7=\begin{bmatrix} C & 0 & DK & 0 & F \end{bmatrix},$$

$$\Theta_8=\begin{bmatrix} (PBK)^T & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\Theta_9=\begin{bmatrix} (Z_1BK)^T & (Z_2BK)^T & (MBK)^T \end{bmatrix}^T.$$

To compare Lemma 3 with Theorem 1, it is necessary to simplify inequality (22).

After some manipulation including the Schur complement, one can get that $\Theta<0$ is equivalent to

$$\mathscr{H}+\mathscr{J}^T\mathscr{J}+\mathscr{L}^T\Big(\tau Z_1+(\eta-\tau)Z_2+\eta M\Big)\mathscr{L} - \begin{bmatrix} \bar{\Theta}_3^T & 0 & 0 \end{bmatrix}^T\Theta_4^{-1}\begin{bmatrix} \bar{\Theta}_3^T & 0 & 0 \end{bmatrix}<0, \tag{23}$$

where

$$\mathscr{H}=\begin{bmatrix} \mathscr{H}_{11} & PBK & \frac{1}{\tau}Z_1 & \frac{1}{\eta}M & PBK & PE \\ * & \mathscr{H}_{12} & \frac{1}{\eta-\tau}Z_2 & \frac{1}{\eta}Z_2 & 0 & 0 \\ * & * & \mathscr{H}_{13} & 0 & 0 & 0 \\ * & * & * & \mathscr{H}_{14} & 0 & 0 \\ * & * & * & * & -W & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\mathscr{H}_{11} = PA + A^T P + Q + R - \frac{1}{\tau} Z_1 - \frac{1}{\eta} M,$$
$$\mathscr{H}_{12} = \Delta^2 W - \frac{1}{\eta - \tau} Z_2 - \frac{1}{\eta} Z_2,$$
$$\mathscr{H}_{13} = -Q - \frac{1}{\tau} Z_1 - \frac{1}{\eta - \tau} Z_2,$$
$$\mathscr{H}_{14} = -R - \frac{1}{\eta}(Z_2 + M),$$
$$\mathscr{J} = \begin{bmatrix} C & DK & 0 & 0 & DK & F \end{bmatrix},$$
$$\mathscr{L} = \begin{bmatrix} A & BK & 0 & 0 & BK & E \end{bmatrix},$$
$$\bar{\Theta}_3 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \times \left(\Theta_3 + \begin{bmatrix} \frac{1}{\tau} Z_1 & 0 & 0 & \frac{1}{\eta} M \\ -\frac{1}{\tau} Z_1 & \frac{1}{\eta - \tau} Z_2 & 0 & 0 \\ 0 & -\frac{1}{\eta-\tau} Z_2 & \frac{1}{\eta} Z_2 & 0 \\ 0 & 0 & -\frac{1}{\eta} Z_2 & -\frac{1}{\eta} M \end{bmatrix} \right).$$

It is obvious that $\mathscr{H} + \mathscr{J}^T \mathscr{J} + \mathscr{L}^T \Big(\tau Z_1 + (\eta - \tau) Z_2 + \eta M \Big) \mathscr{L} < 0$ if $\Theta < 0$ holds. Contrarily, $\Theta < 0$ is also true if $\mathscr{H} + \mathscr{J}^T \mathscr{J} + \mathscr{L}^T \Big(\tau Z_1 + (\eta - \tau) Z_2 + \eta M \Big) \mathscr{L} < 0$ by letting $\bar{\Theta}_3 = 0$.

So, $\Theta < 0$ is equivalent to $\mathscr{H} + \mathscr{J}^T \mathscr{J} + \mathscr{L}^T \Big(\tau Z_1 + (\eta - \tau) Z_2 + \eta M \Big) \mathscr{L} < 0$, which implies that S, T, U and V are all redundant in Lemma 3. Thus, a simplified version of Lemma 3 can be derived.

Similarly, one can see that $\Gamma < 0$ with $\Pi = 0$ in (10) is equivalent to

$$\mathscr{M} + \mathscr{J}^T \mathscr{J} + \mathscr{L}^T \Big(\tau Z_1 + (\eta - \tau) Z_2 + \eta Z_3 \Big) \mathscr{L} < 0, \quad (24)$$

where

$$\mathscr{M} = \begin{bmatrix} \mathscr{M}_{11} & PBK & \frac{1}{\tau}(Z_1 + Z_3) & 0 & PBK & PE \\ * & \mathscr{M}_{22} & \frac{1}{\eta - \tau} Z_2 & \frac{1}{\eta - \tau} Z_2 & 0 & 0 \\ * & * & \mathscr{M}_{33} & \frac{1}{\eta - \tau} Z_3 & 0 & 0 \\ * & * & * & \mathscr{M}_{44} & 0 & 0 \\ * & * & * & * & -2S & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$
$$\mathscr{M}_{11} = PA + A^T P + Q + R - \frac{1}{\tau}(Z_1 + Z_3),$$
$$\mathscr{M}_{22} = 2\Delta^2 S - \frac{2}{\eta - \tau} Z_2,$$
$$\mathscr{M}_{33} = -Q - \frac{1}{\tau}(Z_1 + Z_3) - \frac{1}{\eta - \tau}(Z_2 + Z_3)$$
$$\mathscr{M}_{44} = -R - \frac{1}{\eta - \tau}(Z_2 + Z_3).$$

Let $Z_3 = M$ and $S = \frac{1}{2} W$, one can get that $\mathscr{M} - \mathscr{H} \leq 0$ holds. Thus, the following theorem is immediately obtained.

Theorem 3. Inequality (10) with $\Pi = 0$ is also feasible if inequality (22) is feasible.

Theorem 4. Inequality $\bar{\Phi} < 0$ with $\Pi = 0$ in (20) is feasible if the inequality in Theorem 4 of [18] is feasible.

TABLE I

COMPARISONS OF MINIMUM FEASIBLE γ FOR VARIOUS τ AND η

Methods	η	0.43		0.63	
	τ	0.1	0.2	0.2	0.3
[18]	$\gamma_{\min}$	2.8113	2.5233	11.2314	6.0780
Theorem 1	$\gamma_{\min}$	2.6511	2.3527	6.9674	4.5564

TABLE II

THE MINIMUMS OF γ FOR VARIOUS τ AND η

Method	η	0.43		0.63	
	τ	0.1	0.2	0.2	0.3
Theorem 1	$\gamma_{\min}$	3.6816	2.9769	64.7621	8.6297

V. ILLUSTRATIVE EXAMPLES

Example 1. [18] Consider system (5) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; E = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; D = 0.3, \; F = 0.5, \; K = \begin{bmatrix} -1 & 1 \end{bmatrix}.$$

The parameters for the quantizer $f(\cdot)$ are given by $\rho_1 = 0.9$ and $\rho_2 = 0.8$. As the same as in [18], the maximum number of data packet dropouts is 2, and the sampling period is 10ms. When the upper and lower bounds of delays, τ_k, are 0.6s and 0.3s, respectively, that is, $\tau = 0.3$s and $\eta = 0.63$s, the minimum guaranteed closed-loop H_∞ performance obtained by Theorem 1 is $\gamma_{\min} = 4.5564$ for the case of only state quantizations (let $\Pi = 0$), while the minimum was $\gamma_{\min} = 6.0780$ given in [18]. For various τ, η $(0 < \tau < \eta)$, the calculating results obtained by Theorem 1 and by the method in [18] are listed in Table 1.

When the control signals are also quantized, and the quantizer density of $g(\cdot)$ is given by $\rho_1 = 0.9$, then the minimums of γ obtained by Theorem 1 for various τ and η are listed in Table 2.

Example 2. [18] Consider system (5) with

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.3 & 0.3 & -0.004 & 0.004 \\ 0.3 & -0.3 & 0.004 & -0.004 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$
$$E = \begin{bmatrix} 0 & 0 & 0 & 0.1 \end{bmatrix}^T, \; C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix},$$
$$D = 0, \; F = 0.$$

Firstly, we consider the case of only state quantizations as [18] did. The parameters for the quantizer $f(\cdot)$ are given by $\rho_i = 0.9$ $(i = 1,\ 2,\ 3,\ 4)$. The maximum number of data packet dropouts is assumed to be 2, and the sampling period is 10ms. When the upper and lower bounds of delays, τ_k, are 40ms and 10ms, respectively, the minimum of γ obtained by Theorem 2 is 0.3451 with $\theta_i = 1$, and is 0.2259 with $\theta_i = 0.4$ $(i = 1,\ 2,\ \cdots,\ 5)$, respectively, while the minimum of γ obtained in [18] was 0.7864.

Then, when the control signals are also quantized, and the logarithmic quantizer $g(\cdot)$ is of quantization density $\rho_1 = 0.8$, then the minimum of γ obtained by Theorem 2 is $\gamma_{\min} =$

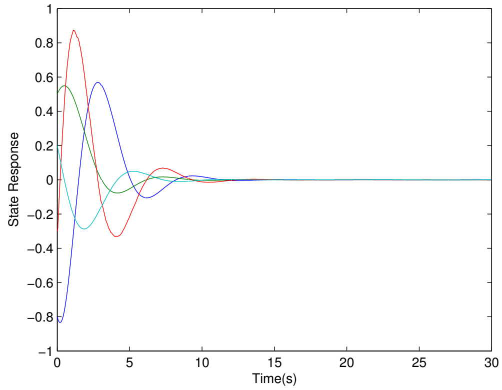

Fig. 1. State response with controller K

0.3582 with $\theta_i = 1$ $(i = 1,\ 2,\ \cdots,\ 5)$, and the corresponding controller gain is given by

$$K = \begin{bmatrix} -3.2619 & -0.9193 & -2.7348 & -7.7960 \end{bmatrix}.$$

Next, we will illustrate the simulating effect. If the initial condition is $\begin{bmatrix} -0.8 & 0.5 & -0.3 & 0.2 \end{bmatrix}^T$ and $\omega(t) = 0$, the network-induced delays and the packet dropouts are the same as in [18], then the state responses are depicted in Fig. 1.

If the disturbance signal is

$$\omega(t) = \begin{cases} \sin(0.2t), & 5 \le t \le 15 \text{ s}, \\ 0 & \text{otherwise}, \end{cases} \tag{25}$$

the network-induced delays and the data packet dropouts are the same as in [18], then the state responses are depicted in Fig. 2 under zero initial conditions and controller gain K.

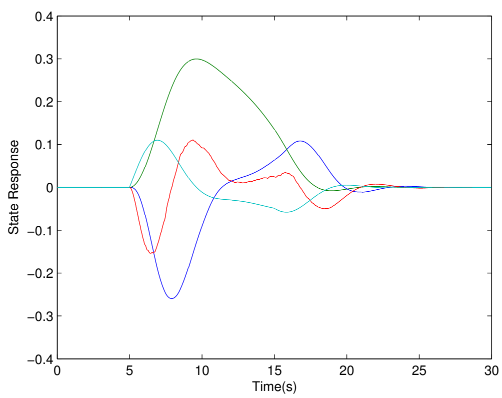

Fig. 2. State response with controller K

The above data and figures show that the H_∞ controller design method presented in this paper is more effective and less conservative.

VI. Conclusions

This paper investigates the problems of H_∞ stability and stabilization for NCSs with quantized state feedback. By proposing a novel sector bound-based method, the transformation of system models to uncertain systems is not needed, then the proposed controller design process is simpler than the existing ones. Even for the case of only state quantizations, the newly obtained results are less conservative than the existing ones. The complexity of the proposed results is also reduced greatly since fewer decision variables are involved. Numerical examples have illustrated the effectiveness and less conservativeness of the proposed results.

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