

Nonlinear Observers for Closed-Loop Control of a Combustion Engine Test Bench

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Abstract—In this article we compare the performance of four nonlinear state observers for a combustion engine test bench simulator including combustion oscillations, noisy measurements and disturbed inputs. These observers are the high-gain observer (HGO), the sliding-mode observer (SMO), the nonlinear extended state observer (NESO) and the extended Kalman Filter (EKF). The different observers are compared in open-loop in terms of the mean quadratic estimation error, computing time and convergence rate. A first important result obtained is that the NESO performance is good, although it does not need to know the engine friction model. Then the best of these is compared in closed-loop with a partial Luenberger observer which requires the knowledge of the model of the combustion oscillations. It turns out that we can achieve similar tracking results without the knowledge of the combustion oscillations model.

I. INTRODUCTION

A combustion engine test bench mainly consists of a combustion engine, a dynamometer and a connection shaft. An interesting experiment that is usually performed with an engine test bench is the reference tracking of the engine speed and torque. In general, for nonlinear systems, the tracking problem is solved using a state feedback controller. This approach requires that all states are known, but this is not always possible through measurements and therefore observers are needed.

As in [1], the state variables of a combustion engine test bench are the engine torque and speed, the dynamometer speed and the torsion angle of the connecting shaft. Usually, the torque and the torsion angle are quite hard to measure. On the contrary, the engine and dynamometer speed are readily available, but they are perturbed by measurement noise and especially engine speed is corrupted by the combustion oscillations.

A possible approach to solve the unknown state problem is to use a state observer. Since the two speeds are known a reduced observer would be possible. This solution was adopted in [1]. However, in this case, since the combustion oscillations strongly affect the observed variables, an observer based on an internal model of the combustion oscillations is necessary.

In this paper different nonlinear full state observers are proposed: the Extended Kalman Filter (EKF), the High Gain Observer (HGO), the Sliding Mode Observer (SMO) and the Nonlinear Extended State Observer (NESO). These do not require the knowledge of the combustion oscillations model,

which is difficult to compute and, only for the NESO, not even the knowledge of the engine friction part of the model is necessary.

The EKF is the most widely used [2], although a well-known difficulty arising through its application is that is often necessary either to have a good initial condition so that the initial estimation error is sufficiently small or to permit functions only weakly nonlinear. To increase the domain of attraction and to reduce the time for error decay in [3] a slight modification of the extended Kalman Filter is shown. A typical observer is the HGO. It was first introduced in [4] for the design of output feedback controllers due to its ability to robustly estimate the unmeasured states while asymptotically attenuating disturbances. Then, there is the SMO. Similar to the sliding mode controller, it is designed by using a sliding surface and offers robustness against both parametric uncertainties and external disturbances. The robustness characteristic is achieved by using a high-speed switching function, which forces the system to remain on the sliding surface [5–7]. Instead, a class of nonlinear extended state observers was proposed in [8]. It is rather independent of the mathematical model of the plant, thus achieving inherent robustness. Furthermore it was also tested and verified in key industrial control problems [9, 10].

The paper presents a comparison of performances of these observers in terms of the quadratic state estimation error, computing time and convergence rate for a combustion engine test bench model. Then, some of these observers are tested in closed-loop, thus constructing a so-called output feedback controller which is used for set point tracking of the test bench. Hence, these different control systems are compared, together with the one presented in [1], in terms of reference tracking error. We prove that the results are quite comparable, although, in our case, the observer design is simpler because it does not require the knowledge of the combustion oscillations model.

The paper is organized as follows. In Section II we introduce the mathematical model of the combustion engine test bench and a brief reminder of the controller adopted. The Section III explains the structure of the observers used (EKF, HGO, SMO and NESO). In the Section IV simulation results of these observers in open-loop are presented. A comparison in terms of mean quadratic error is also performed. A comparison in closed-loop, between two of these observers and the partial observer of [1], is presented in Section V. Finally, conclusions are given in Section VI.

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II. ENGINE TEST BENCH MODEL

The typical structure of the combustion engine test bench is illustrated in Figure 1.

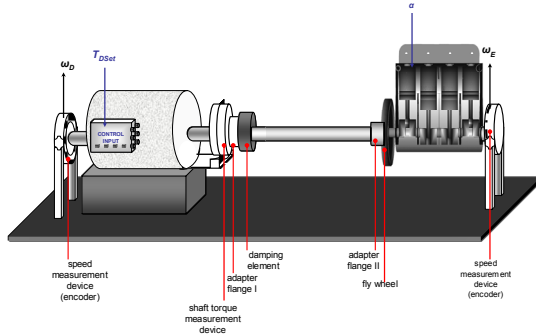


Figure 1 - Combustion Engine Test Bench System

The main parts of such a system are the dynamometer, used to simulate the load, the connection shaft and the combustion engine itself. The engine and dynamometer speed are measured using incremental encoders and, therefore, are known quantities. The engine torque and the torsion angle are hard to measure and therefore unknown.

A typical control design objective for such a system is the reference tracking of the engine torque and speed, by controlling the air gap torque of the dynamometer and the throttle pedal of the combustion engine.

According to [11], the model of the system can be approximated by:

$$\begin{cases} \dot{T}_E = -(c_0 + c_1\omega_E + c_2\omega_E^2)T_E + m(\omega_E, T_E, \alpha) \\ \Delta\dot{\psi} = \omega_E - \omega_D \\ \dot{\omega}_E = \frac{1}{\theta_E}(T_E - c\Delta\psi - d(\omega_E - \omega_D)) \\ \dot{\omega}_D = \frac{1}{\theta_D}(c\Delta\psi + d(\omega_E - \omega_D) - T_{DSet}) \end{cases} \quad (1)$$

where T_E is the engine torque, the first part of the torque equation is an approximation of the engine friction part, ω_E and ω_D are the engine and the dynamometer speed, $\Delta\psi$ is the torsion angle of the shaft, $m(\omega_E, T_E, \alpha)$ is a nonlinear function with α being the throttle pedal angle and T_{DSet} is the torque of the dynamometer; c_0 , c_1 and c_2 are positive constants, θ_E and θ_D are the inertia of the engine and the dynamometer, respectively, c and d are the stiffness and damping coefficients of the shaft.

The measured signals, ω_E and ω_D , are affected by the batch behavior of the combustion, which depends on the crankshaft angle [12, 13]. Exactly, since each cylinder fires every 720° crankshaft angle (720° CA), which means in a four strokes engine there is combustion every 180° CA that causes the combustion oscillation which is considered as a periodic noise to the engine speed.

In [1] a partial observer based on an internal model of combustion oscillations was designed. Exactly, a frequency varying internal model observer was designed to reconstruct

the estimated signal including the periodical signals. In this way, from the state of the internal model it is possible to calculate the mean value of the reconstructed signals, which are necessary for the controller.

Instead, in this paper four different full state observers have been designed without the need of the knowledge of the model of the combustion oscillations, because the state variables ω_E and ω_D are measured and observed which leads to a kind of filtering. Therefore less model information is needed for our proposed observers.

A. Reminder of the controller

Recall that the control design objective is to track a desired T_E and ω_E reference trajectory by controlling the inputs α and T_{DSet} . To simplify the control project the idea has been to approximate the system with an input affine system.

As in [1] and [14], the system (1) being already in Hammerstein form, the idea was to approximate the nonlinear map m with a smooth invertible nonlinear function \tilde{m} , whose inverse, with respect to the control input, exists. Thus, as can be seen in [14], a nonlinear input affine system is obtained where, now, the input is the map m .

Once the system is obtained in affine form, it is easy to use dynamic inversion control laws [15].

Therefore the system takes the following form:

$$\begin{cases} \dot{x} = f(x) + g(x)(u + w) \\ y = h(x) \end{cases} \quad (2)$$

where x , u and y being the state, the input and the output, respectively, while w is input disturbance.

In [16] a robust state feedback controller was designed that guarantees asymptotic stability of the system (1), for a set point tracking problem of the engine. Exactly, the feedback controller is an extension of a robust stabilizing controller, given from the following law:

$$u = -R^{-1}g(x)^T \frac{\partial V(x)^T}{\partial x} \quad (3)$$

where R is a positive definite weights matrix, $g(x)$ is the input nonlinear function of the system and where $V(x)$ is a control Lyapunov function, which is the solution of the partial differential equation of Hamilton Jacobi Bellman.

Then in [1] the original state has been substituted by the state estimate to construct an output feedback controller for the engine.

To obtain a more realistic model, (1) has been augmented with a combustion oscillations model. For this reason, in [1] an observer including the combustion oscillations model (therefore based on internal model principle) has been implemented. In this way, it is possible to estimate these combustion oscillations in order to compensate the noise presents on estimated state.

Since an observer with internal combustion oscillations model is hard to obtain in real applications, in [14] full state observers were designed, that do not require the knowledge of the combustion oscillations model.

III. NONLINEAR OBSERVERS DESIGN

We briefly introduce the nonlinear observers used in this paper.

A. Extended Kalman Filter

The EKF is a commonly used method for estimating the state of a nonlinear system. The method consists of designing an observer for a linearization of the true system along an estimated trajectory [17-19].

For a nonlinear system, input affine, with input disturbance neglected, written as:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (4)$$

the EKF presents in the following form:

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + K(t)(y - h(\hat{x})) \\ \hat{y} = h(\hat{x}) \end{cases} \quad (5)$$

where \hat{x} is the estimated state and $K(t)$ is the observer gain. The gain has to be updated online via the solution of a Riccati differential equation and, for this reason, this filter is computationally onerous.

The main drawback of this observer is that if the initial estimate state is wrong, or if the process is modeled incorrectly, the filter may quickly diverge.

Therefore, to improve the stability of this observer a slight modification to the EKF was made. To calculate the observer gain $K(t)$ a Riccati differential equation which is similar to that one for the EKF is introduced. This equation is the following:

$$\dot{P}(t) = (A(t) + \gamma I)P(t) + P(t)(A(t)^T + \gamma I) - P(t)C(t)^T R^{-1}C(t)P(t) + Q \quad (6)$$

where γ is a positive real number.

In [20] it has been proven that, with this modification, an increased domain of convergence can be obtained. The disadvantage of this filter is that the choice of the initial condition $P(0)$ of the Riccati differential equation has much more effect on the performance of the this observer than on the performance of the traditional EKF.

B. High Gain Observer

This observer works for a wide class of nonlinear systems and guarantees that the output feedback controller recovers the performance of the state feedback controller when the observer gain is sufficiently high.

The observer implementation is simple and it requires small computational effort. More, an advantageous feature of this observer is that it shows robust performance in the presence of model uncertainties.

For the system (4), the HGO has the following form:

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + O^{-1}(\hat{x})K_\theta(y - h(\hat{x})) \\ \hat{y} = h(\hat{x}) \end{cases} \quad (7)$$

where O is the Jacobian of the following transformation:

$$T(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \dots \\ \dots \\ L_f^{r-1} h(x) \end{bmatrix} \quad (8)$$

where L is the Lie operator. K_θ is the observer gain to project. The coefficients of the gain, used for the simulations, are the same shown in [14]. Exactly, the choice of the coefficients guarantees the asymptotic convergence of the estimated state.

C. Sliding Mode Observer

The sliding mode observer is known for its robustness and insensitivity with respect to unknown parameter variations. The fundamental difference between the sliding mode observer and other observers is that the sliding mode observer is usually discontinuous and the state error trajectories are more onto a special attractive so-called sliding surface. The main part of the design is the specification of a switching function, which is selected to guarantee desirable performance exhibited by the system of interest.

For an observer problem, the switching function may mostly be defined as the error between system and observer output. By forcing this switching function to zero, the observer output is compelled to equal the system output and, so, a set of estimated states that yield the measured system output are obtained.

Exactly, the SMO, for the system (4), has the following form:

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}) + g(\hat{x})u + O^{-1}(\hat{x}) \cdot K_{SMO} \operatorname{sgn}(y - h(\hat{x})) \\ \hat{y} = h(\hat{x}) \end{cases} \quad (9)$$

where O is always the Jacobian of (8) and where K_{SMO} is the gain, computed by stability analysis. As for the HGO, the terms of K_{SMO} are chosen equal to [14].

D. Nonlinear Extended State Observer

The previous methods depend on the full knowledge of the plant dynamics. An alternative method is the NESO.

A first step to realize this observer is that to augment the state of the system. In particular the nonlinear part of the system, that in our case is the engine friction part, is treated as an extended state. In this way, it is now possible to estimate this part, by using a simple state estimator. Calling n the nonlinear part of the system and e the error between the system output y and the observer output \hat{y} , the NESO structure is the following:

$$\begin{cases} \dot{\hat{z}} = A\hat{z} + Bu + \beta r(e) \\ \hat{y} = C\hat{z} \end{cases} \quad (10)$$

where $\hat{z} = [\hat{x} \quad \hat{n}]^T$ is the estimated state, with $\hat{x} \in R^m$, $\hat{n} \in R$, $\hat{y} \in R^p$ A , B and C are the dynamic, input and output matrices of the system, respectively; $\beta \in R^{(m+1) \times p}$ is a constant gain, finally $r \in R^p$ is defined as a modified exponential gain function:

$$r(e, \rho, \delta) = \begin{cases} |e|^\rho \operatorname{sign}(e), & |e| > \delta \\ \frac{e}{\delta^{1-\rho}}, & |e| \leq \delta \end{cases} \quad (11)$$

where ρ is chosen between 0 and 1 and δ is a positive small number used to limit the gain in the neighborhood of origin of the error.

It can be seen that for $\rho=0$ the gain function r in (11) is equal to SMO one, while for $\rho=1$ r is a simple proportional observer, like in a Luenberger filter.

A possible way to compute β is that to use the pole placement method, considering the gain linear, i.e. $r = e$.

Further, from experiments, it has been observed that good results can be achieved for $\rho > 1$.

Exactly, we have taken as nonlinear part n of NESO design this term:

$$n = (c_0 + c_1 \omega_E + c_2 \omega_E^2) T_E \quad (12)$$

From here, the NESO structure for model (1) is the following:

$$\begin{cases} \dot{\hat{T}}_E = -\hat{n} + m + \beta_{11} r_1 + \beta_{12} r_2 \\ \Delta \dot{\hat{\psi}} = \hat{\omega}_E - \hat{\omega}_D + \beta_{21} r_1 + \beta_{22} r_2 \\ \dot{\hat{\omega}}_E = \frac{1}{\theta_E} (\hat{T}_E - c \Delta \hat{\psi} - d (\hat{\omega}_E - \hat{\omega}_D)) + \beta_{31} r_1 + \beta_{32} r_2 \\ \dot{\hat{\omega}}_D = \frac{1}{\theta_D} (c \Delta \hat{\psi} + d (\hat{\omega}_E - \hat{\omega}_D) - T_{DSet}) + \beta_{41} r_1 + \beta_{42} r_2 \\ \dot{\hat{n}} = \beta_{51} r_1 + \beta_{52} r_2 \end{cases} \quad (13)$$

where:

$$\beta = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \vdots & \vdots \\ \beta_{51} & \beta_{52} \end{bmatrix} \quad (14)$$

and

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \quad (15)$$

where r_1 and r_2 are computed by (11).

With respect to the choice of parameters ρ and δ , they have been tuned by experimental via, in order to minimize the estimate error:

$$\delta = 1 \quad (16)$$

$$\rho = 0.5$$

Starting with a linear gain $r = e$, the pole placement method has been used to compute the gain β .

IV. OPEN-LOOP COMPARISON

In the case of open-loop tests, the observers above are evaluated according to their mean squared state estimation error, computing time and convergence rate.

The parameters of the test bench and of the controller are the same of [1]. The simulation is obtained with a fixed step

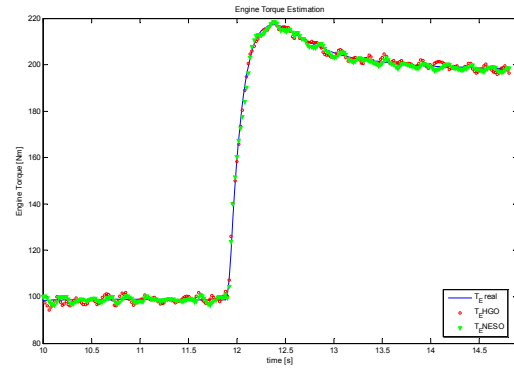
solver. The mean squared estimation error is computed in the interval time of 11-15 seconds.

The observer design parameters for the EKF, HGO and SMO are the same of [14] and thus, for the HGO, the gain is the result of a optimization routine that minimizes the quadratic error between the measured and the estimated mean torque; for the EKF, the output weighting matrix is such that the measured engine speed is more penalized than the dynamometer speed, while the state weighting matrix is such that only the engine torque and disturbance equations are penalized. Finally, for the SMO, as well as the NESO, the gain has been obtained by hand, always taking into account to minimize the estimated state error.

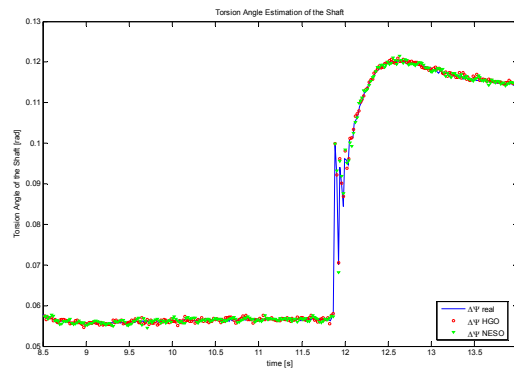
Since a typical problem is to track an engine torque reference, Table 1 shows the mean squared estimation error for each observer, computed on the engine torque and where with EKF2 has been indicated the second version of EKF.

OBSERVERS	MEAN SQUARE ESTIMATION ERROR
EKF	3.37
EKF 2	3.52
HGO	3.27
SMO	5.03
NESO	3.07

Table 1 - Mean Squared Estimation Error of the Engine Torque



(a)



(b)

Figure 2 - Engine Torque (a) and Rotation Angle (b) Estimation

These results show a good performance of NESO. Besides, it is to remember that the NESO does not know part of the plant dynamics on the contrary of the others observers.

This aspect is to take into account for a future test on the real system. In fact, we remember that the first equation of the model (1) is approximated, in order to put the model in Hammerstein form. Since all observers have similar performance, Figure 2 shows the engine torque and rotation angle estimation of NESO and HGO.

Instead, the Figure 3 shows the comparison between the two speeds of the system and the observed ones.

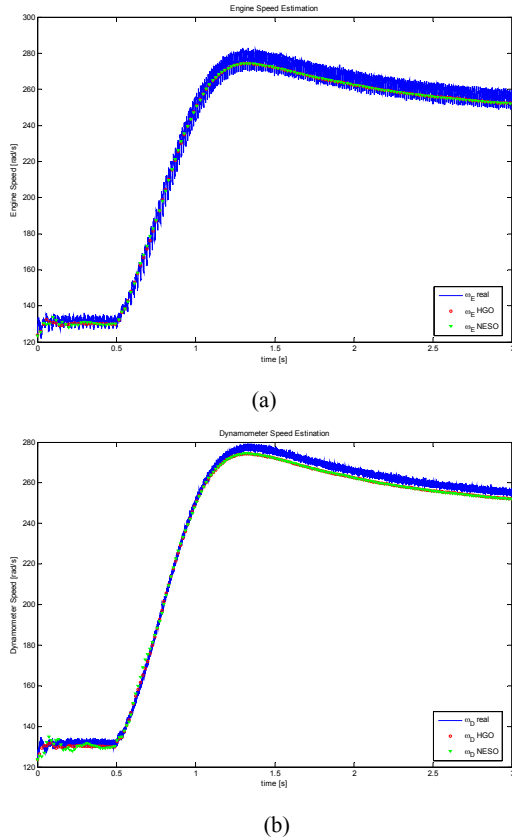


Figure 3 – Comparison between Engine and Observed Engine Speed (a) and Comparison between Dynamometer and Observed Dynamometer Speed (b)

By Figure 3 it is possible to see as the outputs are very noisy. In particular, the engine speed is corrupted by the combustion oscillations. Instead, the estimated outputs have less oscillations. This proves that the observers realize a filtering of the signals and, consequently, also the others two state variables, T_E and $\Delta\psi$, are filtered.

Since we are mainly interesting to the engine torque estimate, in Figure 4 the initial engine torque estimate errors of all observers are compared. We can note as the convergence time is almost similar for all observers. NESO is initially the worst. This was expected because this observer does not assume the knowledge of a part of plant. Therefore, this transient could be the time necessary to the NESO for estimating the nonlinear part.

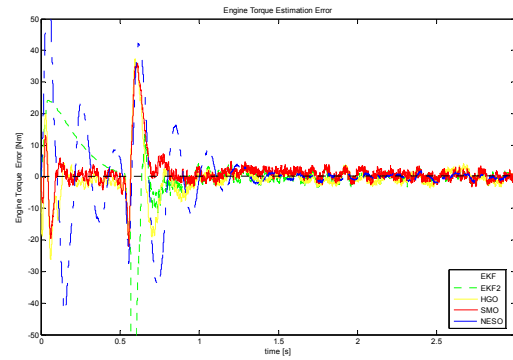


Figure 4 - Convergence Rate

The computing time of the observer depends by type and number of instructions of the algorithm. EKF is computationally onerous. The main aspect of this observer is the gain K . In fact it is a dynamic gain. For every iteration it is necessary to linearize the model equations, to resolve the Riccati differential equation and, finally, to compute the gain. It is evident that these operations require many time.

With respect to the HGO observer, it seems to be the best, considering the computing aspect. In fact, the gain is a static gain. It is computed off line and only once.

Same considerations can be seen about the SMO. In fact the structure of this observer is similar to HGO one.

Finally, the NESO observer presents a computing time slightly worst, compared to the HGO and SMO. In fact, the observer gain is variable and it depends of the output error value. In fact, if the output error is smaller of a certain threshold, the gain structure changes.

We can conclude that the HGO is the best with respect to the computing time.

By results obtained above, we can conclude that all observers show a good performance in term of estimation state and convergence rate. Instead, in term of computing time, the HGO shows the best performance.

While the HGO, the SMO and the EKF are based on the knowledge of the model equations, the NESO estimates the nonlinear part, which is also the uncertain part of the model. For these reasons, it seems reasonable to choose, for a comparison in closed-loop with the reduced order observer, the HGO and NESO.

V. CLOSED-LOOP COMPARISON

In this section a comparison in closed loop among the HGO, NESO and reduced order observer is shown.

In particular, the estimated state is substituted to the original state for constructing an output feedback controller. The control problem is the tracking of ω_E and T_E reference signals. Figure 5 shows the tracking of the engine torque and speed, with the different control systems. The results are very similar while the mean squared tracking errors show that the minimum tracking error is given from the reduced order observer.

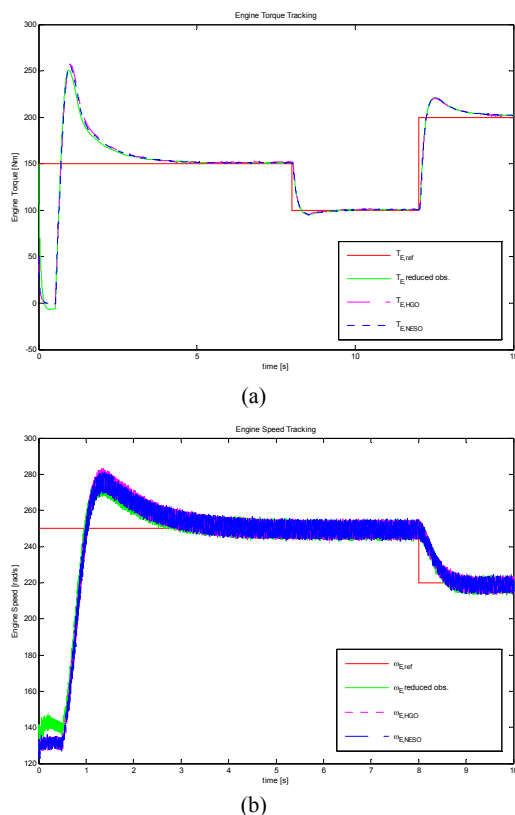


Figure 5 – Tracking of the engine torque (a) and speed (b) using different control systems

The tracking error for HGO control system is +0.3% compared to reduced order observer, while for NESO the tracking error is +0.5%.

However, the surprising result is that the performances are very similar although the NESO and HGO are much less complicated respect to the reduced observer.

VI. CONCLUSIONS

A comparison study of nonlinear observers, including the extended Kalman Filter, the high gain observer, the sliding mode observer and the nonlinear extended state observer, was performed on a combustion engine test bench model. The different observers are compared, in open-loop, in terms of convergence rate, mean squared state estimation error and computing time.

The extended Kalman filter is computationally onerous, therefore it is very hard to use it on a real application. This last aspect has been mainly taken into account for choosing the observers to test in closed loop.

A comparison in closed loop is shown among the high gain observer, the nonlinear extended state observer and the reduced state observer. They are compared in terms of tracking error.

The first important result of this work is that for full order observers the combustion oscillations do not negatively affect their performance, therefore it is not necessary to construct a filter based on combustion oscillations model, which is difficult to obtain in a real application.

The second important result is that the nonlinear extended state observer works well although it does not require the perfect knowledge of the model. Taking into account of a future application on real engine test bench, this aspect is very important.

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