2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Inverted Pendulum with Moving Reference for Benchmarking Control Systems Performance

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Abstract— **This paper presents inverted pendulum with moving reference for the testing of stabilizing control algorithms. The proposed system extends classical inverted pendulum by incorporating two moving masses. The motion of the two masses, that slide along the horizontal plane, is controllable for the purpose of platform stabilization. The usefulness of the idea presented is demonstrated using computer simulations by employing Proportional Integral Derivative (PID) control law.**

I. Introduction

In the field of engineering and technology the importance of benchmarks needs no explanation. In control systems these are used to simulate test environments for control methodologies and to understand the dynamics of complicated systems. They make it easy to check whether a particular algorithm is giving the requisite results. Some of these benchmarks are spring mass system, inverted pendulum, ball and beam, the tank filling process [1], the double pendulum [17] and the multimember pendulum [2].

A lot of work has been carried out on the inverted pendulum in terms of its stabilization. Many attempts have been made to control it using classical control ([5] [6]), non linear control [3] [4], H∞ control [7], control using neural networks [8] and control using fuzzy logic [9]. Thus it is a well pursued and understood problem, knowledge of which has been widely used in the study of stabilization of space vehicles [10] as well as in the study of biped robots ([11] - [14]). It has been used as a test apparatus in computer aided analysis and design thus making it easier to understand and to experiment with different control schemes ([15], [16]).

A number of variants of the inverted pendulum have been developed. The most primitive is the double pendulum ([17], [18]). Difficulty of the problem was further increased by increasing the number of members [19]. Furuta pendulum is a variant of the classical inverted pendulum in which the pendulum is attached to a rotating arm instead of a cart moving on a straight line[20]. Acrobot is a similar arrangement but has two members[21]. Spendulap is also a variant of inverted pendulum but is difficult to fabricate [22].

Above mentioned variant systems have only one output variable therefore they cannot be used as a benchmark for MIMO systems. Other MIMO systems don't render themselves easily to control, analysis and understandability therefore a system which fulfills all these requirements is required

The proposed system derives inspiration from the classical inverted pendulum but makes the problem more challenging by introducing a non-stationary reference, and therefore an additional input. Unlike the moving inverted pendulum robot JOE [23], which uses the velocity of the wheels to stabilize itself resulting in an unpredictable velocity of the wheels. The novelty of the system lies in the manner in which the system is stabilized i-e, that it is stabilized by changing the inertia of the system. The rest of the paper is organized as follows. Section II discusses mechanical construction of the proposed system. Electrical structure, both hardware and software, is described in Section III. The working of proposed system is discussed in Section IV, Computer simulations in Section V demonstrate performance of the proposed system, Section VI discusses some potential applications, Section VII concludes the paper.

II. MECHANICAL CONSTRUCTION

The system comprises of a horizontal plate that is connected to two wheels through a connecting rod. The wheels are independent of each other and placed in the centre of the plate. Thus the platform can move on a horizontal surface and is able to rotate about the axis of wheels. There are two masses on top of the system that can slide along the horizontal plate, one mass on either side. Two masses are used for ease of actuation and control. Both the masses are independent of each other in motion due to independent actuation. The complete arrangement is shown in Fig 1.

The mechanics of the proposed system are highly

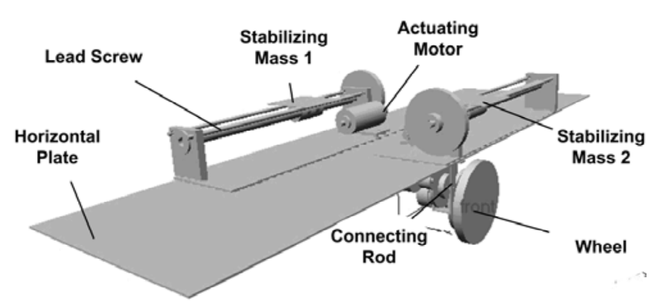

Fig 1 Inverted Pendulum with moving reference.

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nonlinear due to the translating masses (for stabilizing action) and rotating mass of the platform which changes the inertia constantly. The inertia directly depends on the square of the position of mass making the system highly nonlinear. The stabilizing action is also achieved by changing its inertial properties. It is a highly unstable system posing a tough challenge to the controls engineers.

III. ELECTRONIC DESIGN

A. Architecture

Master slave architecture has been used that makes the system modular in architecture. One slave controller serves the Proportional Integral Derivative (PID) control of two motors. The master serves the control algorithm for the stabilizing algorithm of the system.

The controllers are connected using the Philips® I^2C bus [25]. This is a Master Slave bus and can communicate at up to 400 kbps.

B. Microcontrollers

Two PIC 16F876 serve as slaves while 18F452 is used as a master. The master control loop is implemented in the master microcontroller, while the slave controls the four DC motors and provide the firmware for the PID controlling the DC motors. The data for this purpose was supplied by the master microcontroller which is communicated through the I^2C bus.

C. Control Law

The discreet PID control that has been used is adopted from a commercial hardware solution [24]. The proposed system uses a similar algorithm implemented in firmware of the slave microcontrollers. This was done to reduce the cost of the system. The discreet PID control law is given by [24].

$$u(n) = K_p e(n) + K_i \sum_{n=0}^{I} e(n) + K_d \left[e(n) - e(n-1) \right] \qquad (1)$$

where:

u(n) = the target velocity / position

e(n) = the error or difference in the target and reference velocity/position

Kp = proportional constant

Ki = integral constant

Kd = derivative constant

N = Total summation interval

n = Summation variable

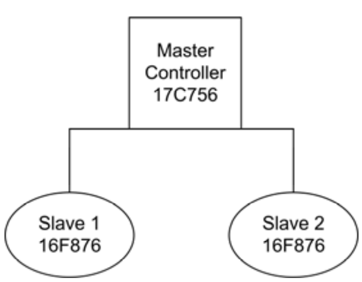

Fig 2 Interconnection of Microcontrollers

D. Orientation Measurement

To measure the orientation of the system a gyro has been used to sense the orientation of the pendulum. Gyrostar ENV05 is used which is a high precision rate gyro based on the Coriolis force generation principle [26]. This is the primary feedback device for the system that senses orientation of the platform.

E. Wheel Encoders

The second feedback input available in the system is the positional feedback of the wheels. 4 x Optical quadrature encoders were installed for full position feedback. They have 288 lines and are installed directly on the wheels to eliminate the inaccuracies caused by the gears. The 288 lines accuracy is increased four fold by using all four states of the encoders.

IV. WORKING OF THE PROPOSED SYSTEM

The objective is to balance the system in the horizontal plane and keep it moving on a surface with a constant velocity. The motors attached to the wheels are responsible for the motion of the platform. The balance is achieved achieved using the moving masses. Movement of the wheels may also be used to stabilize the platform in which case the system would be identical to the primitive inverted pendulum.

The inputs to the system are the positions of both the masses and velocity of the wheels. The outputs are the orientation of the platform and velocity of the platform. If the platform is perfectly balanced, it would acquire the horizontal position and would be moving with the desired velocity.

The stabilizing mass on the platform comprises two independent halves to improve the control of the mass otherwise it would be difficult for the actuation mechanism to move the mass from the extremes due to large forces required. This actuation mechanism has to be very fast since the system is highly unstable and tends to fall very fast if not brought back quickly enough.

The basic problem that the system poses to the controls engineer is the overshoot of the system which, if not controlled, easily blows the system and renders it completely unstable. The overshoot depends on the extent to which the mass should be allowed to move horizontally. If the platform is falling to one side, the control system determines that the mass has to be pushed to the other extreme to bring it back. The system comes from one extreme, crosses the desired position and overshoots to the other side. Sensing this, the control pulls the mass which takes a finite time and in the meanwhile the system reaches a point from where it cannot be brought back. On the other hand if this distance can be decreased the stability of the system can be increased at the cost of recovery speed. From the above scenario the following can be deduced:

1) The system is highly unstable and requires an actuation mechanism with quick response.
2) The control algorithm has to be very accurate and fast to minimize overshoot.
3) In order to minimize the overshoot the algorithm should pull the mass back to the centre once the platform begins to approach the desired angle.
4) The mass aboard the platform needs to be optimized so as to make the actuation efficient.

The control law initially selected was the classical PID however due to complexities as explained in the coming section it was changed to a custom form of the PID to ease the tuning of the coefficients. PID was selected because of its ease and good control for unknown systems.

V. COMPUTER SIMULATION

To gain an understanding of the dynamics and control of the proposed system a numerical model in the form of a simulation was constructed.

The simulation of the system was carried out based on the following simplifying assumptions.

1) The densities of the bodies are constant.
2) All the bodies are rigid.
3) Only the necessary geometries were modeled so that the inertial details are not missed out.
4) Ideal constraints were used to connect the parts.

To check the validity of a control scheme the system was placed at a certain angle as an initial condition. Then the simulation would be run so that this would act as a step and then the control algorithm would act to balance it. During the development of the simulation the following major steps were taken:-

A. Modeling

The system model was developed in Pro Engineer™ for the geometrical details. The same model was imported in Visual Nastran™ to add the mechanical constraints and then subject it to dynamic analysis. The reason for choosing these softwares was their compatibility with each other. Especially Visual Nastran™ integrates well with MATLAB Simulink™. The control system for the said system was implemented in MATLAB Simulink™. Initially a simplified system with single mass was modeled, neglecting inertial details, as shown in the Fig 3.

B. Classical inverted pendulum

To begin with, the system was assumed to be a classical pendulum. The mass was made static at the centre so that its motion would not contribute to the system dynamics. The wheels were actuated as in case of classical inverted pendulum. The system is easily balanced. Fig. 4. shows the orientation of the platform versus time as it is balanced by the actions of the wheels. Fig 5. gives block diagram of the simple control system.

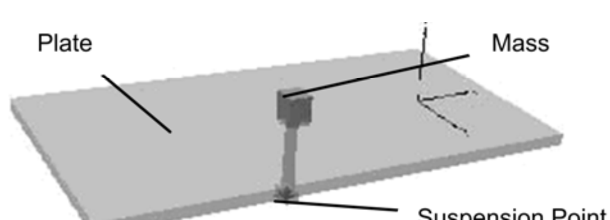

Fig 3 A simplified model of Inverted Pendulum with moving reference.

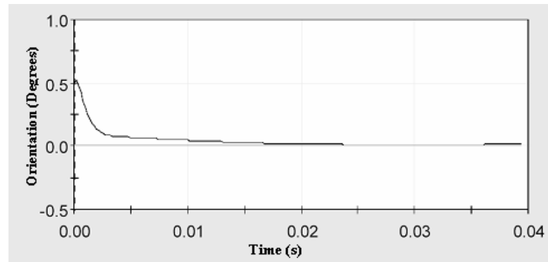

Fig 4 Platform orientation versus time.

C. Moving mass loop

Once the system is stabilized using the wheels motion the action of the moving mass is introduced. To monitor and control the extent to which each control loop has an effect, a saturation block is included for each loop. Once the effect of the wheel loop is minimized the system tries to stabilize but eventually crashes down. Tuning PID parameters for moving mass loop could not stabilize the platform.

D. Remarks

A close examination of the system revealed that the system was destabilized because of the overshoot. Once the system would approach the desired angle it would gain a high velocity and the mass would have to swing to the opposite side it is desirable that when the platform approaches the desired angle, both angular velocity and the angular acceleration should approach zero.

E. Overshoot Compensation

Another control loop is incorporated that runs independent of the moving mass control loop. The action of this loop is to pull the mass back to the origin as the platform begins to stabilize. This would split the control loop into two parts, one pushing the mass to the edge while the other pulling it to the centre. The idea is to decrease the overshoot and to cause the masses to come back to the origin when the system approaches the desired angle. This approach proved useful and led to the stabilization of the system. Fig 6. shows the graph of orientation angle versus time showing how the control system stabilizes the platform.

Once the two PID loops begin to stabilize the system, the saturation limit for the wheel loop is decreased further to

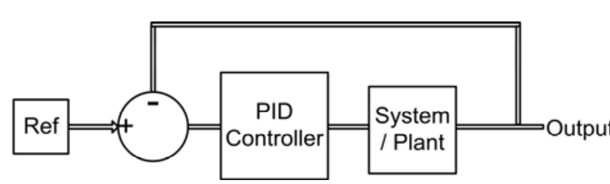

Fig 5 Block diagram of the control system

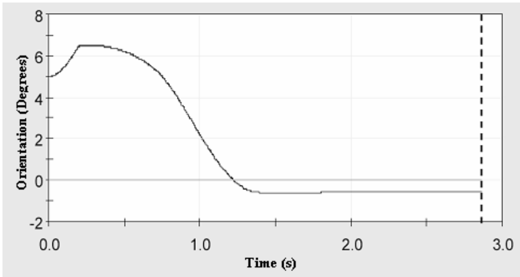

Fig 6 Graph of system orientation versus time

render it ineffective. Finally the system was stabilized only using the moving mass.

Fig 7. shows a block diagram of the control system showing the two PID loops controlling the system.

F. Complete System

After assessing the simplified model of the system the same algorithm was applied to a more realistic model as shown in Fig 1. The only difference is that one mass is moved only to one side and the other to the opposite side. Thus both the masses are used to stabilize the system. This was easily implemented in the simulation by limiting the constraints. The system response in orientation versus time is shown in Fig 8.

VI. APPLICATIONS

The proposed system may be used to test and improve control algorithms designed primarily for space and robotics applications, particularly in the field of missile control and biped robots due to its freely falling nature. Gravity makes the platform fall to sides. This makes it similar to missiles and biped robots which also fall under gravity. Some of these potential applications are discussed below.

A. Walking Robots

Biped robots are also modeled as inverted pendulum, where the center of gravity continuously shifts and the control system has to keep it inside the body to prevent it from falling. The same is the case with the proposed system which also has a displaced centre of gravity. It needs to be shifted using the moving masses to keep the body horizontal.

B. Ground vehicle mounted platforms

Vehicles traveling on ground surface undergo similar rotations while moving on uneven surfaces. Control system

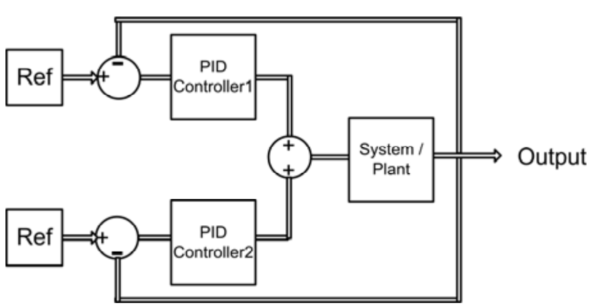

Fig 7 Dual Control Loops based Stabilization

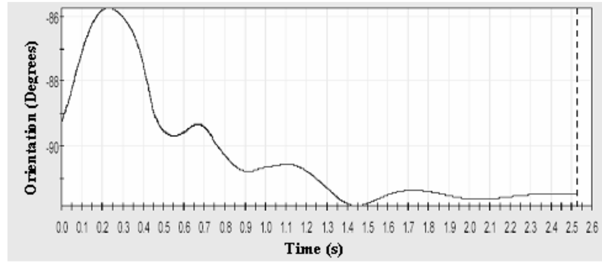

Fig 8 System response of realistic model

for stabilized platforms mounted on such vehicles may also be tested on this platform.

VII. CONCLUSION

The proposed system presents key benefits like multiple input multiple output, highly nonlinear dynamics, non-static reference and ease of fabrication. The system is controllable using a simple arrangement as that of PID, which renders it useful for testing control systems algorithms and benchmarking them. It finds its application as a more realistic apparatus for stabilization algorithm testing in the laboratory environment. The development in this paper is based more on intuition, than mathematical reasoning. The work is intended, to be extended by incorporating mathematical modeling of the underlying mechanism, which is planned to appear in a future paper.

ACKNOWLEDGMENT

The authors would like to thank Mr Amir Hamza at College of Electrical And Mechanical Engineering and Mr Asim Waqas for their continuous help and support.

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