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Centralized and decentralized policies for the containment of moving source in 2D diffusion processes using sensor/actuator network

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Abstract— **In this note we consider centralized and decentralized control policies for the detection and containment of a moving source in 2D diffusion-advection PDE, often describing environmental processes. Such a task is enabled by the employment of a network of sensing devices judiciously located within the 2D spatial domain. These devices are assumed to have actuating capabilities aimed at containing the moving source by minimizing its effects on the process concentration. The source-detecting ability of the sensor network is considerably enhanced when the sensing devices are equipped to measure spatial gradients as opposed to only process concentration. The proposed estimation scheme provides estimates of the process state and at the same time provides an estimate of the proximity of the moving source. An added feature of the supervision and monitoring scheme is a power management scheme whereby a subset of the available sensors within the network are kept active over a time interval while the remaining devices are kept dormant. The resulting hybrid infinite dimensional system switches both the actuator, deemed more suitable to contain the source over the duration of a given time interval, and its associated control signal. Additionally, it switches the set of active sensors that are used by the scheme. The control policy examines two different schemes in which both a centralized and a decentralized scheme are considered. In the centralized scheme, information on the status of the active sensors along with the estimate of the state process are transmitted to the supervisor to feed a dynamic output feedback control signal to the actuator closest to the moving source. In the decentralized scheme, a computationally efficient controller is implemented, whereby the outputs from the active sensors are independently fed to the collocated actuators via static output feedback. Simulation studies utilizing at each time 16% of the total sensors and having either a single actuator with a centralized scheme or 16% of the total actuators with a decentralized scheme and used to minimize the effects of the moving source, are presented.**

I. Introduction

In the past few years there has been a very large body of work on sensor networks (mobile or stationary) and their employment in wireless networks [18] for monitoring spatially distributed processes such as wildfire control in remote areas, border patrol [22], civil infrastructure health monitoring [4], property protection [12], environmental monitoring and weather prediction [21], [24]. Many analytical models for the management (control, resource allocation, foraging, coverage, navigation, collision avoidance) of wireless mobile sensor networks, see [13], [5], [3] and references therein, have also been developed.

More recently, there has been some works of *process-immersion* of the sensing, and possibly actuating, devices into the process at which these networks are called for to perform certain tasks, such as coverage, parameter identification, intrusion detection, etc, [23], [10]. In such applications, the inclusion of the location of the sensing and actuating devices into the process provided a natural setting for the detection of a moving source (intruder). It was argued that fixed-in-space sensors would adequately detect the signature of a source (primarily in triangulation for acoustic source detection, see [9]), but may not adequately detect a moving source. Such a source localization must be viewed within an inverse problem framework [20].

This work is concerned with a methodology that allows for the fixed-in-space scheduling policy of a sensor-and-actuator network used for the detection and containment of a moving source in a process described by a 2D diffusion-advection PDE. It is assumed that the sensor network optimally distributes the sensing devices in the spatial domain and it is desired to *activate only a subset* of such a sensor network during a given time interval while the remaining sensors stay dormant. Once the proximity of the moving source is detected, then the sensor closest to the source is designated the *cluster head* and its collocated actuator can then be used to deliver the control signal to the process in order to contain the source. An alternative containment policy examines the possibility of utilizing the actuators that are collocated to the active sensors and via a static output feedback, to contain the source by minimizing its effects on the process state.

The mathematical formulation for the 2D process is presented in § II with the sensor scheduling, and source and containment detection schema summarized in § III. An example of a 2D PDE with a moving pointwise source and a network of 400 collocated sensors and actuators which engages at most 61 such devices with the remaining 339 kept idle over a given time interval, is presented in § IV. Conclusions and future extensions follow in § V.

II. Problem Formulation

We consider a simplified version of a transport model [19], [15], [14] described by the 2D diffusion-advection PDE

$$\begin{aligned}\frac{\partial c}{\partial t} &= \frac{\partial}{\partial \chi}\left(\kappa_{\chi\chi}\frac{\partial c}{\partial \chi}\right)+\frac{\partial}{\partial \psi}\left(\kappa_{\psi\psi}\frac{\partial c}{\partial \psi}\right)-u_\chi\frac{\partial c}{\partial \chi}\\ &\quad -u_\psi\frac{\partial c}{\partial \psi}+\mu c+b_1(t,\chi,\psi)+b_2(\chi,\psi)u(t),\end{aligned} \tag{1}$$

where $c(t,\chi,\psi)$ denotes the concentration as a function of time t and spatial variables $(\chi,\psi)\in\Omega$. For simplicity of exposition, a rectangular domain is assumed with $\Omega=[0,L_\chi]\times[0,L_\psi]\subset\mathbb{R}^2$. Further, it is assumed that the velocity vector $u=(u_\chi,u_\psi)$ and the (eddy) diffusivities $\kappa_{\chi\chi}(t,\chi,\psi)$,

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$\kappa_{\psi\psi}(t,\chi,\psi)$ are constant [19].

The spatial function $b_2(\chi,\psi)$ describes the spatial distribution of the actuating devices and $u(t)$ the control signal dispensed by these devices to the process. In the decentralized scheme the term $b_2(\chi,\psi)u(t)$ will be given by

$$b_2(\chi,\psi)u(t) = \sum_{i=1}^{m} b_{2i}(\chi,\psi)u_i(t)$$

whereas in the centralized case a single actuating device will be active and therefore the control signal $u(t)$ will be a scalar.

Similarly, the effects of the moving source on the state are described through the term $b_1(t,\chi,\psi)$, which is assumed to be a point source with intensity $f(t)$. The motion of the source term is described by a time-varying 2D delta function $\delta_\chi(\chi-\chi_s)\delta_\psi(\psi-\psi_s)$ [2]. Using the above, then

$$b_1(t,\chi,\psi) = \delta_\chi(\chi-\chi_s(t))\delta_\psi(\psi-\psi_s(t))\, f(t) \qquad (2)$$

where $\theta_s(t) \triangleq (\chi_s(t),\psi_s(t))$ denotes the trajectory of the point source within the spatial domain Ω. Following the earlier work in [9], [11], state measurements are available in the form of pointwise-in-space information of the concentration $c(t,\chi,\psi)$ at the i^{th} spatial location (χ_i,ψ_i)

$$\begin{aligned} y_i(t) &= c(t,\chi_i,\psi_i) \\ &= \int_0^{L_\chi}\int_0^{L_\psi} \delta_\chi(\chi-\chi_i)\delta_\psi(\psi-\psi_i)c(t,\chi,\psi)\,d\chi\, d\psi, \end{aligned} \qquad (3)$$

for $i = 1,\ldots,m$. The system given by (1)-(3) is viewed as an evolution equation in a Hilbert space [6], [8], [16],

$$\begin{aligned} \dot{X}(t) &= \mathcal{A}X(t) + \mathcal{B}_1(t)f(t) + \mathcal{B}_2u(t), \\ y_i(t) &= \mathcal{C}_iX(t), \quad i = 1,\ldots,m, \end{aligned} \qquad (4)$$

where $X(\cdot)$ is the state of the infinite dimensional system and $\mathcal{A}$, $\mathcal{B}_1(t)$, $\mathcal{B}_2$, $\mathcal{C}_i$ are the operators associated with the state operator, the time varying source (moving) spatial distribution function $\delta_\chi(\chi-\chi_s(t))\delta_\psi(\psi-\psi_s(t))$, the actuator distribution function $b_2(\chi,\psi)$ and output measurement distribution function $c(t,\chi_i,\psi_i)$, respectively.

The m-dimensional output associated with the measurement provided by the m sensing devices which are spatially distributed within the 2D spatial domain, is given by

$$y(t) = \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} = \begin{bmatrix} \mathcal{C}_1(t)X(t) \\ \vdots \\ \mathcal{C}_m(t)X(t) \end{bmatrix} = \mathcal{C}(t)X(t). \qquad (5)$$

In the above framework, the motion or scheduling of the sensing agents within a network is naturally represented by the time dependence of the output operator $\mathcal{C}(t)$. Therefore, the advantage of the process-immersion of the sensor scheduling is that now one has to provide the time variation of the operator. Alternatively, this time variation represents the activation/deactivation (scheduling) of the fixed-in-space sensors within the sensor/actuator network.

In view of the above framework and the *process-immersion* of the sensing and actuating devices within the network, the problem under consideration can be stated along with the control and detection objectives:

Problem statement: *It is assumed that a moving source (alternatively thought of as a moving intruder whose signature affects the advection-diffusion process) whose spatial distribution is described by a spatially moving point source and having a constant intensity as given by* (2)*. A supervisory detection scheme is required which must utilize the sensing devices within the network and at the same time provide a power management scheme whereby only a subset of the sensing devices will be active over a given time interval while the remaining devices will be dormant. At the same time, the supervisory scheme must estimate the proximity of the moving source and engage a number of the actuating devices that are collocated to the sensing devices and provide the appropriate control strategy in order to contain the source by minimizing its effects on the state process.*

Remark 2.1: It should be noted that the supervisory scheme has to provide conflicting objectives: of detecting the intruder and of containing the intruder. Once the proximity of the moving source is detected, then the control policy will minimize its effects on the state process. Once this is attained, then the effects of the moving source will not be detected for a given time interval. Consequently the location of the moving source will go undetected till its effects on the process state can be "sensed" by the detection scheme.

The main objectives of the supervisory scheme can now be stated:

1) estimate the process state $c(t,\chi,\psi)$ for all t in a time interval I, $t \in I \subseteq \mathbb{R}^+$ and all spatial points $(\chi,\psi) \in \Omega$
2) estimate the location $\theta_s(t) = (\chi_s(t),\psi_s(t))$ of the unknown moving source
3) provide a computationally feasible containment policy of the moving source via the reconfiguration of both the control signal and the appropriately scheduled actuating device.

In order to simplify the detection scheme, assumptions on the moving source will be considered, while the more general estimation scheme will be the topic of future research.

Assumption 2.1: It is assumed that the intensity of the moving source $f(t) \equiv 1$ and thus the problem of source detection reduces to that of estimating its position or its proximity at each time t within Ω.

Using Assumption 2.1, the abstract representation of (1), with the moving source (2) and the measured output (3), as given by (4) and (5), is now rewritten as

$$\begin{aligned} \dot{X}(t) &= \mathcal{A}X(t) + \mathcal{B}_1(t) + \mathcal{B}_2u(t), \\ y(t) &= \mathcal{C}(t)X(t). \end{aligned} \qquad (6)$$

III. Source detection scheme

The knowledge of the proximity of the moving source will heavily depend on the state estimate where the source position can be viewed within the context of a change (fault) detection. The proposed state estimator takes the form

$$\dot{\widehat{X}}(t) = (\mathcal{A} - \mathcal{L}(t)\mathcal{C}(t))\widehat{X}(t) + \mathcal{L}(t)y(t) + \widehat{\mathcal{B}}_1(t) + \mathcal{B}_2u(t) \quad (7)$$

where $\mathcal{L}(t)$ denotes the observer gain associated with the sensing devices defined by $\mathcal{C}(t)$.

The state estimation error $e(t) = X(t) - \widehat{X}(t)$ which will serve as the residual in the detection scheme, is governed by

$$\dot{e}(t) = (\mathcal{A} - \mathcal{L}(t)\mathcal{C}(t))\, e(t) + \left(\mathcal{B}_1(t) - \widehat{\mathcal{B}}_1(t)\right). \tag{8}$$

Following the earlier work [10], [11], the distributed estimation error can provide the scheduling policy for the sensors by simply activating the sensors situated in the spatial region with the largest (localized) error $e(t,\chi,\psi)$. A similar guidance policy can be used when the sensing devices are capable of moving within the spatial domain Ω. The above assumes that the state estimation error converges to zero asymptotically. Such convergence is attainable when both the moving source and its estimate are square integrable functions in an appropriate space.

An alternative to this approach, which is based on Lyapunov stability arguments, is to use a gradient-type optimization by finding the sensitivity of the state error with respect to the sensor positions. However, in this work, the motivation stems from computationally tractable schemes that propose to change (engage) a set of active sensors at the beginning of a given time interval, and keep them in active mode (transmit and receive) throughout the duration of the time interval while the remaining sensors are kept dormant (idle mode). At the beginning of a new time interval, the process of activating/deactivating is updated so that possibly new sensors that are deemed more relevant are used and sensors that do not provide useful information are turned off.

A significant factor in the ability of the sensor network to effectively detect moving sources, is the type of measurements that the sensing devices provide. Associated with that is the important factor of the location of the sensors within the network, i.e. the position inside the spatial domain Ω.

A. Sensor type

The sensing devices in the network can significantly increase the identifiability by providing information not on the concentration but on the gradients of the concentration. Usually, the sensing devices can measure concentration at a spatial point (χ_i,ψ_i) within the domain Ω, and therefore such devices must be modified so that they can provide information on the spatial gradients of the concentration. To achieve this, a given sensing device can be equipped with a *quadruple probe* that can provide pointwise concentration at four points on the circumference of a ring [17], as shown in Figure 1. In view of (5), the signal from the i^{th} device equipped with four concentration sensors that can be transmitted to the supervisor is given by

$$\begin{bmatrix} c(t,\chi_i+\varepsilon/2,\psi_i) \\ c(t,\chi_i-\varepsilon/2,\psi_i) \\ c(t,\chi_i,\psi_i+\varepsilon/2) \\ c(t,\chi_i,\psi_i-\varepsilon/2) \end{bmatrix}.$$

In essence, the sensor provides concentration at four points on the circumference of the ring. To obtain gradient information, one can use an approximation to arrive at the following triple-signal observation at the sensor location (χ_i,ψ_i)

$$y_i(t) = \begin{bmatrix} c(t,\chi_i,\psi_i) \\ \frac{\partial c}{\partial x}(t,\chi_i,\psi_i) \\ \frac{\partial c}{\partial y}(t,\chi_i,\psi_i) \end{bmatrix} \approx$$

$$\begin{bmatrix} \frac{c(t,\chi_i+\varepsilon/2,\psi_i)+c(t,\chi_i-\varepsilon/2,\psi_i)+c(t,\chi_i,\psi_i+\varepsilon/2)+c(t,\chi_i,\psi_i-\varepsilon/2)}{4} \\ \frac{c(t,\chi_i+\varepsilon/2,\psi_i)-c(t,\chi_i-\varepsilon/2,\psi_i)}{\varepsilon} \\ \frac{c(t,\chi_i,\psi_i+\varepsilon/2)-c(t,\chi_i,\psi_i-\varepsilon/2)}{\varepsilon} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{\varepsilon} & -\frac{1}{\varepsilon} & 0 & 0 \\ 0 & 0 & \frac{1}{\varepsilon} & -\frac{1}{\varepsilon} \end{bmatrix} \begin{bmatrix} c(t,\chi_i+\varepsilon/2,\psi_i) \\ c(t,\chi_i-\varepsilon/2,\psi_i) \\ c(t,\chi_i,\psi_i+\varepsilon/2) \\ c(t,\chi_i,\psi_i-\varepsilon/2) \end{bmatrix}.$$

It is assumed that each sensor in the network can pre-process the four measurements and therefore only three signals can be transmitted to the base station. The radius ε of the circular

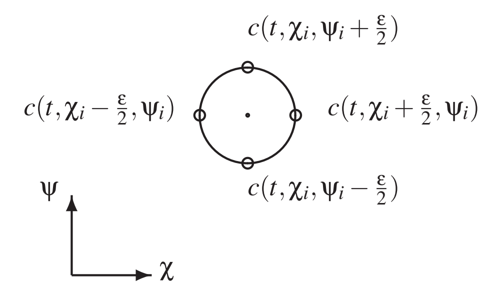

Fig. 1. Cross configuration of a quadruple concentration-measuring probe on a circular ring, placed at location $(\chi_i,\psi_i) \in \Omega$.

ring is dictated by spatial resolution [19] considerations that would justify the approximations

$$\begin{aligned} c(t,\chi_i,\psi_i) \quad &\approx \quad \frac{1}{4}c(t,\chi_i+\varepsilon/2,\psi_i)+\frac{1}{4}c(t,\chi_i-\varepsilon/2,\psi_i) \\ &\quad +\frac{1}{4}c(t,\chi_i,\psi_i+\varepsilon/2)+\frac{1}{4}c(t,\chi_i,\psi_i-\varepsilon/2), \end{aligned}$$

$$\frac{\partial c}{\partial x}(t,\chi_i,\psi_i) \approx \frac{c(t,\chi_i+\varepsilon/2,\psi_i)-c(t,\chi_i-\varepsilon/2,\psi_i)}{\varepsilon},$$

and

$$\frac{\partial c}{\partial y}(t,\chi_i,\psi_i) \approx \frac{c(t,\chi_i,\psi_i+\varepsilon/2)-c(t,\chi_i,\psi_i-\varepsilon/2)}{\varepsilon}.$$

B. Sensor positioning

As was already mentioned in [10], [11], the location of the sensing devices within Ω has a significant effect on the identifiability of the scheme and the quality of the signal, and can significantly affect the detection speed. This is especially so when the detection scheme heavily relies on the fast convergence of the state error to zero.

While it is assumed that a subset of the sensing devices will be active over a given time interval, all devices must be positioned in the spatial domain in an optimal manner. Therefore, it is assumed that the network consists of N sensing devices, of which m will be active at a given time interval $[t_k, t_k+\Delta t]$ (in sensing/transmitting/receiving mode) and with the remaining $(N-m)$ to be disengaged (in sleep mode). Such optimality can be expressed in terms of observability [6].

We denote the set of locations within Ω that are the zeros of the eigenfunctions of the 2D elliptic operator by

$$\Theta_{null} = \left\{(\chi,\psi)\in\Omega : \phi_{kj}(\chi,\psi)=0,\ j,k=1,2\ldots\right\},$$

where $\phi_{jk}(\chi,\psi)$ are the eigenfunctions of the 2D elliptic operator. Hence the sensor locations should be restricted to the set $\Omega_{ob}=\Omega\setminus\Theta_{null}$ which will ensure that no sensing device renders the system unobservable. Choosing N locations from this infinite set can be computationally expensive. However, one may employ computationally efficient schemes to obtain N locations for the sensing devices that would enhance the observability of the network. One such approach is based on spatial norms [1]. It is henceforth assumed that N such optimal sensing devices have been positioned in the spatial domain Ω either using the approach outlined above, or following the alternative methods presented in the earlier work [10], [11]. The set of these locations in denoted by Θ_N.

The next step is then to address the primary objective of state estimation with sensor scheduling that would also provide an estimate of the location of the moving source.

C. Numerical implementation

Using a finite element scheme [7] with the approximation

$$c(t,\chi,\psi)=\sum_{j=1}^{I_1}\sum_{k=1}^{I_2}\phi_{jk}(\chi,\psi)X_{jk}(t)$$

where $\phi_{jk}(\chi,\psi)$ is the 2D approximating function and $X_{jk}(t)$ is the generalized coordinate with jk being the approximation indices in the (χ,ψ) space, the PDE in (1) is semi-discretized in space. By substituting this expansion in (1) and viewing it in weak form with $\phi_{jk}(\chi,\psi)$ as the test functions, one arrives at the matrix representation of (1), given by

$$\dot{x}(t)=Ax(t)+B_1(t)+B_2u(t), \tag{9}$$

where $x(t)$ denotes the state vector whose entries are the generalized coordinates $X_{jk}(t)$, and $A, B_1(t), B_2$ are the matrix representations of the elliptic operator $\mathcal{A}$, source distribution operator $\mathcal{B}_1(t)$, and input distribution operator $\mathcal{B}_2$, respectively. The expression for the i^{th} sensor is given by

$$y_i(t)=\begin{bmatrix} C_{ic}x(t)\\ C_{i\chi}x(t)\\ C_{i\psi}x(t)\end{bmatrix}=\begin{bmatrix}\sum_{j=1}^{I_1}\sum_{k=1}^{I_2}\phi_{jk}(\chi_i,\psi_i)X_{jk}(t)\\ \sum_{j=1}^{I_1}\sum_{k=1}^{I_2}\frac{\partial\phi_{jk}}{\partial\chi}(\chi_i,\psi_i)X_{jk}(t)\\ \sum_{j=1}^{I_1}\sum_{k=1}^{I_2}\frac{\partial\phi_{jk}}{\partial\psi}(\chi_i,\psi_i)X_{jk}(t)\end{bmatrix},$$

where $y_i(t)$ denotes the measurements from the sensor placed at location (χ_i,ψ_i) and which provides information on the concentration $C_{ic}x(t)$, the χ-direction gradient $C_{i\chi}x(t)$ and the ψ-direction gradient $C_{i\psi}y(t)$.

Since a subset of the N available sensing devices will be scheduled at a given time interval, the sensor outputs and the output distribution vectors are parameterized by the locations $\theta_i=(\chi_i,\psi_i)\in\Theta_N$ and therefore

$$y(t;\theta_i)=C(\theta_i)x(t)=\begin{bmatrix} C_c^T(\theta_i) & C_\chi^T(\theta_i) & C_\psi^T(\theta_i)\end{bmatrix}^T x(t)$$

with $C(\theta):\mathbb{R}^n\to\mathbb{R}^3$, $\forall\theta\in\Theta_N$. Due to this parametrization, now any sensor scheduling will be constrained in the set Θ_N.

Using the above approximation, the 2D process with a moving source and a time varying output measurement matrix is given by

$$\begin{aligned}\dot{x}(t)&=Ax(t)+B_1(t)+B_2u(t)\\ y(t)&=C(t)x(t).\end{aligned} \tag{10}$$

Similarly, the finite dimensional representation of the proposed state estimator in (7), is given by

$$\dot{\widehat{x}}(t)=(A-L(t)C(t))\widehat{x}(t)+L(t)y(t)+\widehat{B}_1(t)+B_2u(t), \tag{11}$$

where $\widehat{B}_1(t)$ is the on-line estimate of the moving source vector $B_1(t)$. The above two finite dimensional systems, when combined, yield the state estimation error system

$$\dot{e}(t)=(A-L(t)C(t))\,e(t)+(B_1(t)-\widehat{B}_1(t)). \tag{12}$$

Using the fact that both the filter gain and observation matrix are constant over a given interval $[t_\ell,t_\ell+\Delta t]$ is taken into account, then the state error is conveniently written as

$$\dot{e}(t)=(A-L_\ell C_\ell)\,e(t)+(B_1(t)-\widehat{B}_1^\ell). \tag{13}$$

The above result in a hybrid system and one must ensure that well-posedness and stability under switching can be established. Following [10], one uses the fact that the switching interval Δt is above the *dwell time* and that during any time interval $[t_\ell,t_\ell+\Delta t]$, the term $(B_1(t)-\widehat{B}_1^\ell)$ is square integrable.

In addition to the scheduling of the m sensing devices that will be active over the interval $[t_\ell,t_\ell+\Delta t]$ and the representation of the estimate $\widehat{B}_1^\ell$, one must also provide a containment policy for the effects of the moving source.

Spatial gradient information can be used to detect the moving source location because gradients are sensitive to changes in concentration. In this case, one searches among all m sensors in the set Θ_N and finds the one that has the highest absolute value of either gradient. Such a sensor is termed the *cluster head*. This provides the sensor location closest to the source. Using this information, the source location estimate $\widehat{B}_1^\ell$ over the interval $[t_\ell,t_\ell+\Delta t]$ can then be made. Consequently, the estimated source location θ_{loc}^ℓ-in fact the cluster head closest to the source location-is given by

$$\theta_{loc}^\ell=\arg\max_{\theta_i\in\Theta_N}\left\{\max\left(|C_\chi(\theta_i)x(t)|,|C_\psi(\theta_i)x(t)|\right)\right\}. \tag{14}$$

The above detection scheme compares the absolute value of each of the spatial gradients $C_\chi(\theta_i)x(t)$ and $C_\psi(\theta_i)x(t)$ and chooses the largest one. It then compares all such gradients from the sensor network to find the maximum gradient.

The way one decides which m sensors should be activated over the next interval depends on the average spacing of the sensors within Ω. At a given time t_ℓ, the sensor location θ_{loc}^ℓ will be within a distance ρ from the source position $(\chi_s(t),\psi_s(t))$ where ρ denotes the maximum distance between any two adjacent sensors from the set Θ_N. To incorporate the activation of m devices out of the N available ones, one considers a ball of radius R centered at the cluster head sensor. Then one expresses the radius R in terms of the number m and the sensor spacing ρ. Therefore, for a given spacing ρ and given radius $R\gg\rho$, one can find the number of sensors m that are contained within the ball of radius R.

Once the cluster head sensor $\theta^{\ell}_{loc} = (\chi^{\ell}_{loc}, \psi^{\ell}_{loc})$ has been declared for the interval $[t_\ell, t_\ell + \Delta t]$, then the distribution vector of the estimated source position is given by $[M\widehat{B}_1(t_\ell)] = \phi_{pq}(\chi^{\ell}_{loc}, \psi^{\ell}_{loc})$, where M denotes the mass matrix of the finite dimensional approximation.

Two algorithms are considered for declaring the cluster head and containing the moving source.

Algorithm 1: Moving source estimation with sensor management and centralized source containment; single actuator

1) consider the time interval $[t_0, t_0 + \Delta t)$ and employ all N sensors in the network for the first time interval
2) find sensor location with maximum gradient
$$\theta^0_{grad} = \arg\max_{\theta_i \in \Theta_N} \{\max(|C_\chi(\theta_i)x(t)|, |C_\psi(\theta_i)x(t)|)\}$$
3) find sensor location with maximum concentration
$$\theta^0_{conc} = \arg\max_{\theta_i \in \Theta_N} |C_c(\theta_i)x(t)|$$
4) *if* $|C_c(\theta^0_{conc})x(t)| > |C_c(\theta^0_{grad})x(t)|$
   then set $\theta^0_{loc} = \theta^0_{grad}$
   else set $\theta^0_{loc} = \theta^0_{conc}$
   end loop
5) sensor nearest the source at t_0 is at $\theta^0_{loc} = (\chi^0_{loc}, \psi^0_{loc})$.
6) activate actuator collocated with $\theta^0_{loc} = (\chi^0_{loc}, \psi^0_{loc})$
$$\langle \widehat{B}_2(\theta^0_{loc})u(t), \phi\rangle = \int_0^{L_\chi}\int_0^{L_\psi} \delta_\chi(\chi - \chi^0_{loc})\delta_\psi(\psi - \psi^0_{loc})\phi(\chi,\psi)\, d\chi\, d\psi\, u(t)$$
7) provide control signal to actuator $\widehat{B}_2(\theta^0_{loc})$
$$u^0(t) = -K(\theta^0_{loc})\widehat{x}(t), \quad t \in [t_0, t_0 + \Delta t]$$
8) find set of m sensors within a distance R from θ^0_{loc}
$$\Theta_R(t_0) = \{\theta \in \Theta_N : |\theta - \theta^0_{loc}|_{\mathbb{R}^2} \le R\} \qquad (15)$$
9) consider next interval by setting $t_0 = t_1 \triangleq t_0 + \Delta t$, and perform search in steps 2 and 3 over $\Theta_R(t_0)$ instead of Θ_N. Repeat steps 4,5,6,7,8 for new interval and define the new set $\Theta_R(t_1)$ in (15) as $\Theta_R(t_1) = \{\theta \in \Theta_N : |\theta - \theta^1_{loc}|_{\mathbb{R}^2} \le R\}$. Repeat step (9) for subsequent intervals.

The above policy uses the actuator that is collocated to the cluster head as declared by the supervisor for the duration of the interval $[t_\ell, t_\ell + \Delta t)$. To enhance the containment effects, one may consider activating the actuators that are collocated to the m active sensors. However, this might significantly increase the computational load and therefore one may consider a simplified control policy. This takes the form of static output feedback of all the m collocated sensors/actuators. Such a control policy is decentralized since each actuator only uses information from its collocated sensor.

Algorithm 2: Moving source estimation with sensor management and decentralized source containment; m actuators

1) consider the interval $[t_0, t_0 + \Delta t)$ and employ all N sensors in the network for the first time interval
2) find sensor location with maximum gradient
$$\theta^0_{grad} = \arg\max_{\theta_i \in \Theta_N} \{\max(|C_\chi(\theta_i)x(t)|, |C_\psi(\theta_i)x(t)|)\}$$
3) find sensor location with maximum concentration
$$\theta^0_{conc} = \arg\max_{\theta_i \in \Theta_N} |C_c(\theta_i)x(t)|$$
4) *if* $|C_c(\theta^0_{conc})x(t)| > |C_c(\theta^0_{grad})x(t)|$
   then set $\theta^0_{loc} = \theta^0_{grad}$
   else set $\theta^0_{loc} = \theta^0_{conc}$
   end loop
5) sensor nearest the source at t_0 is at $\theta^0_{loc} = (\chi^0_{loc}, \psi^0_{loc})$.
6) find set of m sensors within a distance R from θ^0_{loc}
$$\Theta_R(t_0) = \{\theta \in \Theta_N : |\theta - \theta^0_{loc}|_{\mathbb{R}^2} \le R\} \qquad (16)$$
7) activate the actuators that are collocated to the $m-1$ sensors encircling the sensor at θ^0_{loc}, i.e. $\theta^0_{loc_i} \in \Theta_R(t_0)$
$$\langle \widehat{B}_2(\theta^0_{loc})u(t), \phi\rangle = \begin{bmatrix} \int_0^{L_\chi}\int_0^{L_\psi} \delta_\chi(\chi - \chi^0_{loc_1})\delta_\psi(\psi - \psi^0_{loc_1})\phi(\chi,\psi)\, d\chi\, d\psi \\ \vdots \\ \int_0^{L_\chi}\int_0^{L_\psi} \delta_\chi(\chi - \chi^0_{loc_m})\delta_\psi(\psi - \psi^0_{loc_m})\phi(\chi,\psi)\, d\chi\, d\psi \end{bmatrix}$$
8) provide decentralized control to m actuators $\widehat{B}_2(\theta^0_{loc})$
$$u^0(t) = -\mathrm{diag}[k_1, \ldots, k_m]y(t; \theta^0_{loc}), \quad t \in [t_0, t_0 + \Delta t]$$
9) consider next interval by setting $t_0 = t_1 \triangleq t_0 + \Delta t$, and perform search in steps 2 and 3 *over* $\Theta_R(t_0)$ *instead of* Θ_N. Repeat steps 4,5,6,7,8 for new interval and define the new set $\Theta_R(t_1)$ in (16) as $\Theta_R(t_1) = \{\theta \in \Theta_N : |\theta - \theta^1_{loc}|_{\mathbb{R}^2} \le R\}$. Repeat step (9) for subsequent intervals.

TABLE I
$L_2(0,20; L_2(0,\ell)$ AND $L_2(0,20;\mathbb{R}^2)$ NORMS

case	plant	observer	error	traj. error
no control	17.90	16.60	2.31	0.368
1 actuator, dynamic control	3.12	1.51	2.58	0.415
m actuators, static control	16.04	15.52	1.64	0.308

IV. Numerical results

We consider the PDE in (1) with $L_\chi = 0.1$, $L_\psi = 0.06$ and $\kappa_{\chi\chi} = \kappa_{\psi\psi} = 1.5 \times 10^{-5}$, $u_\chi = u_\psi = 10^{-4}$ and $\mu = -3 \times 10^{-4}$. The system was approximated using 20 linear elements in each direction. A uniform grid of 20×20 sensing devices was considered. We consider $c(0,\chi,\psi) = 500^2(\chi\psi)^3(L_\chi - \chi)^3(L_\psi - \psi)^3/(L_\chi L_\psi)^6$ and $\widehat{c}(0,\chi,\psi) = 0$. Using the proposed moving source detection algorithm, the system was simulated with a moving source described by
$$\chi_s(t) = \frac{L_\chi(10 - 9\sin(\frac{5\pi t}{t_f}))}{20}, \psi_s(t) = \frac{L_\psi(10 - 9\cos(\frac{3\pi t}{t_f}))}{20}.$$
The system was simulated in $[0,20]s$ with Δt in Algorithms 1 and 2 taken as $\Delta t = 0.1s$, i.e. the position of a new sensor closer to the moving source was declared every $\Delta t = 0.1s$.

The radius of a circle containing the sensors around θ^k_{loc} was $R = 0.15\sqrt{L^2_\chi + L^2_\psi}$ giving a total of $m = 61$ sensors out of the $N = 400$ available ones in usage, or 15.25%. A snapshot of the source position $\theta_s(t)$ and the active sensors is depicted in Figure 2. The position of θ_{loc} ($\circ$) along with $\theta_s(t)$ ($\square$) are depicted for different time instances. The largest distance between $\theta_s(t)$ and θ^{ℓ}_{loc} was $\max_{t\in[0,20]} |\theta_s(t) - \theta^{\ell}_{loc}|_{\mathbb{R}^2} = 0.0139$. Note that at each time $\theta_s(t)$ ($\square$) is surrounded by the active sensors ($\diamond$) and hence

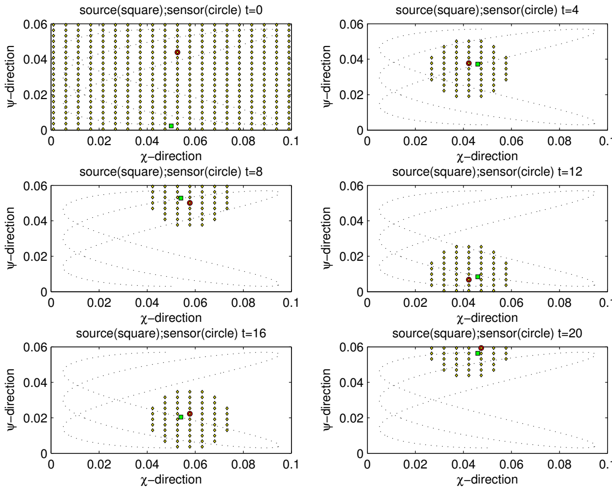

Fig. 2. Source position (green square) and sensor θ_{loc} (○) using Algorithm 1 without containment for $t = 0, 0.4, 0.8, 1.2, 1.6, 2.0$. The active sensors are depicted by (◇).

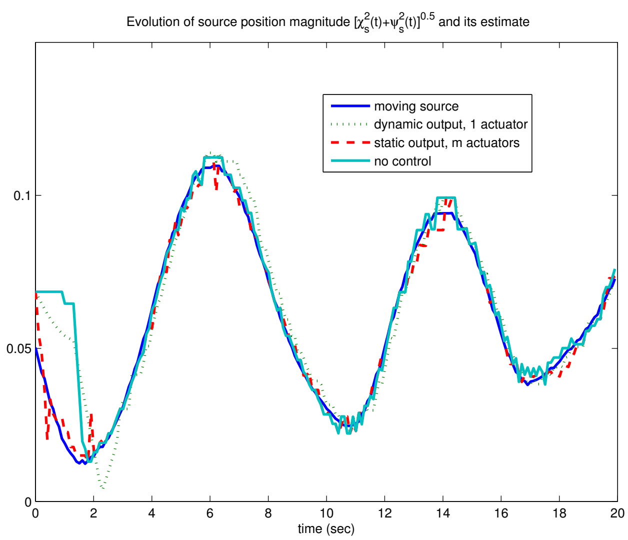

Fig. 3. Evolution of source trajectory $\theta_s(t)$ and its estimates based on cluster head using no containment and Algorithms 1 and 2.

one can detect the region where the moving source resides.

The trace of $\theta_s(t)$ and its time estimate (cluster head sensor) is depicted in Figure 3 for three different containment cases, where it is seen that the no-containment case provides the best estimate of the source position. Additionally, the $L_2(0,20;L_2(\Omega))$ norms for the plant, observer and state error and the $L_2(0,20,\mathbb{R}^2)$ of the trajectory error are presented in Table I.

V. Conclusion

The proposed moving source detection scheme utilizing a sensor/actuator network, examined the possibility of containing the moving source by minimizing its effects on the state. Two different containment strategies were proposed, a centralized one whereby the actuator collocated to the cluster head was used along with a dynamic feedback controller, and a decentralized scheme in which the actuators collocated to the active sensors utilized a static output feedback to contain the source. An extension to the above is the incorporation of a mobile sensor/actuator network that includes collision avoidance and peer-to-peer communication limitations.

References

[1] A. Armaou and M. A. Demetriou, *Optimal actuator/sensor placement for linear parabolic PDE's using $\mathcal{H}_2$ norm*, Chemical Engineering Science, 61, pp. 7351–7367,, year =.

[2] A. G. Butkovskiy and L. M. Pustyl'Nikov, *Mobile Control of Distributed Parameter Systems*, Ellis Horwood Ltd, Chichester, 1987.

[3] C. G. Cassandras and W. Li, *Sensor networks and cooperative control*, European Journal of Control, 11 (2005).

[4] K. Chintalapudi, E. Johnson, and R. Govindan, *Structural damage detection using wireless sensor-actuator networks*, in Proc. of the 13th Med. Conf. on Control and Automation, Cyprus, 2005.

[5] J. Cortès, S. Martínez, T. Karatas, and F. Bullo, *Coverage control for mobile sensor networks*, IEEE Transactions on Robotics and Automation, 20 (2004).

[6] R. F. Curtain and H. J. Zwart, *An Introduction to Infinite Dimensional Linear Systems Theory*, Springer-Verlag, Berlin, 1995.

[7] R. Dautray and J.-L. Lions, *Mathematical Analysis and Numerical Methods for Science and Technology*, vol. 4: Integral Equations and Numerical Methods, Springer, Berlin, 1999.

[8] ———, *Mathematical Analysis and Numerical Methods for Science and Technology*, vol. 6: Evolution Problems II, Springer, Berlin, 2000.

[9] M. A. Demetriou, *Power management of sensor networks for detection of a moving source in 2-D spatial domains*, in Proceedings of the 2006 ACC, Minneapolis, Minnesota, USA, June 14-16 2006.

[10] M. A. Demetriou, *Detection and containment policy of moving source in 2-d diffusion processes using sensor/actuator network*, in roc. of the 2007 European Control Conference (ECC'07), Times Square, New York, NY, July 2-5 2007.

[11] ———, *Process estimation and moving source detection in 2-d diffusion processes by scheduling of sensor networks*, in Proc. of the 2007 American Control Conference, Times Square, New York, NY, July 11-13 2007.

[12] A. Ganguli, J. Cortès, and F. Bullo, *Distributed deployment of asynchronous guards in art galleries*, in Proc. of the 2006 ACC.

[13] V. Giordano, P. Ballal, F. Lewis, B. Turchiano, and J. Zhang, *Supervisory control of mobile sensor networks: math formulation, simulation and implementation*, IEEE Transaction on Systems, Man and Cybernetics-Part B Cybernetics, 36 (2006).

[14] M. Z. Jacobson, *Fundamentals of Atmospheric Modeling*, Cambridge University Press, Cambridge, UK, 2005.

[15] E. Kalnay, *Atmospheric Modeling, Data Assimilation and Predictability*, Cambridge University Press, New York, 2003.

[16] S. Omatu and J. H. Seinfeld, *Distributed Parameter Systems: Theory and Applications*, Oxford University press, New York, 1989.

[17] R. A. Russel, *Odour Detection by Mobile Robots*, World Scientific, Singapore-New Jersey-London-Hong Kong, 1999.

[18] P. Santi, *Topology Control in Wireless Ad Hoc and Sensor Networks*, John Wiley and Sons, Ltd, Chichester, West Sussex, England, 2006.

[19] J. H. Seinfeld and S. N. Pandis, *Atmospheric Chemistry and Physics: From Air Pollution to Climate Change*, Wiley, N.Y., 1997.

[20] I. F. Sivergina, M. P. Polis, and I. Kolmanovsky, *Source identification for parabolic equations*, Mathematics of Control, Signals and Systems, 16 (2003), pp. 141–157.

[21] Z. Song, Y.-Q. Chen, J. Liang, and D. Ucinski, *Optimal mobile sensor motion planning under nonholonomic contraints for parameter estimation of distributed systems*, in Proceedings of the International Conference on Intelligent Robotics and Systems, 2005.

[22] S. Susca, S. Martínez, and F. Bullo, *Monitoring environmental boundaries with a robotic sensor network*, in Proc. of the 2006 ACC.

[23] C. Tricaud, M. Patan, D. Ucinski, and Y. Chen, *D-optimal trajectory design of heterogeneous mobile sensors for parameter estimation of distributed systems*, in Proc. of the 2008 American Control Conference, Seattle, WA, June 11-13 2008.

[24] D. Ucinski and M. A. Demetriou, *An approach to the optimal scanning measurement problem using optimum experimental design*, in Proceedings of the 2004 ACC, 2004, pp. 1616–1621.