2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

The POG Technique for Modelling Engine Dynamics Based on Electrical Analogy

Federica Grossi, Angelo Palladino, Roberto Zanasi, Giovanni Fiengo

Abstract— **In this paper the Power-Oriented Graphs (POG) technique is used for modelling an internal combustion engine. The POG technique is focused on a new modular, physically based and lumped parameter approach, leading to a complete and coherent engine model structure. The aim of the authors, starting from an analogy with electrical systems, is to simplify the approach eliminating the space dynamics (multi-zone combustion and wave effects), while preserving the time dynamics. In this way, it is possible to obtain an engine description similar to an electrical circuit, with all the useful consequences in term of existence and numerical availability of the solution. The advantages are in the specific correspondence that is found between the engine components and variables (as throttle valve, cylinder, inertial flows) with electrical counterparts (current, voltage, resistance). The main benefit achievable with this methodology is the simplicity to compose the whole engine model and customize it including the differential equations of the engine in state space form, using the POG technique. The POG technique is a graphical modelling technique which uses only two basic blocks (the "elaboration" and "connection" blocks) for modelling physical systems. The state space mathematical model of a system can be "directly" obtained from the corresponding POG representation. The POG model of the considered combustion engine shows its internal structure from a "power" point of view.**

GLOSSARY

p	gas pressure $[N/m^2]$
T	gas temperature $[K]$
T_{sl}	temperature of lateral surface of cylinder $[K]$
T_{sb}	temperature of basic surface of cylinder $[K]$
$\dot{m}$	air mass flow rate $[kg/s]$
$\dot{Q}_{ext}$	heat flow rate $[kJ/s]$
V	volume $[m^3]$
θ	spark advance $[deg]$
c_x	specific heat capacities ($x = p$ or v)$[J/(kgK)]$
γ	ratio of specific heats , c_p/c_v $[-]$
h_i	enthalpy $[kJ/kg]$
m_{fuel}	fuel mass $[kg]$
η_{cb}	combustion efficiency $[-]$
λ	air/fuel ratio $[-]$
S	heat release $[W/kg]$
$\dot{m}_c$	choked air mass flow rate $[kg/s]$
$\dot{m}_{nc}$	not choked air mass flow rate $[kg/s]$
C_D	discharge coefficient $[-]$
A_T	effective flow area $[m^2]$

Federica Grossi and Roberto Zanasi are with University of Modena and Reggio Emilia, Via Vignolese 905, 41100 Modena, Italy. E-mail: {federica.grossi, roberto.zanasi}@unimore.it

Angelo Palladino and Giovanni Fiengo are with Dipartimento di Ingegneria, Università degli Studi del Sannio, Piazza Roma 21, 82100 Benevento, Italy. E-mail: {angelo.palladino,gifiengo}@unisannio.it

I. INTRODUCTION

Today's automotive systems face a competitive market that demands excellent performance and limited time of expensive tests, therefore continual improvements are needed on both mechanical components and on system management software, i.e. control strategies.

Regarding the development of new and more efficient control strategies, automotive companies commonly adopt models that can be simulated on a computer to develop new control algorithms in order to reduce the effort and the cost of the testing phase. This phase consists of running the control strategy to verify its performance and limits. To this aim, the models must be reliable and easily achievable.

A modeling technique supported by physical properties together with a schematic representation based on some simple rules, would ease the writing of the models, simplify the formal check of models and allow a common modeling language to share the models. This problem is in common between automotive control systems and many other research fields and many possible solutions have been already proposed: the first one is the Bond Graphs modeling technique, see [1], [2] and the references therein. This modeling technique uses power interaction between subsystems as the basic concept for modeling. It has also a formal language to represent the basic components that may appear in a broad range of physical systems. However this technique has a few drawbacks: the schematic representation needs more than ten symbols and it is not easily readable; the "power" variables are classified in "effort" and "flow" variables (note that this definition does not coincide with the definition of across-variable and through-variable) and finally the implementation of the Bond Graphs on a general purpose computer simulator may require a non trivial 'translation' (causality problem). The modeling technique we propose here is the Power-Oriented Graphs (POG) technique. As for Bond Graphs, the basic idea of the POG modeling technique is to use the power interaction between subsystems as basic concept for modeling [3], [4]. The POG schemes are particularly suitable to electro-hydraulic mechanical systems where the power flows through different energetic domains. Differently from the Bond Graphs technique [5], the POG technique uses only two basic blocks (see Fig. 1), does not need to classify the power variables and always uses the integral causality. By this way, the POG schemes are easily readable, close to the computer implementation and allow reliable simulations using every computer simulator. Many examples involving the electrical, hydraulic and mechanical domains about vehicle

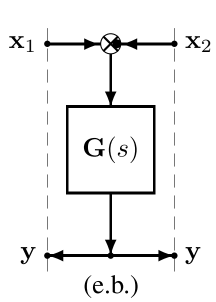

Fig. 1. The POG basic blocks: the elaboration block (e.b.) on the left and the connection block (c.b.) on the right.

systems and components can be found: common rail system in [6], clutches and gearboxes in [7].

In this work, the POG technique is used for modelling an internal combustion engine [8] using the electric analogy, in order to obtain a mathematical approach useful for control purposes.

In literature, particular interest is dedicated to engine modeling, both for analysis and control purpose. As an example, in [9] a cylinder-by-cylinder model of experimental variable valve timing 4-cylinders engine has been developed. In [10] and [11] mathematical engine models are developed to study the dynamics of the HCCI engines. In [11], [12] and [13] engine models are presented to control designs. The main features of these models are the simplicity (typically linear models are adopted) and the accuracy necessary to reach the control goals.

The focus of this paper is to formalize the analogy of the internal combustion engine with electrical systems [14], realize the corresponding POG scheme and obtain from it the differential equations of the engine dynamics in the state space form. Starting from an analogy with electrical systems it can be obtained an engine description similar to an electrical (although not linear) circuit, with all the useful consequences in term of existence and numerical availability of the solution. The advantages are in the specific correspondence that is found between the engine components and variables (as throttle valve, cylinder, inertial flows), with electrical counterparts (current, voltage, resistance).

The main benefit achievable with this methodology is the simplicity to compose the whole engine model and customize it including all the latest devices. Moreover, in order to obtain a new engine mathematical model for control application, the POG technique has been used to model the internal combustion engine analogue to the electric circuit.

The paper is organized as follows. Section II states the basic properties of the POG modeling technique and Section III shows the engine model features. An internal combustion engine model through POG is described in Section IV. Finally, experimental results are shown in Section V and Conclusions end the paper.

II. THE BASES OF POWER-ORIENTED GRAPHS

The "Power-Oriented Graphs" (POG) technique is a graphical modelling technique that uses the "power interaction" between subsystems as basic element for modeling physical systems. The POG technique has a "modular" structure which essentially uses only the two blocks shown in Fig. 1 named "elaboration block" (e.b.) and "connection block" (c.b.). The basic characteristic of this modular structure is the direct correspondence between pairs of system variables and real power flows: the product of the two variables involved in each dashed line of the graph has the physical meaning of "power flowing through the section". The circle present in the e.b. is a summation element and the black spot represents a minus sign that multiplies the entering variable. There is no restriction on variables $\mathbf{x}$ and $\mathbf{y}$ other than the fact that their inner product $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^{\mathrm{T}}\mathbf{y}$ must have the physical meaning of a "power". The e.b. and the c.b. are suitable for representing both scalar and vectorial systems. In the vectorial case, $\mathbf{G}(s)$ and $\mathbf{K}$ are matrices: $\mathbf{G}(s)$ is always square (because its input and output vectors must have the same dimension), $\mathbf{K}$ can also be rectangular. There is a direct correspondence between the POG representations and the corresponding state space descriptions. For example, the system

$$\begin{cases} \mathbf{L}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{B}^{\mathrm{T}}\mathbf{x} \end{cases} \qquad \mathbf{L} = \mathbf{L}^{\mathrm{T}} > 0 \qquad (1)$$

can be represented by the POG scheme shown in Fig. 2. Note that every physical system respecting causality constraints can be written in the form (1). More details on

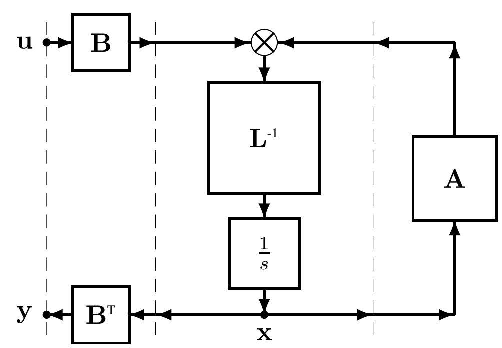

Fig. 2. POG block scheme of a generic dynamic system.

Power-Oriented Graphs are reported in [3], [15] and [4].

III. ENGINE MODEL DESCRIPTION

A physical system can be decomposed in basic elements. Each element is characterized by three variables named [16]:

- q quantity, i.e. energy variable of the element;
- i flow, equal to dq/dt, i.e. the power variable that flows through the element (through-variable);
- v forcing, i.e. the power variable acting at the extremes of the element (across-variable).

As an example, Table I reports these variables for the electric, hydraulic and mechanical elements. The main feature is that each variable can be considered constant despite the others or, alternatively, the following ratios can be considered constant

$$\frac{v}{i}, \ \frac{v}{\frac{di}{dt}}, \ \frac{i}{\frac{dv}{dt}} \qquad (2)$$

as shown in Table II. Moreover, it is possible to demonstrate that, for a network of elements and in particular conditions, the variables are related by Kirchhoff laws, as follows:

- for each node of the network, the algebraic sum of the flow variables is zero.
- for each mesh of the network, the algebraic sum of the forcing variables is zero.

TABLE I
VARIABLES

element	q	i	v
electric	charge / flow $q\ [C]$ / $\phi\ [Vs]$	current $i\ [A]$	voltage $v\ [V]$
hydraulic	volume / hydr. flow V $[m^3]$ / $\phi_h\ [Ns/m^2]$	flow rate $\dot{m}\ [kg/s]$	pressure $p\ [N/m^2]$
mechanic rotational	angle / momentum $\theta\ [rad]$ / $P\ [kg\, rad/s]$	torque $T\ [Nm]$	speed $\omega\ [rad/s]$

TABLE II
PARAMETERS

element	parameters		
electric	resistance $R = \frac{v}{i}$	inductance $L = \frac{v}{\frac{di}{dt}}$	capacity $C = \frac{i}{\frac{dv}{dt}}$
hydraulic	pneumatic resistance $R = \frac{p}{\dot{m}}$	hydraulic inductance $L = \frac{p}{\frac{d\dot{m}}{dt}}$	pneumatic capacity $C = \frac{\dot{m}}{\frac{dp}{dt}}$
mechanic rotational	friction $B = \frac{T}{\omega}$	elasticity $\frac{1}{K} = \frac{\omega}{\frac{dT}{dt}}$	inertia $J = \frac{T}{\frac{d\omega}{dt}}$

Now, considering the engine formed by mechanical components as throttle valve, manifolds, cylinders and crank shaft, crossed by a gas, the approach proposed in this work is based on the analogy among the electrical, more simple to model, and the mechanical and hydraulic elements.

Regarding the mechanical systems, an analogy can be found among speed and torque respectively with electric current and voltage, it results in considering the inverse of the mechanical friction B as an electric resistance R and, similarly, the inertia J as a capacitor C and the inverse of the elasticity K as an electric inductance L. The same considerations can be done for the hydraulic elements. Here the analogies are between the gas flow rate $\dot{m}$ with the current i, the pressure p with the voltage v and the volume V with the charge q. Then, the electrical resistance corresponds to the pneumatic resistance, that is the resistance of gas flowing across an orifice, and the relationship between volume and pressure to an electric capacitor.

In this scenario, all the parts composing the engine can find an equivalent electrical circuit or element as detailed described in the following sections. Moreover, in order to exactly describe the operation of electrical circuit, it is necessary to use the Maxwell equations. These can capture both the dynamics of the electrical quantities, such as currents and voltages, and the related electromagnetical phenomena, as transmission and radiation. Fortunately, if the size of the circuit is small compared to the wavelength of the electrical variables (i.e. the ratio between the light speed and the frequency of the pulsating events), these electromagnetical phenomena can be neglected. As a consequence, the partial differential relationships of the Maxwell equations can be simplified to the widely used electrical engineering equations, that are the Kirchhoff laws and the current/voltage relationships of circuit components. Similarly, the same approach can be extended to the internal combustion engine, providing that the wavelength (in this case the ratio between the sound speed and the frequency of its pulsating events) is large enough compared to the length size of the engine. As an example, a four cylinder four stroke engine, running at 3000 rpm, generates intake pulses at 100 Hz, resulting in a wavelength of 3.4 m. Considering the engine size of approximately 1 m, the lumped parameter approach seems to be reasonable [8], [14].

The engine is seen as an array of cylinders, having common connections with an intake and an exhaust manifold. The connections are regulated by valves opening. According to the previous section, it is possible to distinguish separate subsystems interconnected each others, such as the intake manifold equipped with throttle valve, the exhaust manifold and cylinders. From the phenomenological point of view, the elements composing the engine can be classified in the following categories: volumes, orifices, inertial effects and combustion. In the following, each category is introduced and the relationships among the interested variables are reported. Moreover a brief explanation of the corresponding POG section is given, according to Section IV.

A. Volumes

Here are grouped the intake and exhaust manifold and cylinders, respectively as constant and variable volumes. The electric counterpart is the quantity of charge stored in a capacitor. Applying the corresponding current/voltage relationship and considering the analogies with pressure and temperature inside the volume, it is possible to obtain the classical equations [17]. Starting from ideal gas equation

$$pV = mRT \tag{3}$$

where R is the specific gas constant and m is the mass of the gas, it is possible to obtain the following relations:

$$\dot{p} = \frac{R\gamma}{V}\left[\sum_i \dot{m}_i T_i - T\sum_j \dot{m}_j + \frac{\gamma-1}{R\gamma}\dot{Q}_{ext} - \frac{p\dot{V}}{R}\right] \tag{4}$$

$$\begin{aligned}\dot{T} \;&= \tfrac{R\gamma T}{pV}\Big[\textstyle\sum_i \dot{m}_i T_i(1-\frac{T}{\gamma T_i}) - T\sum_j \dot{m}_j(1-\frac{1}{\gamma T}) + \\ &+ \tfrac{\gamma-1}{R\gamma}\dot{Q}_{ext} - \tfrac{p\dot{V}}{R}(1-\tfrac{1}{\gamma T})\Big]\end{aligned} \tag{5}$$

where i represents the entering mass flow and j the outgoing mass flow. For sake of brevity, the details on how to obtain equations (4) and (5) are omitted.

It is remarked that, regarding the intake and exhaust manifolds, since the volume V is constant, the derivative

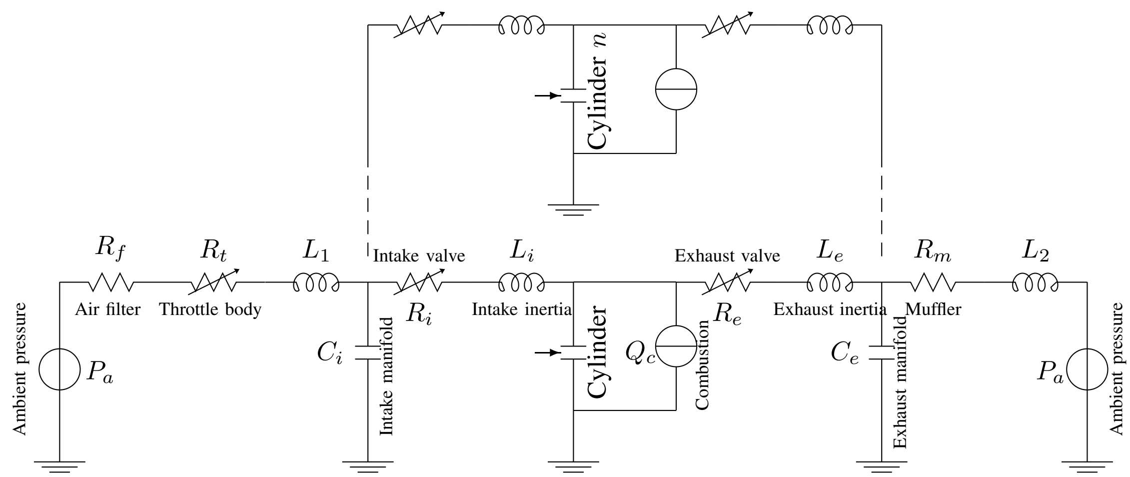

Fig. 3. Internal combustion engine equivalent circuit.

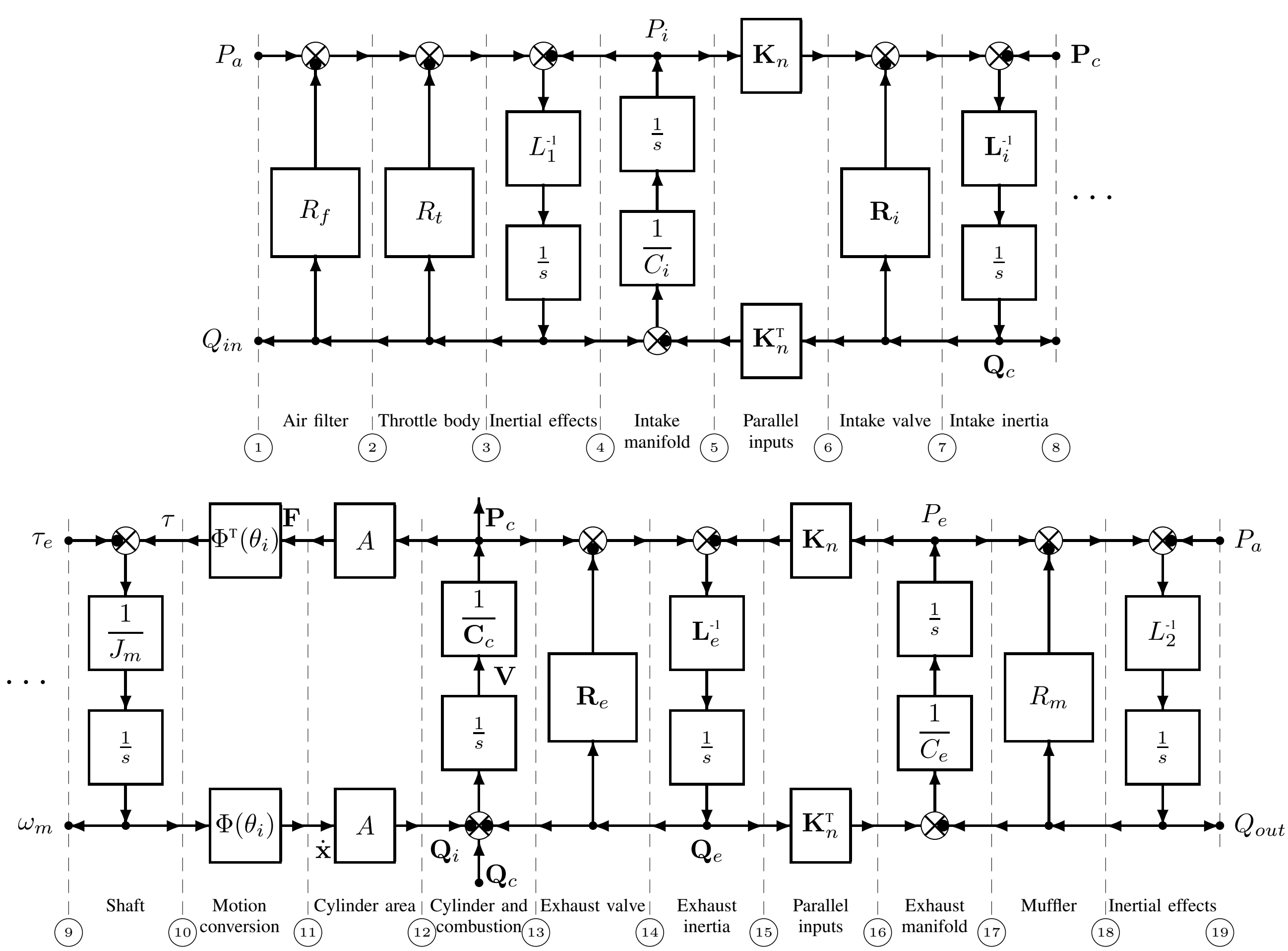

Fig. 4. POG scheme of the combustion engine.

terms in the equation disappear. These elements have their corresponding POG elaboration blocks between power sections ④ and ⑤ for the intake manifold and between sections ⑯ and ⑰ for the exhaust manifold.

B. Orifices

The orifices are responsible of the pressure drops along the gas path. They are modelled as variable resistances causing equivalent voltage drops. The size of the orifice is variable and regulated by valve opening, as throttle valve, air bypass, intake and exhaust valves.

The electrical resistance is governed by a static relationship between voltage and current, corresponding to a static relationship between the analogue variables, i.e. pressure and flow rate, according to the well known equations [18]

$$\begin{cases} \dot{m}_c = \frac{C_D A_T p_0}{\sqrt{RT}} \gamma_{\frac{1}{2}} (\frac{2}{\gamma+1})^{\frac{\gamma+1}{2(\gamma-1)}} & \frac{p_T}{p_0} > 1 \\ \dot{m}_{nc} = \frac{C_D A_T p_0}{\sqrt{RT}} (\frac{p_T}{p_0})^{\frac{1}{\gamma}} \{ \frac{2\gamma}{\gamma-1} [1 - (\frac{p_T}{p_0})^{\frac{\gamma-1}{\gamma}}] \} & \frac{p_T}{p_0} < 1 \end{cases} \tag{6}$$

where p_T and p_0 are respectively the pressure upstream and downstream the orifice. In the POG scheme orifices are represented with static elaboration blocks named R_f, R_t, $\mathbf{R}_i$, $\mathbf{R}_e$ and R_m (see Section IV for symbols explanation).

C. Inertial effects

The inertial phenomena can be considered as minor efforts but not completely negligible. They describe the reduction or the increase of the pressure upstream the valve of a quantity proportional to the derivative of the mass flow through the same valve. Here, they are modelled as a linear inductance, regulated by a differential relationship between voltage and current, corresponding to the following equation

$$p_{corr} = p - k\ddot{m} \tag{7}$$

where p_{corr} is the manifold pressure and k is a parameter to be set. It is remarked that this kind of relationship is not present in literature. In order to justify the adopted choice, both the analogy with the electrical circuit and the simulation results illustrated later on the paper can be adduced. The POG dynamic blocks representing inertial effects are placed between power sections ③ and ④, sections ⑦ and ⑧ (intake inertia), sections ⑭ and ⑮ (exhaust inertia) and between sections ⑱ and ⑲.

D. Combustion description

The combustion process constitutes the most meaningful and complex phenomenon occurring into the engine. In order to model the in-cylinder cycle pressure, an equivalent electric circuit has been adopted. The circuit is formed by a variable capacitor, representing the cylinder volume, equipped by an impulsive current generator. In intake condition, the piston downstroke causes a pressure drop through the valve (it generates a voltage difference at the capacitor extremities and, consequently, generates a current flow through the capacitor). During the combustion phase, when the intake and exhaust valves are closed, the current generator develops an impulsive current flow, simulating the pressure increase in combustion chamber during this phase. This phenomenon corresponds to the well known combustion process, i.e. an impulsive increase of the in-cylinder pressure caused by the combustion results in a torque generation and in mass flow through the exhaust valves. The equation regarding this process is described by the following relationship

$$\begin{aligned} \dot{Q}_e &= h_i A_l(\theta)(T_{sl} - T) + h_2 A_b(T_{sb} - T) \\ &+ m_{fuel}\eta_{cb}\eta_{burn}(\lambda, p)S(\theta, p)Q_{HV} \end{aligned} \tag{8}$$

where h_1 and h_2 are parameters to be set and A_l and A_b the lateral and base area respectively. It is remarked that equation (8) represents the heat power generated by the combustion affecting the pressure (4) and temperature (5). The cyclic variation has been implemented as a function of the operating conditions (η_{burn} in (8)) of engine and, partially, equipped with a random behavior needed to represent the combustion cycle-by-cycle and cylinder-by-cylinder irregularity. In the POG scheme the dynamic elaboration block modeling the array of cylinders and combustion is given between sections ⑫ and ⑬.

E. Equivalent circuit

Based on the analogies depicted in the previous paragraph, the whole engine can be represented by the circuit shown in Fig. 3.

The model starts describing the dynamic of the air crossing the intake manifold, i.e. driven by the ambient pressure (a voltage generator), the air mass passes the filter (a resistance) and the throttle body (a variable resistance) and arrives into the cylinder through the intake valves (a new variable resistance).

The cylinders are described by a parallel of "n" combustion equivalent circuits (capacitors), with "n" the number of cylinders composing the engine.
Finally, the gas mixture is discharged into the exhaust manifold through the exhaust valves (a variable resistance) and ends into the ambient crossing the muffler (a resistance). For sake of completeness, it is possible to introduce the inductors representing the inertial effects of the current as it is explained in Section III-C and are able to describe the analogue effects of the fluid columns. The validity of the modeling choices adopted in this work has been tested with experimental data of Fiat engine 1.8 liters with 8 valves and with a Variable Valve Timing (VVT).

IV. POG MODEL OF THE INTERNAL COMBUSTION ENGINE

The POG scheme of the combustion engine is reported in Fig. 4. The meaning of the model parameters is the following:

R_f	:	filter resistance;
R_t	:	throttle variable resistance;
L_1, L_2	:	inductors representing inertial effects;
C_i	:	intake manifold capacity;
$\mathbf{R}_i$	:	intake valve variable resistance;
$\mathbf{L}_i$	:	intake valve inertia;
$\mathbf{C}_c$	:	cylinder capacity;
$\mathbf{R}_e$	:	exhaust valve variable resistance;
$\mathbf{L}_e$	:	exhaust valve inertia;
C_e	:	exhaust manifold capacity;
R_m	:	muffler resistance;
J_m	:	motor inertia;
A	:	cylinder area.

Note that upright type denotes scalar parameters, while bold type denotes matrices. The scheme blocks between power sections ① to ⑧ and ⑫ to ⑲ are in the same order as the components appear in the equivalent electric circuit of Fig.3, according to the direction of the power flow. The power flows into this scheme according to the following rule: for each section of the POG scheme the power is entering (outgoing) if the path from the input to the output has an even (odd) number of 'minus' signs. Blocks between power sections ⑨ and ⑫ represent the mechanical part that is in parallel with the hydraulic part between sections ① and ⑧ (note that they have the same input across-variable $\mathbf{P}_c$ and give in output the sum of the two through-variables $\mathbf{Q}_c$ and $\mathbf{Q}_i$). This part is not considered in the equivalent electric circuit of Fig. 3, but here it is necessary in order to give the exact energetic meaning to the model.

The model has eight dynamic elements. The dynamic elements such as inductances integrate the across-variable and give the through-variable, while dynamic elements such as capacitors and inertias integrate the through-variable and give the across-variable. The state vector components can be chosen as the power variables in output from the dynamic elements taken in the order as they appear in the POG scheme:

$$\mathbf{x} = \left[\begin{array}{cccccccc} Q_{in} & P_i & \mathbf{Q}_c & \mathbf{P}_c & \mathbf{Q}_e & P_e & Q_{out} & \omega_m \end{array} \right]^{\mathrm{T}},$$

where Q_{in} is the mass flow in the air filter and throttle body, P_i is the pressure given by the intake manifold, $\mathbf{Q}_c$ is the vector of mass flows in the intake valve entering the cylinder, $\mathbf{P}_c$ is the cylinder pressure, $\mathbf{Q}_e$ is the vector of mass flows in the exhaust valve outgoing the cylinder, P_e is the pressure given by the exhaust manifold, Q_{out} is the mass flow passing through the muffler and ω_m is the velocity of the motor shaft. The input vector is:

$$\mathbf{u} = \left[\begin{array}{cc} P_a & \tau_e \end{array} \right]^{\mathrm{T}},$$

where P_a is the ambient pressure and τ_e is the external torque applied to the motor shaft. The connection matrix $\mathbf{K}_n$ is a column unitary vector of dimension n:

$$\mathbf{K}_n = \left[\begin{array}{ccccc} 1 & 1 & 1 & \dots & 1 \end{array} \right]^{\mathrm{T}}$$

where n is the number of cylinders. This matrix allows to pass from a scalar to a vectorial system (and viceversa) that is the parallel of n cylinders. The connection blocks between power sections ⑩ and ⑫ allow to pass from the hydraulic to the mechanical domain and viceversa. Function $\Phi(\theta_i)$ is defined as

$$\Phi(\theta_i) = \left[\begin{array}{c} \varphi(\theta_1) \\ \varphi(\theta_2) \\ \vdots \\ \varphi(\theta_n) \end{array} \right]$$

where θ_i is the angle between the piston rod and the motion direction of each cylinder, $\varphi(\theta)$ converts the rotational motion of the shaft into the linear motion of the piston (and viceversa) and it is defined as:

$$\varphi(\theta) = -l_1 \sin(\theta) \left(1 + \frac{\cos(\theta)}{l_2 \sqrt{1 - \left(\frac{l_1}{l_2} \sin(\theta) \right)^2}} \right),$$

where l_1 and l_2 are the lengths of the piston rod and the crank respectively.

The differential equations of the motor can be written in the state space form as:

$$\mathbf{L}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$

and the system matrices can be obtained straightforward from the POG scheme. Matrix $\mathbf{L}$, named *energy matrix*, is diagonal and it can be obtained putting on its diagonal the coefficients of the dynamic elements of the system, it is to say all inductors, capacitors and inertias taken in the order as they appear in the POG scheme from left to right (as the mechanical part is in parallel, the inertia can be put in the last position in matrix $\mathbf{L}$). The input matrix $\mathbf{B}$ can be obtained from the scheme following the paths connecting the three inputs to each component of vector $\dot{\mathbf{x}}$. The system matrices are given by:

$$\mathbf{L} = \left[\begin{array}{cccccccc} L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{L}_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C}_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{L}_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_r \end{array} \right], \mathbf{B} = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{array} \right] \quad (9)$$

and

$$\mathbf{A} = \left[\begin{array}{cccccccc} -(R_f+R_t) & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\mathbf{K}_n^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{K}_n & -R_i & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & -\varphi(\theta)A \\ 0 & 0 & 0 & 1 & -R_e & -\mathbf{K}_n & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{K}_n^{\mathrm{T}} & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -R_m & 0 \\ 0 & 0 & 0 & \varphi(\theta)A & 0 & 0 & 0 & 0 \end{array} \right]. \quad (10)$$

The system matrix $\mathbf{A}$ can always be represented as the sum of a symmetric part $\mathbf{A}_s = \frac{A+A^{\mathrm{T}}}{2}$ and a skew-symmetric part $\mathbf{A}_w = \frac{A-A^{\mathrm{T}}}{2}$. The symmetric part $\mathbf{A}_s$ is a function

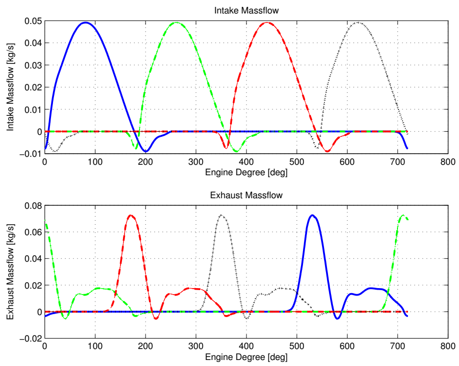

Fig. 5. Experiment at 1500 rpm and WOT. Intake and exhaust mass flows for the four cylinders.

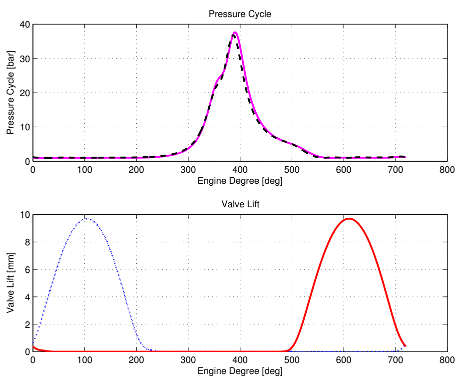

Fig. 6. Experiment at 1500 rpm and WOT. The first plot compares experimental data (dotted-black line) of the pressure inside cylinder and simulated results (solid-magenta line); the second plot reproduces the intake and exhaust valves lift (experimental data).

of the static parameters of the system and it represents the system dissipations. The dissipating power of the system is given by a quadratic form function of matrix $\mathbf{A}_s$ and state vector $\mathbf{x}$. The skew-symmetric part $\mathbf{A}_w$ is a function of the connecting parameters between the system elements and it represents energy redistribution within the system, so it is not responsible for neither storing nor dissipating energy and the quadratic form based on this matrix is always zero.

V. Simulation Results

In this section an experiment is illustrated at 1500 rpm. Figures 5 and 6 report the experiment results. In particular, Fig. 5 shows the simulated intake and exhaust mass flows for each cylinder. Fig. 6 completes the experiment comparing the simulated in-cylinder pressure cycle with experimental data highlighting the good performance. The intake and exhaust valve lift ends the Fig. 6.

VI. Conclusion and Future Work

The paper has described the Power-Oriented Graphs (POG) technique for modeling Spark-Ignition engine based on the equivalence with electric circuit. This approach exhibits some advantages in comparison with other graphical techniques and allows to realize very compact schemes which can be easily translated into Simulink models. The resulting mean value model has been tested by comparing with experimental data, showing a good level of reliability and accuracy. The POG scheme will be translated into Simulink models and, after having exploited the parameters identification, simulation results will be compared with the same data. Thanks to its properties, the POG modeling technique is a good alternative to model automotive systems and to define the differential equations of the engine in the state space form in simple tasks.

References

[1] H. Paynter, "Analysis and design of engineering systems," *MIT-press, Camb., MA*, 1961.

[2] D. Karnopp, D. Margolis, and R. Rosemberg, *Analysis and Design of Engineering Systems*. Wiley Interscience, ISBN 0-471-33301-8, 3rd ed. 2000.

[3] R. Zanasi, "Power oriented modelling of dynamical system for simulation," *IMACS Symposium on Modelling and Control of Technological System*, 1991.

[4] R. Zanasi and F. Grossi, "The pog technique for modelling multi-phase permanent magnet synchronous motors," *6th EUROSIM Congress on Modelling and Simulation*, 2007.

[5] R. Zanasi, G. Geitner, A. Bouscayrol, and W. Lhomme, "Different energetic techniques for modelling traction drives," *ELECTRIMACS, Canada*, 2008.

[6] R. Morselli, E. Corti, and G. Rizzoni, "Energy based model of a common rail injector," *Conference on Control Applications*, 2002.

[7] R. Morselli, R. Zanasi, and P. Ferracin, "Modeling and simulation od static and coulomb friction in a class of automotive systems," *International Journal of Control, Vol.79, No.5, pp.508-520*, 2006.

[8] A. Palma, A. Palladino, G. Fiengo, F. D. Cristofaro, and F. Garofalo, "A modeling approach for engine dynamics based on electrical analogy," *Accepted to IFAC*, 2008.

[9] L. Mianzo and H. Peng, "Modelling and control of a variable valve timing engine," *American Control Conference*, 2000.

[10] J. Bengtsson, M. Gafvert, and P. Strandh, "Modelling of hcci engine combustion for control analysis," *43rd IEEE Conference of Decision and Control*, 2004.

[11] D. Rausen, A. Stefanopoulou, J. Kang, J. Eng, and T. Kuo, "A mean-value model for control of homogeneous charge compression ignition (hcci) engines," *American Control Conference*, 2004.

[12] J. Cook and B. Powell, "Modelling of an internal combustion engine for control analysis," *Control Systems Magazine*, vol. I, pp. 20–31, 1988.

[13] B. Powell, "A dynamic model for automotive engine control analysis," *18th IEEE Conference on Decision and Control*, 1979.

[14] A. Palma, A. Palladino, G. Fiengo, F. D. Cristofaro, and F. Garofalo, "Air mass flow analysis for si engine: Egr and scavenging," *Accepted to IFAC*, 2008.

[15] R. Morselli and R. Zanasi, "Modeling of automotive control systems using power oriented graphs," *32nd Annual Conference of the IEEE Industrial Electronics Society*, 2006.

[16] A. Balestrino and G. Celentano, *Teoria dei sistemi*. Liguori Editore, 1997.

[17] M. Muller, E. Hendricks, and S. Sorenson, "Mean value modelling of turbocharged spark ignition engine," *SAE Paper 980784*, 1998.

[18] E. Hendricks, "The analisys of mean value engine models," *SAE Paper 980563*, 1989.