2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Reliable Stabilization and H_∞ Control for Switched Systems with Faulty Actuators: An Average Dwell Time Approach

Li-li Li, Jiaxin Feng, Georgi M. Dimirovski and Jun Zhao

***Abstract*— This paper deals with the issues of observer-based reliable stabilization and H_∞ control for a class of continuous-time switched Lipschitz nonlinear systems in the sense that actuators suffer a "destabilizing failure". When the never-faulty actuators cannot stabilize the corresponding system, the closed-loop switched systems can still be exponentially stable based on the average dwell time scheme. Under the condition requiring that activation time ratio between stabilizable subsystems and unstabilizable ones is not less than a specified constant, sufficient condition is derived for the switched systems to be exponentially stabilizable for all admissible actuator failures via switching and associated observer-based feedback controllers. The result is also extended to solve the observer-based reliable H_∞ control problem.**

I. INTRODUCTION

The last decade has witnessed rapidly increasing interest in switched systems [1-6]. Dwell time approach has been developed in several references [7-10] to be one of effective tools of constructing some proper switching law. In particular, exponential stability of a switched system is considered in [7] if all subsystems are stable and the dwell time is set sufficiently large. The concept of dwell time was extended to average dwell time by Hespanha and Morse [8] with switching among stable subsystems. Furthermore, [9,10] generalized the results to the case where stable and unstable subsystems co-exist.

On the other hand, the popularity of studying reliable control problem is raised for the growing demands of system reliability in aerospace and industrial process. When controlling a real plant with failures of control components, classical control methods may not achieve satisfactory performance. To overcome this problem, reliable control has made great progress recently. Among the existing studies, [11] presented the reliable control via a robust pole region assignment scheme. [12] solved the reliable H_∞ control problem for affine nonlinear systems with actuator and sensor failures via Hamilton-Jacobi inequality. However, these design methods are all based on a basic assumption that the never-faulty actuators must stabilize the given system. These design methods of existing reliable control do not work, when actuators suffer a "destabilizing failure"–the never-faulty actuators are not assumed to be able to stabilize the considered system. For this case, [13] dealt with exponential stability for a class of linear systems with faulty actuators using switching control technique.

The information of state variable is usually unavailable or not fully available in engineering practice. So the feedback control and the switching law can not be designed depending on state variable. Inspired by this fact, this paper focuses on the observer-based reliable control problems for a class of switched Lipschitz nonlinear systems with destabilizing actuator failure by exploiting the average dwell time approach. Due to the complexity of switched systems, few results have been devoted to reliable control for switched systems until now [15-17]. Unlike the previous works on the reliable control for switched systems, this paper owns three features. First, only the estimate state $\hat{x}$ rather than the state x is available for designing the control feedback. Secondly, a class of time dependent switching signals is employed via average dwell time scheme. Thirdly, the differentiable Lipschitz nonlinearity allows large values of the Lipschitz constant compared with the classical ones.

Throughout this paper, $\bar{\lambda}(\cdot)(\underline{\lambda}(\cdot))$ is the largest (smallest) eigenvalue of a symmetric matrix, the set $Co(a,b)=\{\lambda a+(1-\lambda)b, 0\le\lambda\le1\}$ is the convex hull of a,b, $e_s(i) s\ge1$ are vectors of the canonical basis of $\Re^s$.

II. PRELIMINARIES

In this paper, we consider the class of switched nonlinear systems represented by the following state-space description:

$$\begin{aligned}\dot{x}(t) &= A_\sigma x(t) + B_\sigma u_\sigma + D_\sigma f_\sigma(x(t), y(t), u_\sigma),\\ y(t) &= g_\sigma(x(t), u_\sigma),\end{aligned} \tag{1}$$

where $\sigma:\Re^+ \mapsto M=\{1,2,\ldots,m\}$ is the right continuous piecewise constant switching signal to be designed, $x\in\Re^n$ is the state vector, $u_i\in\Re^{m_i}$ and $y\in\Re^{p_i}$ denote the control input and measured output, A_i, B_i and D_i are constant matrices of appropriate dimensions, the functions f_i and g_i satisfy:

Assumption 1: The nonlinear functions $f_i:\Re^n\times\Re^{p_i}\times\Re^{m_i}\mapsto\Re^{q_i}$ and $g_i:\Re^n\times\Re^{m_i}\mapsto\Re^{p_i}$ are differentiable with respect to x, and satisfy $\underline{f}^i_{jk}\le\frac{\partial f_{ij}}{\partial x_k}(x,y,u_i)\le\bar{f}^i_{jk}$, $\underline{g}^i_{jk}\le\frac{\partial g_{ij}}{\partial x_k}(x,u_i)\le\bar{g}^i_{jk}$,where $\underline{g}^i_{jk}=\inf_{Z\in\Re^n\times\Re^{m_i}}\big(\frac{\partial g_{ij}}{\partial x_k}(Z)\big)$, $\bar{g}^i_{jk}=\sup_{Z\in\Re^n\times\Re^{m_i}}\big(\frac{\partial g_{ij}}{\partial x_k}(Z)\big)$, $\underline{f}^i_{jk}=\inf_{Z\in\Re^n\times\Re^{p_i}\times\Re^{m_i}}\big(\frac{\partial f_{ij}}{\partial x_k}(Z)\big)$, $\bar{f}^i_{jk}=\sup_{Z\in\Re^n\times\Re^{p_i}\times\Re^{m_i}}\big(\frac{\partial f_{ij}}{\partial x_k}(Z)\big)$, f_{ij}, g_{ij} and x_j denote

This work was supported by Dogus University Fund for Science, the NSFC under Grants 60874024 and 9081602, and the SRFDP under Grant 200801450019.

Li-li Li and Jun Zhao are with the Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang, 110004, P. R. China, and Jun Zhao is also with the Department of Information Engineering, Research School of Information Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia tolilili@gmail.com,zhaojun@ise.neu.edu.cn

Jiaxin Feng is with the School of Information Management and Engineering, Shanghai University of Finance & Economics, Shanghai, 200433, China jiaxinf@126.com

Georgi M. Dimirovski is with the Department of Computer Engineering, Dogus University, Kadikoy, TR-34722, Istanbul, Turkey gdimirovski@dogus.edu.tr

978-1-4244-4524-0/09/$25.00 ©2009 AACC

the j-th components of f_i, g_i and x respectively. Moreover, $f_i(0, y, u_i) \equiv 0$.

Actuator failures are assumed to occur within a prescribed subset of control input channel. We classify actuators of the i-th subsystem of the system (1) into two groups. One is a set of actuators susceptible to failures, denoted by $\Theta_i \subseteq \{1, 2, \cdots, m_i\}$, $i \in M$. The actuators in this set may occasionally fail. The other is a set of actuators robust to failures, denoted by $\bar{\Theta}_i \subseteq \{1, 2, \cdots, m_i\} - \Theta_i$, $i \in M$. According to the classification of actuators, we have the decomposition $B_i = B_{\bar{\Theta}_i} + B_{\Theta_i}, i \in M$, where $B_{\bar{\Theta}_i}$ and B_{Θ_i} are formed from B_i by zeroing out columns corresponding to Θ_i and $\bar{\Theta}_i$, respectively.

Define the set of actual actuator failures of the system (1) as w_i, obviously $w_i \subseteq \Theta_i$, and the outputs of faulty actuators are assumed to be zero. For $w_i \subseteq \Theta_i$, introduce the decomposition $B_i = B_{w_i} + B_{\bar{w}_i}, i \in M$, where B_{w_i} and $B_{\bar{w}_i}$ are formed from B_i by zeroing out columns corresponding to $\bar{w}_i$ and w_i, respectively. Thus the following inequalities are obvious and will be used in the sequel

$$B_{\bar{\Theta}_i} B_{\bar{\Theta}_i}^T \leq B_{\bar{w}_i} B_{\bar{w}_i}^T, \quad B_{w_i} B_{w_i}^T \leq B_{\Theta_i} B_{\Theta_i}^T. \tag{2}$$

Consider the following standard state observers

$$\begin{aligned} \dot{\hat{x}}(t) = & A_\sigma \hat{x}(t) + B_\sigma u_\sigma + D_\sigma f_\sigma(\hat{x}(t), y(t), u_\sigma) \\ & - L_\sigma (g_\sigma(\hat{x}, u_\sigma) - g_\sigma(x, u_\sigma)), \end{aligned} \tag{3}$$

where $\hat{x}(t)$ denotes the estimate of the state $x(t)$ and observer gain matrices $L_i \in \Re^{n \times p_i}$ will be determined later. The estimation error $e(t) = \hat{x}(t) - x(t)$ satisfies

$$\begin{aligned} \dot{e}(t) = & A_\sigma e(t) + D_\sigma (f_\sigma(\hat{x}, y, u_\sigma) - f_\sigma(x, y, u_\sigma)) \\ & - L_\sigma (g_\sigma(\hat{x}, u_\sigma) - g_\sigma(x, u_\sigma)). \end{aligned} \tag{4}$$

Definition 1: ([11]) For any switching signal $\sigma(t)$ and any $t > \tau \geq 0$, let $N_\sigma(\tau, t)$ denote the number of switchings of $\sigma(t)$ on the interval (τ, t). If

$$N_\sigma(\tau, t) \leq N_0 + \frac{t - \tau}{\tau_a} \tag{5}$$

holds for $N_0 \geq 0$, $\tau_a > 0$. The constant τ_a is called average dwell time and N_0 is the chatter bound. As commonly used in the literature, we choose $N_0 = 0$.

The reliable control problem can be easily solved if all subsystems are stabilizable. Therefore, we consider the case that stabilizable and unstabilizable subsystems coexist. Let M_p denote a proper nonempty subset of M, $\tilde{M}_p$ denote a complement of M_p with respect to M. If $i \in M_p$, then i-th subsystem is stabilizable and satisfies the reliable control, otherwise, if $i \in \tilde{M}_p$, then i-th subsystem is unstabilizable.

For any switching signal and any $0 \leq \tau < t$, we let $T^+(\tau, t)$ (resp., $T^-(\tau, t)$) denote the total activation time of unstabilizable subsystems (resp., stabilizable subsystems) during the interval $[\tau, t)$. Denote $\lambda^+ = \max_{i \in \tilde{M}_p}\{-\lambda_i\}$, $\lambda^- = \min_{i \in M_p}\{\lambda_i\}$. Then, for any given $\lambda \in (0, \lambda^-)$, we choose an arbitrary $\lambda^* \in (\lambda, \lambda^-)$. Motivated by the idea in [9], we propose the following switching law:

(S) Determine the switching signal $\sigma(t)$ such that

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{6}$$

holds for any given initial time t_0.

III. RELIABLE EXPONENTIAL STABILIZATION

Define sets $\mathscr{H}_{q_i,n}^i = \{v^i = (v_{11}^i \dots v_{1n}^i \dots v_{q_in}^i) : \underline{f}_{jk}^i \leq v_{jk}^i \leq \bar{f}_{jk}^i, j = 1, \dots, q_i, k = 1, \dots, n\}, \forall i \in M$. Each set $\mathscr{H}_{q_i,n}^i$ is a bounded convex domain whose vertices set is $\mathscr{V}_{q_i,n}^i = \{\alpha^i = (\alpha_{11}^i \dots \alpha_{1n}^i \dots \alpha_{q_in}^i) : \alpha_{jk}^i \in \{\underline{f}_{jk}^i, \bar{f}_{jk}^i\}\}$. Define the affine matrix functions

$$\mathscr{A}_i(v^i) = A_i + D_i \sum_{j,k=1}^{q_i,n} v_{jk}^i e_{q_i}(j) e_n^T(k), \ v^i \in \mathscr{H}_{q_i,n}^i. \tag{7}$$

By the differential mean value theorem [14], there exist $z_j(t), \bar{z}_j(t) \in Co(x(t), \hat{x}(t))$ such that

$$\begin{aligned} & f_i(\hat{x}, y, u_i) - f_i(x, y, u_i) \\ & \qquad = \Big(\sum_{j,k=1}^{q_i,n} e_{q_i}(j) e_n^T(k) \frac{\partial f_{ij}}{\partial x_k}(z_j, y, u_i)\Big) e, \end{aligned} \tag{8}$$

$$g_i(\hat{x}, u_i) - g_i(x, u_i) = \Big(\sum_{j,k=1}^{p_i,n} e_{p_i}(j) e_n^T(k) \frac{\partial g_{ij}}{\partial x_k}(\bar{z}_j, u_i)\Big) e. \tag{9}$$

With (7), (8) and $h^i(t) = (h_{11}^i(t) \dots h_{1n}^i(t) \dots h_{q_in}^i(t))$, $h_{jk}^i(t) = \frac{\partial f_{ij}}{\partial x_k}(z_j, y, u_i)$, (4) can be rewritten as

$$\dot{e}(t) = (\mathscr{A}_\sigma(h^\sigma(t)) - L_\sigma \mathscr{G}_\sigma(\rho^\sigma(t))) e(t), \tag{10}$$

where $\mathscr{G}_i(\cdot)$ are given by $\mathscr{G}_i(\rho^i(t)) = \sum_{j,k=1}^{p_i,n} \rho_{jk}^i e_{p_i}(j) e_n^T(k)$, with $\rho^i(t) = (\rho_{11}^i(t) \dots \rho_{1n}^i(t) \dots \rho_{p_in}^i(t))$, $\rho_{jk}^i(t) = \frac{\partial g_{ij}}{\partial x_k}(\bar{z}_j, u_i)$. From Assumption 1, $\rho^i(\cdot)$ remains in a bounded domain $\mathscr{F}_{p_i,n}^i$ of which vertices set is $\mathscr{W}_{p_i,n}^i = \{\beta^i = (\beta_{11}^i \dots \beta_{1n}^i \dots \beta_{p_in}^i) : \beta_{jk}^i \in \{\underline{g}_{jk}^i, \bar{g}_{jk}^i\}\}$. It follows from $f_i(0, y, u_i) = 0$ and (8) that there exist $\tilde{z}_j(t) \in Co(0, \hat{x})$ (without loss of generality, suppose that $\tilde{z}_j(t) = z_j(t)$), such that $D_i f_i(\hat{x}, y, u_i) = (\mathscr{A}_i(h^i(t)) - A_i)\hat{x}$. Then, the closed-loop system composed of (3), (10) and $u_\sigma = K_\sigma \hat{x}(t)$ is:

$$\dot{\tilde{x}}(t) = \tilde{A}_\sigma \tilde{x}(t), \tag{11}$$

where

$$\tilde{A}_i = \begin{bmatrix} \mathscr{A}_i(h^i(t)) + B_i K_i & -L_i \mathscr{G}_i(\rho^i(t)) \\ 0 & \mathscr{A}_i(h^i(t)) - L_i \mathscr{G}_i(\rho^i(t)) \end{bmatrix}, \tilde{x}(t) = \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix}.$$

The observer-based reliable stabilization problem is to construct switching signals associated with observer-based output feedback controllers $u_\sigma = K_\sigma \hat{x}$ under which system (1) is exponentially stabilizable for actuator failures corresponding to any $w_i \subseteq \Theta_i$, i.e., there exist positive constants c and λ such that $\|\tilde{x}(t)\| \leq c \|\tilde{x}(t_0)\| e^{-\lambda(t - t_0)}, \forall t > t_0$ for all trajectories of the closed-loop system (11).

The following result is used to develop the main result.

Lemma 1: Given positive constants λ_i for $i \in M_p$ and negative constants λ_i for $i \in \tilde{M}_p$, if

(a) There exist matrices $P_i > 0$ such that

$$\text{Block-diag}\{\Psi_i(\alpha_1^i), \Psi_i(\alpha_2^i), \dots, \Psi_i(\alpha_{2^{q_in}}^i)\} < 0 \tag{12}$$

hold for $\forall\ i = M, j = 1, \dots, 2^{q_in}$, $\alpha_j^i \in \mathscr{V}_{q_i,n}^i$, where

$$\begin{aligned} \Psi_i(\alpha_j^i) = & \mathscr{A}_i^T(\alpha_j^i) P_i + P_i \mathscr{A}_i(\alpha_j^i) - 2 P_i B_{\bar{\Theta}_i} B_{\bar{\Theta}_i}^T P_i \\ & + P_i L_i L_i^T P_i + 2\lambda_i P_i. \end{aligned}$$

(b) There exists a matrix $S>0$ and matrices R_i such that

$$\text{Block-diag}\big\{\Gamma_i(\alpha_1^i,\beta_1^i),\ldots,\Gamma_i(\alpha_{2^{q_in}}^i,\beta_1^i),\Gamma_i(\alpha_1^i,\beta_2^i),\ldots,\Gamma_i(\alpha_{2^{q_in}}^i,\beta_{2^{p_in}}^i)\big\}<0 \quad (13)$$

hold for $\forall\ i=M, j=1,\ldots,2^{q_in}, k=1,\ldots,2^{p_in}$, $\alpha_j^i\in\mathscr{V}_{q_i,n}^i, \beta_k^i\in\mathscr{W}_{p_i,n}^i$, where

$$\Gamma_i(\alpha_j^i,\beta_k^i)=\begin{bmatrix}\Xi_i(\alpha_j^i,\beta_k^i) & \mathscr{G}_i^T(\beta_k^i)\\ \mathscr{G}_i(\beta_k^i) & -I\end{bmatrix},$$
$$\Xi_i(\alpha_j^i,\beta_k^i)=\mathscr{A}_i^T(\alpha_j^i)S-\mathscr{G}_i^T(\beta_k^i)R_i+S\mathscr{A}_i(\alpha_j^i)-R_i^T\mathscr{G}_i(\beta_k^i)+2\lambda_iS.$$

Then, there exist feedback controllers $u_i=K_i\hat{x}$ such that

$$V_i(t)\le e^{-2\lambda_i(t-t_0)}V_i(t_0) \quad (14)$$

hold along the trajectory of system (11) for actuator failures corresponding to any $w_i\subseteq\Theta_i$, where the controller and observer gain matrices are $K_i=-B_i^TP_i$, $L_i=S^{-1}R_i^T$, $i\in M$.

Proof: Choose a piecewise Lyapunov function candidate for closed-loop system (11) of form

$$V(t)=V_{\sigma(t)}(\tilde{x})=\tilde{x}^T\tilde{P}_{\sigma(t)}\tilde{x}=\tilde{x}^T\begin{bmatrix}P_{\sigma(t)} & 0\\ 0 & S\end{bmatrix}\tilde{x}, \quad (15)$$

where $P_i(i\in M), S$ are positive definite matrices to be determined later. In view of the zero outputs of the faulty actuators and observer-based controllers $u_i=K_i\hat{x}$, one has $B_iK_i=-B_{\bar{w}_i}B_{\bar{w}_i}^TP_i$. It follows from (2) that the derivative of each V_i in (15) along the trajectory of the corresponding subsystem satisfies

$$\begin{aligned}&\dot{V}_i+2\lambda_iV_i\\ &\le\hat{x}^T\big(\mathscr{A}_i^T(h^i)P_i+P_i\mathscr{A}_i(h^i)-2P_iB_{\bar{\Theta}_i}B_{\bar{\Theta}_i}^TP_i\\ &\quad+P_iL_iL_i^TP_i+2\lambda_iP_i\big)\hat{x}+e^T\big(\mathscr{A}_i^T(h^i)S-\mathscr{G}_i^T(\rho^i)R_i\\ &\quad+S\mathscr{A}_i(h^i)-R_i^T\mathscr{G}_i(\rho^i)+\mathscr{G}_i^T(\rho^i)\mathscr{G}_i(\rho^i)+2\lambda_iS\big)e. \quad (16)\end{aligned}$$

On the other hand, denote $\Psi_i(h^i)=\mathscr{A}_i^T(h^i)P_i+P_i\mathscr{A}_i(h^i)-2P_iB_{\bar{\Theta}_i}B_{\bar{\Theta}_i}^TP_i+P_iL_iL_i^TP_i+2\lambda_iP_i$, which are affine in $h^i(t)$. (12) implies that $\Psi_i(\alpha^i)<0$ for all $\alpha^i\in\mathscr{V}_{q_i,n}^i$. Using the convexity principle (see [18] for more details), we deduce that $\Psi_i(h^i(t))<0$ for all $h^i(t)\in\mathscr{H}_{q_i,n}^i$, which means that

$$\mathscr{A}_i^T(h^i(t))P_i+P_i\mathscr{A}_i(h^i(t))-2P_iB_{\bar{\Theta}_i}B_{\bar{\Theta}_i}^TP_i+P_iL_iL_i^TP_i+2\lambda_iP_i<0. \quad (17)$$

Similarly, condition (b) implies

$$\mathscr{A}_i^T(h^i(t))S-\mathscr{G}_i^T(\rho^i(t))R_i+S\mathscr{A}_i(h^i(t))-R_i^T\mathscr{G}_i(\rho^i(t))+\mathscr{G}_i^T(\rho^i(t))\mathscr{G}_i(\rho^i(t))+2\lambda_iS<0 \quad (18)$$

holds for all $h^i(t)\in\mathscr{H}_{q_i,n}^i$, $\rho^i(t)\in\mathscr{F}_{p_i,n}^i$. Substituting (17) and (18) into (16), we have

$$\dot{V}_i+2\lambda_iV_i<0. \quad (19)$$

The differential inequality theory and (19) gives (14). ■

Remark 1: It follows from $f_i(0,y,u_i)\equiv 0$ and (7) that there exist $\tilde{z}_j\in Co(0,\hat{x})$ such that $D_if_i(\hat{x},y,u_i)=\big(\mathscr{A}_i(\tilde{h}^i(t))-A_i\big)\hat{x}(t)$, where, similarly to $h^i(t)$, $\tilde{h}^i(t)$ can be defined as $\tilde{h}^i(t)=(\tilde{h}_{11}^i(t)\ldots\tilde{h}_{1n}^i(t)\ldots\tilde{h}_{q_in}^i(t))$, $\tilde{h}_{jk}^i(t)=\frac{\partial f_{ij}}{\partial x_k}(\tilde{z}_j,y,u_i)$. A common convex set $\mathscr{H}_{q_i,n}^i$ for both $\tilde{h}^i(t)$ and $h^i(t)$ can be easily found. The conditions of Lemma 1 need only the values of vertices in sets $\mathscr{V}_{q_i,n}^i$. Therefore, we can suppose that $\tilde{z}_j(t)=z_j(t)$ without loss of generality as foregoing statement.

Next, we present solvability condition and a design method for the observer-based reliable stabilization of system (1).

Theorem 1: For given constants λ_i positive for $i\in M_p$ and negative for $i\in\tilde{M}_p$, suppose that there exist positive definite matrices P_i and S, matrices R_i such that (12) and (13) hold, then for actuator failures corresponding to any $w_i\subseteq\Theta_i$, the observer-based reliable stabilization of system (1) is solved under the observer-based output feedback controllers $u_\sigma=K_\sigma\hat{x}$ for any switching signal $\sigma(t)$ with average dwell time (5) and switching condition (S) satisfying

$$\tau_a\ge\tau_a^*=\frac{\ln\mu}{2(\lambda^*-\lambda)}, \quad (20)$$

where the controller and observer gain matrices are $K_i=-B_i^TP_i$ and $L_i=S^{-1}R_i^T$. Moreover,

$$\|\tilde{x}(t)\|\le\sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\lambda(t-t_0)}\|\tilde{x}(t_0)\|,\ \forall t>t_0, \quad (21)$$

where $\mu\ge 1$ satisfies

$$P_i\le\mu P_j,\ \forall i,j\in M, \quad (22)$$
$$\lambda_1=\min\{\min_{i\in M}\underline{\lambda}(P_i),\underline{\lambda}(S)\},\ \lambda_2=\max\{\max_{i\in M}\bar{\lambda}(P_i),\bar{\lambda}(S)\}. \quad (23)$$

Proof: According to (22) and the definition of piecewise Lyapunov function candidate in (15), we can easily obtain

$$V_i\le\mu V_j,\ \forall i,j\in M, \quad (24)$$
$$\lambda_1\|\tilde{x}(t)\|^2\le V(t),\ V_{\sigma(t_0)}(t_0)\le\lambda_2\|\tilde{x}(t_0)\|^2. \quad (25)$$

For any given $t>t_0$, let $0=t_0<t_1<\cdots<t_k=t_{N_\sigma(t_0,t)}$ be the switching points of $\sigma(t)$ over the interval (t_0,t). Then, from Lemma 1, we know that for any $t\in[t_k,t_{k+1})$ $(0\le k\le N_\sigma(t_0,t))$, the piecewise Lyapunov function candidate (15) satisfies

$$V(t)=V_{\sigma(t)}(t)\le\begin{cases}e^{-2\lambda^-(t-t_k)}V_{\sigma(t_k)}(t_k),\ i\in M_p,\\ e^{2\lambda^+(t-t_k)}V_{\sigma(t_k)}(t_k),\ i\in\tilde{M}_p.\end{cases}$$

Since $V_{\sigma(t_k)}(t_k)\le\mu V_{\sigma(t_k^-)}(t_k^-)$ is true from (24) at the switching point t_k, where $t_k^-=\lim_{t\to t_k}t$, we obtain by induction that

$$\begin{aligned}V(t)&\le e^{2\lambda^+T^+(t_k,t)-2\lambda^-T^-(t_k,t)}V_{\sigma(t_k)}(t_k)\\ &\le e^{2\lambda^+T^+(t_k,t)-2\lambda^-T^-(t_k,t)}\mu V_{\sigma(t_k^-)}(t_k^-)\\ &\le e^{2\lambda^+T^+(t_{k-1},t)-2\lambda^-T^-(t_{k-1},t)}\mu V_{\sigma(t_{k-1})}(t_{k-1})\\ &\le\cdots\le e^{2\lambda^+T^+(t_0,t)-2\lambda^-T^-(t_0,t)}\mu^{N_\sigma(t_0,t)}V_{\sigma(t_0)}(t_0)\\ &=e^{2\lambda^+T^+(t_0,t)-2\lambda^-T^-(t_0,t)+N_\sigma(t_0,t)\ln\mu}V_{\sigma(t_0)}(t_0). \quad (26)\end{aligned}$$

Combining (25) and (26) leads to

$$\|\tilde{x}(t)\|\le\sqrt{\frac{\lambda_2}{\lambda_1}}e^{\lambda^+T^+(t_0,t)-\lambda^-T^-(t_0,t)+N_\sigma(t_0,t)\frac{\ln\mu}{2}}\|\tilde{x}(t_0)\|. \quad (27)$$

Therefore, when $\mu = 1$, which is a trivial case,

$$\|\tilde{x}(t)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\lambda^*(t-t_0)} \|\tilde{x}(t_0)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\lambda(t-t_0)} \|\tilde{x}(t_0)\|,$$

which means the switched system is globally exponentially stabilizable with actuator failures under switching law (S) without considering the average dwell time.

Next, we consider the nontrivial case of $\mu > 1$. It follows from (20) with average dwell time (5) and the switching condition (S) that

$$-\lambda^*(t-t_0) + \frac{\ln \mu}{2} N_\sigma(t_0, t) \le -\lambda(t-t_0)$$

for any $t \ge t_0$. Thus (27) implies (21). ■

Remark 2: When $\mu = 1$, we have $\tau_a^* = 0$, which implies that switching signals can be arbitrary and a common Lyapunov function is formed. In this case, the observer-based reliable stabilization problem of system (1) is solvable under arbitrary switching.

IV. EXTENSION TO RELIABLE H_∞ CONTROL

Consider the switched systems described by the equations

$$\begin{aligned} \dot{x}(t) &= A_\sigma x(t) + B_\sigma u_\sigma + D_\sigma f_\sigma(x(t), y(t), u_\sigma) + W_{1\sigma}\omega(t), \\ y(t) &= g_\sigma(x(t), u_\sigma) + W_{2\sigma}\omega(t), \\ z(t) &= \begin{bmatrix} E_\sigma x(t) \\ u_\sigma \end{bmatrix}, \end{aligned} \tag{28}$$

where $z \in \Re^{r_i}$ are the controlled output, $\omega(t) \in \Re^{h_i}$ which belongs to $L_2[0, \infty)$ denotes the disturbance input, W_{1i}, W_{2i} and E_i are constant matrices of appropriate dimensions. The estimation error $e(t) = \hat{x}(t) - x(t)$ satisfies

$$\begin{aligned} \dot{e}(t) = {} & A_\sigma e(t) + D_\sigma (f_\sigma(\hat{x}, y, u_\sigma) - f_\sigma(x, y, u_\sigma)) \\ & - L_\sigma(g_\sigma(\hat{x}, u_\sigma) - g_\sigma(x, u_\sigma)) + (L_\sigma W_{2\sigma} - W_{1\sigma})\omega(t). \end{aligned} \tag{29}$$

Then combining (3), (29) and $u_\sigma = K_\sigma \hat{x}(t)$ gives the closed-loop system

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}_\sigma \tilde{x}(t) + \tilde{B}_\sigma \omega(t), \\ z(t) &= \tilde{C}_\sigma \tilde{x}(t), \end{aligned} \tag{30}$$

where

$$\tilde{A}_i = \begin{bmatrix} \mathscr{A}_i(h^i(t)) + B_i K_i & -L_i \mathscr{G}_i(\rho^i(t)) \\ 0 & \mathscr{A}_i(h^i(t)) - L_i \mathscr{G}_i(\rho^i(t)) \end{bmatrix},$$
$$\tilde{B}_i = \begin{bmatrix} L_i W_{2i} \\ L_i W_{2i} - W_{1i} \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} E_i & -E_i \\ K_i & 0 \end{bmatrix}.$$

Now, the observer-based reliable H_∞ control problem for the switched system (28) is stated as follows: Given a constant $\gamma > 0$, for actuator failures corresponding to any $w_i \subseteq \Theta_i$, find observer-based output feedback controllers $u_i = K_i \hat{x}$ for all subsystems associated with a class of switching signals such that

(i) System (30) is exponentially stable when $\omega(t) = 0$.

(ii) System (30) has finite L_2-gain γ from the disturbance input $\omega(t)$ to the controlled output $z(t)$, i.e., $\int_{t_0}^{T} z^T(t) z(t)\,\mathrm{d}t \le \gamma^2 \int_{t_0}^{T} \omega^T(t)\omega(t)\,\mathrm{d}t + \upsilon(x(t_0))$ holds for all $T > 0$, where $x(t_0)$ is the initial state, $t_0 = 0$ is the initial time, $\upsilon(\cdot)$ is some real-valued function.

The following results are used to develop the main result.

Lemma 2: Given positive constant γ, positive constants λ_i for $i \in M_p$ and negative constants λ_i for $i \in \tilde{M}_p$, if

(a) There exist matrices $P_i > 0$ such that

$$\text{Block-diag}\left\{\Psi_i(\alpha_1^i), \Psi_i(\alpha_2^i), \ldots, \Psi_i(\alpha_{2^{q_i n}}^i)\right\} < 0 \tag{31}$$

hold for $\forall\, i = M, j = 1, \ldots, 2^{q_i n}, \alpha_j^i \in \mathscr{V}_{q_i, n}^i$, where

$$\begin{aligned} \Psi_i(\alpha_j^i) = {} & \mathscr{A}_i^T(\alpha_j^i) P_i + P_i \mathscr{A}_i(\alpha_j^i) - P_i B_{\bar{\Theta}_i} B_{\bar{\Theta}_i}^T P_i + P_i L_i L_i^T P_i \\ & + 2\gamma^{-2} P_i L_i W_{2i} W_{2i}^T L_i^T P_i + 2E_i^T E_i + 2\lambda_i P_i. \end{aligned}$$

(b) There exists a matrix $S > 0$ and matrices R_i such that

$$\begin{aligned} \text{Block-diag}\{ & \Gamma_i(\alpha_1^i, \beta_1^i), \ldots, \Gamma_i(\alpha_{2^{q_i n}}^i, \beta_1^i), \Gamma_i(\alpha_1^i, \beta_2^i), \\ & \ldots, \Gamma_i(\alpha_{2^{q_i n}}^i, \beta_{2^{p_i n}}^i)\} < 0, \end{aligned} \tag{32}$$

hold for $\forall\ i = M, j = 1, \ldots, 2^{q_i n}, k = 1, \ldots, 2^{p_i n}$, $\alpha_j^i \in \mathscr{V}_{q_i, n}^i, \beta_k^i \in \mathscr{W}_{p_i, n}^i$, where

$$\Gamma_i(\alpha_j^i, \beta_k^i) = \begin{bmatrix} \Xi_i(\alpha_j^i, \beta_k^i) & \mathscr{G}_i^T(\beta_k^i) & R_i^T W_{2i} - S W_{1i} \\ \mathscr{G}_i(\beta_k^i) & -I & 0 \\ W_{2i}^T R_i - W_{1i}^T S & 0 & -\frac{1}{2}\gamma^2 I \end{bmatrix}$$
$$\begin{aligned} \Xi_i(\alpha_j^i, \beta_k^i) = {} & \mathscr{A}_i^T(\alpha_j^i) S - \mathscr{G}_i^T(\beta_k^i) R_i + S \mathscr{A}_i(\alpha_j^i) - R_i^T \mathscr{G}_i(\beta_k^i) \\ & + 2E_i^T E_i + 2\lambda_i S. \end{aligned}$$

Then, there exist feedback controllers $u_i = K_i \hat{x}$ such that

$$V_i(t) \le e^{-2\lambda_i(t-t_0)} V_i(t_0) - \int_{t_0}^{t} e^{-2\lambda_i(t-\tau)} \Gamma(\tau)\,\mathrm{d}\tau \tag{33}$$

hold along the trajectory of system (30) for actuator failures corresponding to any $w_i \subseteq \Theta_i$, where $\Gamma(\tau) = z^T(\tau) z(\tau) - \gamma^2 \omega^T(\tau)\omega(\tau)$, the controller and observer gain matrices are $K_i = -B_i^T P_i$ and $L_i = S^{-1} R_i^T$, $i \in M$.

Proof: The derivative of $V_i = \tilde{x}^T \tilde{P}_i \tilde{x}$ in (15) along the trajectory of the corresponding subsystem of (30) satisfies

$$\begin{aligned} & \dot{V}_i + z^T z - \gamma^2 \omega^T \omega \\ & \le \hat{x}^T \big(\mathscr{A}_i^T(h^i) P_i + P_i \mathscr{A}_i(h^i) - P_i B_{\bar{\Theta}_i} B_{\bar{\Theta}_i}^T P_i + P_i L_i L_i^T P_i \\ & \quad + 2\gamma^{-2} P_i L_i W_{2i} W_{2i}^T L_i^T P_i + 2E_i^T E_i + 2\lambda_i P_i \big) \hat{x} \\ & \quad + e^T \big(\mathscr{A}_i^T(h^i) S - \mathscr{G}_i^T(\rho^i) R_i + S \mathscr{A}_i(h^i) - R_i^T \mathscr{G}_i(\rho^i) \\ & \quad + 2\gamma^{-2} (R_i^T W_{2i} - S W_{1i})(W_{2i}^T R_i - W_{1i}^T S) \\ & \quad + \mathscr{G}_i^T(\rho^i) \mathscr{G}_i(\rho^i) + 2E_i^T E_i + 2\lambda_i S \big) e. \end{aligned} \tag{34}$$

From a similar proof in Lemma 1, (31) and (32) implies that

$$\dot{V}_i + 2\lambda_i V_i + z^T z - \gamma^2 \omega^T \omega < 0. \tag{35}$$

The differential inequality theory and (35) gives (33). ■

Sufficient condition guaranteeing the solvability of the observer-based reliable H_∞ control problem of (28) is proposed via Lemma 2 in the following theorem.

Theorem 2: For given positive constant γ, positive constants λ_i for $i \in M_p$ and negative constants λ_i for $i \in \tilde{M}_p$, suppose that there exist positive definite matrices P_i and S, matrices R_i such that (31) and (32) hold, then for actuator failures corresponding to any $w_i \subseteq \Theta_i$, the observer-based reliable H_∞ control problem of system (28) is solved under the observer-based output feedback controllers $u_\sigma = K_\sigma \hat{x}$ for any switching signal $\sigma(t)$ with average dwell time (5) and switching condition (S) satisfying (20), where $\mu \ge 1$ satisfies

(22) and (23), the controller and observer gain matrices are $K_i = -B_i^T P_i$ and $L_i = S^{-1} R_i^T$, $i \in M$.

Proof: When $\omega(t) = 0$, (12) and (13) imply (31) and (32) and thus exponential stabilizability follows from Theorem 1.

Next, we show that the closed-loop system has finite L_2-gain. It can be easily seen from Lemma 2 that for any $t \in [t_k, t_{k+1})$ $(0 \le k \le N_\sigma(t_0, t))$, the piecewise Lyapunov function candidate (15) satisfies

$$V(t) = V_{\sigma(t)}(t) \le \begin{cases} e^{-2\lambda^-(t-t_k)} V_{\sigma(t_k)}(t_k) - \int_{t_k}^{t} e^{-2\lambda^-(t-\tau)} \Gamma(\tau)\,\mathrm{d}\tau, i \in M_p, \\ e^{2\lambda^+(t-t_k)} V_{\sigma(t_k)}(t_k) - \int_{t_k}^{t} e^{2\lambda^+(t-\tau)} \Gamma(\tau)\,\mathrm{d}\tau, i \in \tilde{M}_p. \end{cases}$$

Since $V_{\sigma(t_k)}(t_k) \le \mu V_{\sigma(t_k^-)}(t_k^-)$ is true from (24) at the switching point t_k, we obtain by induction that

$$\begin{aligned} V(t) &\le e^{2\lambda^+ T^+(t_k,t) - 2\lambda^- T^-(t_k,t)} V_{\sigma(t_k)}(t_k) \\ &\quad - \int_{t_k}^{t} e^{2\lambda^+ T^+(\tau,t) - 2\lambda^- T^-(\tau,t)} \Gamma(\tau)\,\mathrm{d}\tau \\ &\le e^{2\lambda^+ T^+(t_k,t) - 2\lambda^- T^-(t_k,t)} \mu V_{\sigma(t_k^-)}(t_k^-) \\ &\quad - \int_{t_k}^{t} e^{2\lambda^+ T^+(\tau,t) - 2\lambda^- T^-(\tau,t)} \Gamma(\tau)\,\mathrm{d}\tau \\ &\le e^{2\lambda^+ T^+(t_{k-1},t) - 2\lambda^- T^-(t_{k-1},t)} \mu V_{\sigma(t_{k-1})}(t_{k-1}) \\ &\quad - e^{2\lambda^+ T^+(t_k,t) - 2\lambda^- T^-(t_k,t)} \mu \\ &\quad \cdot \int_{t_{k-1}}^{t_k} e^{2\lambda^+ T^+(\tau,t) - 2\lambda^- T^-(\tau,t)} \Gamma(\tau)\,\mathrm{d}\tau \\ &\quad - \int_{t_k}^{t} e^{2\lambda^+ T^+(\tau,t) - 2\lambda^- T^-(\tau,t)} \Gamma(\tau)\,\mathrm{d}\tau \\ &\le \cdots \le e^{2\lambda^+ T^+(t_0,t) - 2\lambda^- T^-(t_0,t)} \mu^{N_\sigma(t_0,t)} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} \mu^{N_\sigma(\tau,t)} e^{2\lambda^+ T^+(\tau,t) - 2\lambda^- T^-(\tau,t)} \Gamma(\tau)\,\mathrm{d}\tau \\ &= e^{2\lambda^+ T^+(t_0,t) - 2\lambda^- T^-(t_0,t) + N_\sigma(t_0,t)\ln\mu} V_{\sigma(t_0)}(t_0) \\ &\quad - \int_{t_0}^{t} e^{2\lambda^+ T^+(\tau,t) - 2\lambda^- T^-(\tau,t) + N_\sigma(\tau,t)\ln\mu} \Gamma(\tau)\,\mathrm{d}\tau. \end{aligned} \tag{36}$$

For the trivial case of $\mu = 1$, (36) gives that $V(t) \le e^{-2\lambda^*(t-t_0)} V_{\sigma(t_0)}(t_0) - \int_{t_0}^{t} e^{-2\lambda^*(t-\tau)} \Gamma(\tau)\,\mathrm{d}\tau$, which implies

$$\int_{t_0}^{t} e^{-2\lambda^*(t-\tau)} z^T(\tau) z(\tau)\,\mathrm{d}\tau \le e^{-2\lambda^*(t-t_0)} V_{\sigma(t_0)}(t_0) + \gamma^2 \int_{t_0}^{t} e^{-2\lambda^*(t-\tau)} \omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau.$$

Integrating both sides of this inequality from $t = 0$ to ∞ results in $\int_{t_0}^{\infty} z^T(\tau) z(\tau)\,\mathrm{d}\tau \le \gamma^2 \int_{t_0}^{\infty} \omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau + V_{\sigma(t_0)}(t_0)$ for $\forall\, \omega(t) \in L_2[0,\infty)$, which means that the closed-loop system (30) has finite L_2-gain under switching law (S) without considering the average dwell time.

For the case $\mu > 1$, combining the average dwell time (5) and switching condition (S) satisfying (20), applying (36) leads to $V(t) \le e^{-2\lambda(t-t_0)} V_{\sigma(t_0)}(t_0) - \int_{t_0}^{t} e^{-2\lambda(t-\tau)} \Gamma(\tau)\,\mathrm{d}\tau$, which implies

$$\int_{t_0}^{t} e^{-2\lambda(t-\tau)} z^T(\tau) z(\tau)\,\mathrm{d}\tau \le e^{-2\lambda(t-t_0)} V_{\sigma(t_0)}(t_0) + \gamma^2 \int_{t_0}^{t} e^{-2\lambda(t-\tau)} \omega^T(\tau)\omega(\tau)\,\mathrm{d}\tau.$$

Integrating both sides of this inequality from $t = 0$ to ∞ yields that system (30) has finite L_2-gain under average dwell time (5) and switching condition (S) satisfying (20). ■

V. EXAMPLE

Consider the switched system (28) with $M = \{1, 2\}$ and

$$A_1 = \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -1 \\ -1 & 1 & -0.9 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -0.8 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.8 & -2 \\ 1.3 & 1 \\ 1.5 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} -2 & 1 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}, \Theta_1 = \{2\}, \Theta_2 = \{1\},$$

$$B_{\bar{\Theta}_1} = \begin{bmatrix} -0.8 & 0 \\ 1.3 & 0 \\ 1.5 & 0 \end{bmatrix}, B_{\bar{\Theta}_2} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 0 & 2 \end{bmatrix}, D_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -0.4 \\ -0.3 \\ -0.4 \end{bmatrix}, W_{11} = \begin{bmatrix} -0.2 \\ -0.2 \\ -0.3 \end{bmatrix}, W_{12} = \begin{bmatrix} -0.4 \\ 0.2 \\ -0.2 \end{bmatrix}, W_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$W_{22} = \begin{bmatrix} 1 & 1.8 \end{bmatrix}^T, E_1 = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}, E_2 = \begin{bmatrix} -0.2 & 1 & -1 \end{bmatrix},$$

$$f_1 = \begin{bmatrix} 0.4\sin x_1 \\ 0.2\sin x_3 \end{bmatrix}, f_2 = 0.6\sin x_2,$$

$$g_1 = \begin{bmatrix} x_1 - 0.5e^{-t} x_2 + x_3 \\ -x_1 + x_2 + x_3 \end{bmatrix}, g_2 = \begin{bmatrix} x_1 + x_3 \\ -x_2 + x_3 \end{bmatrix}.$$

Then we have the sets of vertices $\mathscr{V}_{2,3}^1 = \{\alpha_1^1, \alpha_2^1, \alpha_3^1, \alpha_4^1\}$, $\mathscr{V}_{1,3}^2 = \{\alpha_1^2, \alpha_2^2\}$, $\mathscr{W}_{2,3}^1 = \{\beta_1^1, \beta_2^1\}$, $\mathscr{W}_{2,3}^2 = \{\beta_1^2\}$, where

$$\alpha_1^1 = \begin{bmatrix} 0.4 & 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}, \alpha_2^1 = \begin{bmatrix} 0.4 & 0 & 0 & 0 & 0 & -0.2 \end{bmatrix},$$

$$\alpha_3^1 = \begin{bmatrix} -0.4 & 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}, \alpha_4^1 = \begin{bmatrix} -0.4 & 0 & 0 & 0 & 0 & -0.2 \end{bmatrix},$$

$$\alpha_1^2 = \begin{bmatrix} 0 & 0.6 & 0 \end{bmatrix}, \alpha_2^2 = \begin{bmatrix} 0 & -0.6 & 0 \end{bmatrix},$$

$$\beta_1^1 = \begin{bmatrix} 1 & -0.5 & 1 & -1 & 1 & 1 \end{bmatrix}, \beta_2^1 = \begin{bmatrix} 1 & 0 & 1 & -1 & 1 & 1 \end{bmatrix},$$

$$\beta_1^2 = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}.$$

Take $\lambda_1 = \lambda^- = -1.4, \lambda_2 = \lambda^+ = 0.1$ and the disturbance attenuation level is given by $\gamma = 1$. Solving (31) and (32), we get the following positive definite matrices

$$P_1 = \begin{bmatrix} 0.5778 & -0.3386 & 0.1550 \\ -0.3386 & 0.9049 & 0.0660 \\ 0.1550 & 0.0660 & 0.5014 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.6845 & 0.1515 & -0.0112 \\ 0.1515 & 0.9529 & 0.4324 \\ -0.0112 & 0.4324 & 0.6802 \end{bmatrix},$$

$$S = \begin{bmatrix} 9.6739 & -3.2100 & 3.0320 \\ -3.2100 & 34.6600 & -15.2359 \\ 3.0320 & -15.2359 & 17.1998 \end{bmatrix}.$$

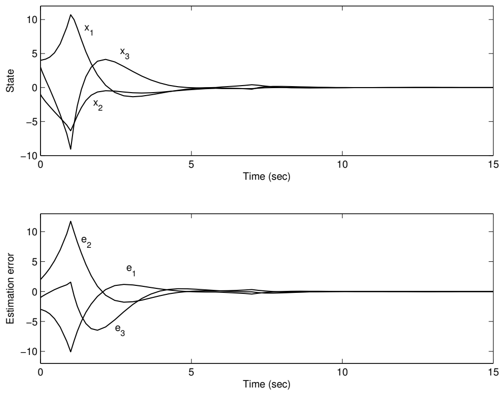

Fig. 1. The state response of the system (28) and system (29).

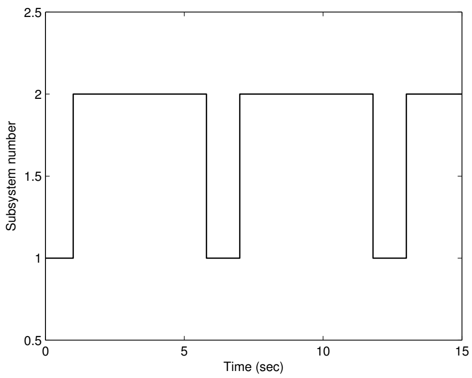

Fig. 2. The switching signal.

Then, we can obtain that $\mu = \sup_{i,j \in M} \frac{\bar{\lambda}(P_i)}{\underline{\lambda}(P_j)} = 5.0998$. Choosing $\lambda = 0.1$, $\lambda^* = 0.8$,

$$\frac{T^-(0,t)}{T^+(0,t)} \geq 1.5,\ \tau > \tau_a^* \geq 1.1637$$

hold under the average dwell time scheme with (6) and (20). The observer-based reliable H_∞ control problem of system (28) is solved under this switching law, where gain matrices

$$K_1 = \begin{bmatrix} 0.6700 & -1.5463 & -0.7139 \\ 1.3393 & -1.6482 & -0.2574 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 1.2063 & -0.2176 & 0.2254 \\ -0.5106 & -0.0633 & -0.9168 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.1364 & -0.5063 & -1.2737 \\ -0.0495 & 0.3205 & 0.9840 \end{bmatrix}^T,$$

$$L_2 = \begin{bmatrix} 0.1059 & 0.6255 & 0.2088 \\ -0.2816 & -0.2367 & -0.2280 \end{bmatrix}^T$$

are given by applying the conditions of Theorem 2. Fig.1 shows the state and estimation error trajectories of the closed-loop system with the initial state $x(0) = \begin{bmatrix} 4 & -1 & 3 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^T$. The corresponding switching law is given by Fig.2.

VI. CONCLUSION

The observer-based reliable stabilization control problem has been solved for switched Lipschitz nonlinear systems consisting of stabilizable and unstabilizable subsystems with actuators failure. Attention is particularly concentrated on actuators suffering "destabilizing failures". In terms of average dwell time approach, we have design hybrid observer-based output feedback controllers and a class of switching signals under which the switched system is exponentially stabilizable for all admissible actuator failures. Moreover, as an extension, sufficient condition has been also presented to solve the observer-based reliable H_∞ control problem for switched Lipschitz nonlinear systems.

REFERENCES

[1] D. Liberzon, *Switching in Systems and Control*, Birkhauser, Boston, 2003.

[2] Z.D. Sun and S.S. Ge, "Analysis and synthesis of switched linear control systems", *Automatica*, vol. 41, 2005, pp. 181-195.

[3] R.A. Decarlo, M.S. Branicky, S. Pettersson and B. Lennartson, "Perspectives and results on the stability and stabilization of hybrid systems", *Proceedings of IEEE*, vol. 88, 2000, pp. 1069-1082.

[4] J. Zhao and D.J. Hill, "Passivity and stability of switched systems: a multiple storage function method", *Systems and Control Letters*, vol. 57, 2008, pp. 158-164.

[5] D.Z. Cheng, L. Guo, Y.D. Lin and Y. Wang, "Stabilization of switched linear systems", *IEEE Transactions on Automatic Control*, vol. 50, 2005, pp. 661-666.

[6] G.M. Xie and L. Wang, "Controllability and stabilizability of switched linear systems", *Systems and Control Letters*, vol. 48, 2003, pp. 135-155.

[7] A.S. Morse, "Supervisory control of families of linear set-point controllers-Part I: exact matching", *IEEE Transactions on Automatic Control*, vol. 41, 2004, pp. 960-966.

[8] J.P. Hespanha and A.S. Morse, "Stability of switched systems with average dwell-time", *Proceeding of the 38th IEEE Conference on Decision and Control*, Phoenix, USA, 1999, pp. 2655-2660.

[9] G.S. Zhai, B. Hu, K. Yasuda and A.N. Michel, "Piecewise Lyapunov functions for switched systems with average dwell time", *Asian Journal of Control*, vol. 2, 2000, pp. 192-197.

[10] G.S. Zhai, B. Hu, K. Yasuda and A.N. Michel, "Disturbance attenuation properties of time-controlled switched systems", *Journal of the Franklin Institute*, vol. 338, 2001, pp. 765-779.

[11] Q. Zhao and J. Jiang, "Reliable state feedback control system design against actuator failures", *Automatica*, vol. 34, 1998, pp. 1267-1272.

[12] G.H. Yang, J. Lam and J.L. Wang, "Reliable H_∞ control for affine nonlinear systems", *IEEE Transactions on Automatic Control*, vol. 43, 1998, pp. 1112-1117.

[13] R. Wang, J. Zhao, G.M. Dimirovski and G.P. Liu, "Output feedback control for uncertain linear systems with faulty actuators based on a switching method", *International Journal of Robust and Nonlinear Control*, DOI: 10.1002/rnc.1379, 2008.

[14] A. Zemouche, M. Boutayeb, and G.I. Bara, "Observers for a class of Lipschitz systems with extension to H_∞ performance analysis", *Systems and Control Letters*, vol. 57, 2008, pp. 18-27.

[15] M.A. Demetriou, "Adaptive reorganization of switched systems with faulty actuators", *Proceeding of the 40th IEEE Conference on Decision and Control*, Orlando, FL, 2001, pp. 1879-1884.

[16] R. Wang and J. Zhao, "Robust fault-tolerant control for a class of switched nonlinear systems in lower triangular form", *Asian Journal of Control*, vol. 9, 2007, pp. 70-74.

[17] R. Wang, M. Liu and J. Zhao, "Robust H_∞ control for a class of switched nonlinear systems with actuator failures", *Nonlinear Analysis: Hybrid Systems*, vol. 1, 2007, pp. 317-325.

[18] S. Boyd and L. Vandenberghe, *Convex optimization with engineering applications*, Lecture Notes, Stanford University, Stanford, 2001.