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Adaptive Control of a Class of Strict-Feedback Discrete-Time Nonlinear Systems with Unknown Control Gains and Preceded by Hysteresis

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***Abstract*— In this paper, adaptive control is studied for a class of discrete-time nonlinear systems in strict-feedback form. The systems are with unknown control gains and are proceeded by hysteresis. Prandtl-Ishlinskii (PI) model is used to describe the hysteresis. The control design is based on the predicted future states and discrete Nussbaum gain is employed in the parameter update law to deal with the unknown control directions. All the closed-loop signals are guaranteed to be bounded and the output tracking error is made to be within a neighborhood around zero ultimately. The effectiveness of the proposed control law is demonstrated in the simulation.**

I. Introduction

Recently, adaptive control of discrete-time nonlinear systems in the lower triangular form have attracted much research interest. In [1], backstepping in discrete-time was developed for a class of parametric-strict-feedback systems based on the proposed coordinate transformation. This result was further explored in [2] using parameter projection for robustness. A novel parameter estimation for parameter-strict-feedback systems was proposed in [3], in which the estimation law guaranteed the convergence of estimates to the real values in finite steps when the system was in the absence of any disturbance or uncertainties. However, it has been pointed out in [4] that these results on parameter-strict-feedback systems are not directly applicable to more general strict-feedback systems with unknown control gains. When the signs of control gains are unknown, the adaptive control of the strict-feedback systems becomes difficult since the direction along which the control operates cannot be determined. Combining future state/output prediction method with discrete Nussbaum gain to deal with unknown control direction, adaptive control has been designed for strict-feedback systems with unknown control gains [4], [5]. It is worth to mention that, lower triangular systems with unknown system functions have also been intensively studied using adaptive neural network control in [6], [7], [8], [9], where prediction function based system transformation have been utilized to avoid noncausal problem in control design. Using neural network approximation approach, adaptive single-input and single-output (SISO) parameter-strict-feedback systems with unknown system functions was studied in [6], where the system transformation was carried out before applying backstepping design. This method was further developed in [7], [8] for multi-input and multi-output (MIMO) systems.

On the other hand, control design for system with hysteresis input has received much attention in the adaptive control literature, because hysteresis phenomenon occurs in a wide range of physical systems and devices. But it is not a easy task to control systems with input of hysteresis nonlinearities [10], [11]. The existence of hysteresis in input can result in undesirable inaccuracies or oscillations, which damages the closed-loop control system's performance and can even lead to instability [10]. To control systems with hysteresis, an inverse operator was constructed to eliminate the effects of the hysteresis in [10]. In addition, various models have been proposed to describe the hysteresis, such as Preisach model [12], Prandtl-Ishlinskii (PI) model [13], and Krasnosel'skii-Pokrovskii model [14]. In recent years, PI model has been extensively used to study in the adaptive control literature [11], [13], [15], [16], in which the control directions are assumed to be known. One recent attempt to control system in continuous-time with unknown control directions using PI model has been made in [17]. However, due to some inherent difficulties in discrete-time models many controls designed for continuous-time systems may be not suitable for discrete-time systems, and in most cases, adaptive control design for discrete-time systems is much more difficult.

In this paper, we are going to study adaptive control of a class of discrete-time parameter-strict-feedback nonlinear systems with unknown control directions and proceeded by hysteresis. PI model is used to described hysteresis and discrete Nussbaum gain is exploited to deal with unknown control gains.

The main contributions of the paper lie in:

(i) Adaptive control is developed for a class of strict-feedback systems with unknown control gains and proceeded by hysteresis.
(ii) To tackle the difficulty caused by hysteresis input, Prandtl-Ishlinskii (PI) model is exploited in the adaptive control design.
(iii) Combined with deadzone method, discrete Nussbaum gain is utilized in the presence of external disturbance and hysteresis.

Throughout this paper, the following notations are used.

- $\|\cdot\|$ denotes the Euclidean norm of vectors and induced norm of matrices.
- $(\hat{\cdot})$ and $(\tilde{\cdot})$ denote the estimate of parameters and estimation error, respectively.
- N^+ denotes the set of all nonnegative integers.

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II. Problem Formulation and Preliminaries

A. System Representation

Consider a class of strict-feedback nonlinear discrete-time systems in the following form:

$$\begin{cases} \xi_1(k+1) = \Theta_1^T \Phi_1(\bar{\xi}_1(k)) + g_1 \xi_2(k) \\ \xi_2(k+1) = \Theta_2^T \Phi_2(\bar{\xi}_2(k)) + g_2 \xi_3(k) \\ \qquad \vdots \\ \xi_n(k+1) = \Theta_n^T \Phi_n(\bar{\xi}_n(k)) + g_n u(k) + d(k) \\ u(k) = H[v](k) \\ y(k) = \xi_1(k) \end{cases} \tag{1}$$

where $\bar{\xi}_i(k) = [\xi_1(k), \xi_2(k), \ldots, \xi_i(k)]^T$ are system states, $\Theta_i \in R^{p_i}$ and $g_i \in R$, $i = 1, 2, \ldots, n$, are unknown system parameters (p_i's are positive integers), $\Phi_i(\bar{\xi}_i(k)) : R^i \to R^{p_i}$ are known vector-valued functions. The control objective is to make the output $y(k)$ track a bounded reference trajectory $y_d(k)$ and to guarantee the boundedness of all the closed-loop signals.

The hysteresis is denoted by the operator $u(k) = H[v](k)$, where $v(k)$ is the input and $u(k)$ is the output of the hysteresis, which is represented by discrete-time Prandtl-Ishlinskii (PI) model as follows [16]:

$$\begin{aligned} u(k) &= \int_0^\infty p(r) E_r[v](k) dr \\ E_r(0) &= e_r(v(0) - u(-1)) \\ E_r(k) &= e_r[v(k) - v(k_i) + E_r[v](k_i)] \\ e_r(v) &= \min(r, \max(-r, v)) \end{aligned} \tag{2}$$

where $p(r)$ is an unknown density function satisfying $p(r) \geq 0$ with $\int_0^\infty rp(r)dr < \infty$, and $E_r(\cdot)$ is called as stop operator. When the value r is large enough, the density function $p(r)$ will vanishes, i.e., there exists a constant R such that $p(r) = 0$, $\forall r > R$, and thus the integral $\int_0^\infty p(r)E_r[v](k)dr$ is replaced by $\int_0^R p(r)E_r[v](k)dr$ in the sequel.

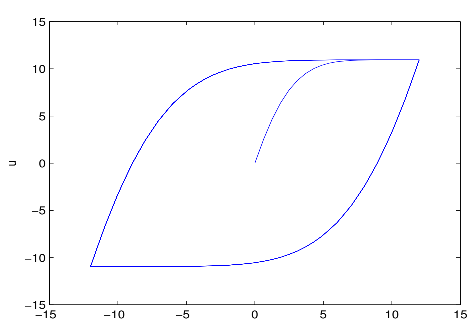

Fig. 1. Hysteresis curve give by the PI model in (2)

Figure 1 illustrates the input (v) and output (u) relationship of the PI model in (2). The density function used is $p(r) = e^{-0.07(r-1)^2}$ with $R = 10$. The input is chosen as $v(k) = 12.0 \sin(\frac{\pi}{60} k)$ with $k = 1, 2, \ldots, 360$.

Assumption 2.1: The system functions $\Phi_i(\bar{\xi}_i(k))$ are Lipschitz functions, i.e., $\|\Phi_i(\varepsilon_1) - \Phi_i(\varepsilon_2)\| \leq L_i \|\varepsilon_1 - \varepsilon_2\|$, $\forall \varepsilon_1, \varepsilon_2 \in R^i$, $1 \leq i \leq n$ and L_i are the Lipschitz coefficients. The control gains $g_i \neq 0$. In addition, the external disturbance is bounded by a constant $\bar{d}$, i.e., $|d_0(k)| \leq \bar{d}$.

B. Useful Definitions and Lemmas

Definition 2.1: [18] Let $x_1(k)$ and $x_2(k)$ be two discrete-time scalar or vector signals, $\forall k \in N^+$.

- We denote $x_1(k) = O[x_2(k)]$, if there exist positive constants m_1, m_2 and k_0 such that $\|x_1(k)\| \leq m_1 \max_{k' \leq k} \|x_2(k')\| + m_2, \quad \forall k > k_0$.
- We denote $x_1(k) = o[x_2(k)]$, if there exists a discrete-time function $\alpha(k)$ satisfying $\lim_{k\to\infty} \alpha(k) \to 0$ and a constant k_0 such that $\|x_1(k)\| \leq \alpha(k) \max_{k' \leq k} \|x_2(k')\|$, $\forall k > k_0$.
- We denote $x_1(k) \sim x_2(k)$ if they satisfy $x_1(k) = O[x_2(k)]$ and $x_2(k) = O[x_1(k)]$.

Lemma 2.1: [9] Under Assumptions 2.1, the states and input of system (1) satisfy

$$\begin{aligned} \bar{\xi}_i(k) &= O[y(k+i-1)], \ i = 1, 2, \ldots, n-1 \\ u(k) &= O[y(k+n)] \end{aligned} \tag{3}$$

Definition 2.2: [19] Consider a discrete nonlinear function $N(x(k))$ defined on a sequence $x(k)$ with $x_s(k) = \sup_{k' \leq k}\{x(k')\}$. $N(x(k))$ is a discrete Nussbaum gain if and only if it satisfies the following two properties:

(i) If $x_s(k)$ increases without bound, then

$$\sup_{x_s(k) \geq \delta_0} \frac{S_N(x(k))}{x_s(k)} = +\infty, \inf_{x_s(k) \geq \delta_0} \frac{S_N(x(k))}{x_s(k)} = -\infty$$

(ii) If $x_s(k) \leq \delta_1$, then $|S_N(x(k))| \leq \delta_2$ with some positive constants δ_1 and δ_2.

where $S_N(x(k))$ is defined with $\Delta x(k) = x(k+1) - x(k)$ as follows:

$$S_N(x(k)) = \sum_{k'=0}^{k} N(x(k')) \Delta x(k') \tag{4}$$

In this paper, for adaptive control of system (1), the discrete Nussbaum gain $N(x(k))$ proposed in [20] is exploited, which requires the sequence $x(k)$ to satisfy

$$x(k) \geq 0, \quad \forall k, |\Delta x(k)| = |x(k+1) - x(k)| \leq \delta_0 \tag{5}$$

Lemma 2.2: [4] Let $V(k)$ be a positive definite function defined $\forall k$, $N(x(k))$ be a discrete Nussbaum gain, and θ be a nonzero constant. If the following inequality holds, $\forall k$

$$V(k) \leq \sum_{k'=k_1}^{k} (c_1 + \theta N(x(k'))) \Delta x(k') + c_2 x(k) + c_3 \tag{6}$$

where c_1, c_2 and c_3 are some constants, k_1 is a positive integer, then $V(k)$, $x(k)$ and $\sum_{k'=k_1}^{k} (c_1 + \theta N(x(k'))) \Delta x(k') + c_2 x(k) + c_3$ must be bounded, $\forall k$.

C. Future States Prediction

Future states prediction proposed in [4] is employed here to facilitate the control design. Define $\bar{\hat{\Theta}}_i(k) = [\hat{\Theta}_i^T(k), \hat{g}_i(k)]^T \in R^{p_j+1}$ and $\bar{\tilde{\Theta}}_i(k) = [\tilde{\Theta}_i^T(k), \tilde{g}_i(k)]^T$, where $\hat{\Theta}_i(k)$ and $\hat{g}_i(k)$ denote the estimate of Θ_i and g_i at the k-th, respectively, and $\tilde{\Theta}_i(k) = \hat{\Theta}_i(k) - \Theta_i$ and $\tilde{g}_i(k) = \hat{g}_i(k) - g_i$ are estimate errors.

Then, the prediction laws of the one-step ahead future states $\xi_i(k+1)$, are given as

$$\hat{\xi}_i(k+1|k) = \bar{\hat{\Theta}}_i^T(k-n+2)\Psi_i(k),\ i = 1, 2, \ldots, n-1$$

where $\Psi_i(k) = [\Phi_i^T(\bar{\xi}_i(k)), \xi_{i+1}(k)]^T \in R^{p_i+1}$.

The j-step ahead future states $\xi_i(k+j)$, $j = 2, 3, \ldots, n-1$, are predicted as

$$\hat{\xi}_i(k+j|k) = \bar{\hat{\Theta}}_i^T(k-n+j+1)\hat{\Psi}_i(k+j-1|k) \tag{7}$$

for $i = 1, 2, \ldots, n-j$, where

$$\hat{\Psi}_i(k+j) = [\Phi_i^T(\bar{\hat{\xi}}_i(k+j-1|k)),\quad \hat{\xi}_{i+1}(k+j-1|k)]^T$$

$$\bar{\hat{\xi}}_i(k+j-1|k) = [\hat{\xi}_1(k+j-1|k), \ldots, \hat{\xi}_i(k+j-1|k)]^T.$$

The parameter estimates are obtained by

$$\bar{\hat{\Theta}}_i(k+1) = \bar{\hat{\Theta}}_i(k-n+2) - \frac{\tilde{\xi}_i(k+1|k)\Psi_i(k)}{1+\Psi_i^T(k)\Psi_i(k)} \tag{8}$$

for $i = 1, 2, \ldots, n-1$.

Remark 2.1: The parameter update law (8) is presented at the $(k+1)$-th step when $\bar{\xi}_n(k+1)$ are all available. The control input $u(k)$ is designed at the k-th step and only depends on $\hat{\Theta}_i(j)$, $j \leq k$.

Lemma 2.3: [4] The parameter estimates $\bar{\hat{\Theta}}_i(k)$ ($i = 1, 2, \ldots, n-1$) in (8) are bounded and the estimate errors satisfy $\bar{\tilde{\xi}}_i(k+n-i|k) = o[O[y(k+n-1)]]$ where

$$\bar{\tilde{\xi}}_i(k+n-i|k) = \bar{\hat{\xi}}_i(k+n-i|k) - \bar{\xi}_i(k+n-i)$$

$$\bar{\hat{\xi}}_i(k+n-i|k) = [\hat{\xi}_1(k+n-1|k), \ldots, \hat{\xi}_i(k+n-i|k)] \tag{9}$$

III. Adaptive Control Design

A. System Transformation

Let us rewrite system (1) as

$$\begin{cases} \xi_1(k+n) = \Theta_1^T\Phi_1(\bar{\xi}_1(k+n-1)) + g_1\xi_2(k+n-1) \\ \qquad\vdots \\ \xi_n(k+1) = \Theta_n^T\Phi_n(\bar{\xi}_n(k)) + g_n u(k) + d(k) \\ y(k) = \xi_1(k) \end{cases}$$

and then we combine the n equations above together by iterative substitution and consider hysteresis behavior (2), thus we obtain

$$\begin{aligned} y(k+n) \;=\; & \Theta_f^T\Phi(k+n-1) \\ & + g\int_0^R p(r)E_r[v](k)dr + d_0(k) \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\Theta_f = [\Theta_{f1}^T, \ldots, \Theta_{fn}^T]^T \in R^p,\ \Theta_{f_1} = \Theta_1 \\ &\Theta_{f_i} = \Theta_i\prod_{j=1}^{i-1} g_j,\ i = 2, 3, \ldots, n,\ \underline{g} = \prod_{j=1}^{n}\underline{g}_j \\ &g = \prod_{j=1}^{n} g_j,\ d_o(k) = \frac{g}{g_n}d(k) \\ &\Phi(k+n-1) = [\Phi_1^T(\bar{\xi}_1(k+n-1)), \\ &\qquad \Phi_2^T(\bar{\xi}_2(k+n-2)), \ldots, \Phi_n^T(\bar{\xi}_n(k))]^T \in R^p \end{aligned} \tag{11}$$

B. Control and Parameter Estimation

Using the predicted future states, the future states dependent function $\Phi(k+n-1)$ in (11) can be estimated as

$$\hat{\Phi}(k+n-1|k) = [\Phi_1^T(\hat{\xi}_1(k+n-1|k))\ldots, \Phi_n^T(\bar{\xi}_n(k))]^T$$

where $\bar{\hat{\xi}}_i(k+n-i|k)$, $i = 1, 2, \ldots, n-1$, are defined in (9).

Denote $\hat{\Theta}_{fg}(k)$ and $\hat{g}_I(k)$ as the estimates of $g^{-1}\Theta_f$ and g^{-1}, respectively. Using the predicted function $\hat{\Phi}(k+n-1|k)$, let us define

$$u'(k) = -\hat{\Theta}_{fg}^T(k)\hat{\Phi}(k+n-1|k) + \hat{g}_I(k)y_d(k+n) \tag{12}$$

Let $[v_{\min},\ v_{\max}]$ be the practical input range to the hysteresis operator, which is a strict subset of $[-R,\ R]$, and the saturation output of $\int_0^R \hat{p}(r,k)E_r[v^*](k)dr$ be $\hat{u}'_{\text{sat}}(k)$, in which these notations are borrowed from [16]. $v^*(k)$ is derived as follows [16]. If $u'(k) < -\hat{u}'_{\text{sat}}(k)$, then $v^*(k) = v_{\min}$; if $u'(k) > \hat{u}'_{\text{sat}}(k)$, then $v^*(k) = v_{\max}$; otherwise, following the algorithm proposed in [16] (Section C), it can be obtained a $v^*(k)$ such that

$$\begin{aligned} \mu(k) \;&=\; \int_0^R \hat{p}(r,k)E_r[v^*](k)dr - u'(k) \\ |\mu(k)| \;&\leq\; \bar{\mu} \end{aligned} \tag{13}$$

where $\bar{\mu}$ is an assigned admissible error, $\hat{p}(r,k)$ is the estimate of $p(r)$ defined in (17) and it is nonnegative.

In this paper, the adaptive control input is considered as

$$v(k) = v^*(k) \tag{14}$$

Substituting the adaptive control (14) into the n-step predictor (10) and subtracting $y_d(k+n)$ on both hand sides, it follows that the error dynamics is given by

$$\begin{aligned} e(k+n) \;&=\; y(k+n) - y_d(k+n) \\ &=\; -g\tilde{\Theta}_{fg}^T(k)\Phi(k+n-1) + g\tilde{g}_I(k)y_d(k+n) \\ &\qquad -g\int_0^R \tilde{p}(r,k)E_r[v^*](k)dr \\ &\qquad -g\beta(k+n-1) + g\mu(k) + d_0(k) \end{aligned} \tag{15}$$

where $\tilde{\Theta}_{fg}(k)$, $\tilde{p}(r,k)$, $\mu(k)$ and $\beta(k)$ are defined as

$$\begin{aligned} \tilde{\Theta}_{fg}(k) \;&=\; \hat{\Theta}_{fg}(k) - \Theta_{fg} \\ \tilde{p}(r,k) \;&=\; \hat{p}(r,k) - p(r) \\ \mu(k) \;&=\; \int_0^R \hat{p}(r,k)E_r[v^*](k)dr - u'(k) \end{aligned} \tag{16}$$

$$\beta(k+n-1) = \hat{\Theta}_{fg}^T(k)[\hat{\Phi}(k+n-1|k) - \Phi(k+n-1)]$$

The parameters estimates in the control law are updated by the following adaptation law

$$\begin{aligned}
\epsilon(k) &= \frac{\gamma e(k) + N(x(k))\psi(k)\beta(k-1)}{G(k)} \\
\hat{\Theta}_{fg}(k) &= \hat{\Theta}_{fg}(k-n) + \gamma\frac{a(k)N(x(k))}{D(k)}\Phi(k-1)\epsilon(k) \\
\hat{g}_I(k) &= \hat{g}_I(k-n) - \gamma\frac{a(k)N(x(k))}{D(k)}y_d(k)\epsilon(k) \\
\hat{p}'(r,k) &= \hat{p}(r,k-n) \\
&\quad +\gamma\frac{a(k)N(x(k))}{D(k)}E_r[v^*](k-n)\epsilon(k) \\
\hat{p}(r,k) &= |\hat{p}'(r,k)| \qquad (17) \\
\Delta\psi(k) &= \psi(k+1) - \psi(k) \\
&= \frac{-a(k)N(x(k))\beta(k-1)\epsilon(k)}{D(k)} \\
\Delta z(k) &= z(k+1) - z(k) = \frac{a(k)G(k)\epsilon^2(k)}{D(k)} \\
x(k) &= z(k) + \frac{\psi^2(k)}{2} \\
G(k) &= 1 + |N(x(k))| \\
D(k) &= (1+|\psi(k)|)(1+|N(x(k))|^3) \\
&\quad \times(1+\|\Phi(k-1)\|^2 + y_d^2(k) + \beta^2(k-1) \\
&\quad + \epsilon^2(k) + \int_0^R E_r^2[v^*](k-n)dr) \\
a(k) &= \begin{cases} 1 & \text{if } |\epsilon(k)| > \lambda \\ 0 & \text{others} \end{cases}
\end{aligned}$$

where $z(0) = \psi(0) = 0$ and $\epsilon(k)$ is introduced as an augmented error and $\gamma > 0$ is the tuning rate to be specified by the designer. It should be mentioned that the requirement on sequence $x(k)$ in the definition of discrete Nussbaum gain in (5) is satisfied by the sequence $x(k)$ defined in (17).

Remark 3.1: It can be shown later that using the discrete Nussbaum gain, there is no need to know the sign of control gain g in (10) and it can be guaranteed that the estimate $\hat{p}(r,k)$ is nonnegative, such that the algorithm solving for $v^*(k)$ from (13) proposed in [16] can be applied.

Lemma 3.1: Consider the parameters $\hat{p}(r,k)$ and $\hat{p}'(r,k)$ in (17), we have $\int_0^R \tilde{p}'^2(r,k)dr \geq \int_0^R \tilde{p}^2(r,k)dr$, where $\tilde{p}'(r,k) = \hat{p}'(r,k) - p(r)$ and $\tilde{p}(r,k) = \hat{p}(r,k) - p(r)$.

Proof: According to (17), we can see that $|\tilde{p}'(r,k)| = |\tilde{p}(r,k)|$ when $\hat{p}'(r,k) \geq 0$. Now, considering the case that $\hat{p}'(k) < 0$ and noting that $p(r) > 0$ defined in (2), thus we have

$$\begin{aligned}
|\tilde{p}(r,k)| &= |-\hat{p}'(r,k) - p(k)| \\
&\leq -\hat{p}'(r,k) + p(r) = |\tilde{p}'(r,k)|
\end{aligned}$$

In summary, we always have $|\tilde{p}'(r,k)| \geq |\tilde{p}(r,k)|$, which implies $\int_0^R \tilde{p}'^2(r,k)dr \geq \int_0^R \tilde{p}^2(r,k)dr$. This completes the proof. ■

C. Stability Analysis

Theorem 3.1: Consider the adaptive closed-loop system consisting of system (1) under Assumption 2.1, states prediction laws defined in (7) with parameter estimation law (8), control law (14) and parameter adaptation law (17). If there exists an integer $k_1 > 0$ such that $|u'(k)| \leq \hat{u}'_{\text{sat}}(k)$, $\forall k > k_1$, then all the signals in the closed-loop system are bounded and $G(k) = 1 + |N(x(k))|$ will converge to a constant. Denote $C = \lim_{k\to\infty} G(k)$, then the tracking error satisfies $\lim_{k\to\infty}\sup|e(k)| < \frac{C\lambda}{\gamma}$, where γ and λ are the tuning factor and the threshold value specified by the designer.

Proof: In the proof, it is supposed that $|u'(k)| \leq \hat{u}'_{\text{sat}}(k)$ [16]. Substituting the error dynamics (15) into the augmented error $\epsilon(k)$, it can be obtained that

$$\begin{aligned}
&\gamma\tilde{\Theta}_{fg}^T(k-n)\Phi(k-1) - \gamma\tilde{g}_I(k-n)y_d(k) \\
&\quad +\gamma\int_0^R \tilde{p}(r,k-n)E_r[v^*](k-n)dr \\
&= -\frac{1}{g}G(k)\epsilon(k) - \gamma\beta(k-1) + \gamma\mu(k-n) \\
&\quad +\gamma\frac{1}{g}d_0(k-n) + \frac{1}{g}N(x(k))\psi(k)\beta(k-1) \qquad (18)
\end{aligned}$$

Choose a positive definite function $V(k)$ as

$$\begin{aligned}
V(k) &= \sum_{j=1}^n \|\tilde{\Theta}_{fg}(k-n+j)\|^2 + \sum_{j=1}^n \tilde{g}_I^2(k-n+j) \\
&\quad +\sum_{j=1}^n \int_0^R \tilde{p}^2(r,k-n+j)dr \qquad (19)
\end{aligned}$$

From the adaptation law (17), it is clear that

$$\begin{aligned}
&a(k)N(x(k))(\mu(k-n) + \frac{1}{g}d_o(k-n))\epsilon(k) \\
&\leq a(k)d_b|N(x(k))|\epsilon^2(k) \\
&\Delta x(k) = \Delta z(k) + \psi(k)\Delta\psi(k) + \frac{[\Delta\psi(k)]^2}{2} \qquad (20) \\
&0 \leq \Delta z(k) < 1, \quad 0 \leq \Delta\psi(k) < 1 \\
&|N(x(k))|[\Delta\psi(k)]^2 \leq \Delta z(k) \\
&\frac{a(k)N(x(k))\epsilon^2(k)}{D(k)} \leq \Delta z(k)
\end{aligned}$$

where $d_b = \frac{1}{\lambda}(\frac{\bar{d}}{|g_n|} + \bar{\mu})$. Considering (17), (18), (20), and using Lemma 3.1, it can be obtained that the difference equation of $V(k)$ is given by

$$\begin{aligned}
&\Delta V(k) = V(k) - V(k-1) \\
&= \tilde{\Theta}_{fg}^T(k)\tilde{\Theta}_{fg}(k) - \tilde{\Theta}_{fg}^T(k-n)\tilde{\Theta}_{fg}(k-n) \\
&\quad +\tilde{g}_I^2(k) - \tilde{g}_I^2(k-n) \\
&\quad +\int_0^R \tilde{p}'^2(r,k)dr - \int_0^R \tilde{p}^2(r,k-n)dr \\
&= \gamma^2 a^2(k)\frac{N^2(x(k))\epsilon^2(k)}{D^2(k)} \\
&\quad \times(\|\Phi(k-1)\|^2 + y_d^2(k) + \int_0^R E_r^2[v^*](k-n)dr)
\end{aligned}$$

$$
\begin{aligned}
&+2\gamma a(k)N(x(k))\frac{\tilde{\Theta}_{fg}^T(k-n)\Phi(k-1)}{D(k)}\epsilon(k)\\
&-2\gamma a(k)N(x(k))\frac{\tilde{g}_I(k-n)y_d(k)}{D(k)}\epsilon(k)\\
&+2\gamma a(k)N(x(k))\frac{\int_0^R \tilde{p}(r,k-n)E_r[v^*](k-n)dr}{D(k)}\epsilon(k)\\
\leq\ & \gamma^2\frac{a(k)G(k)\epsilon^2(k)}{D(k)}+2\gamma\frac{a(k)d_b|N(x(k))|\epsilon^2(k)}{D(k)}\\
&-\frac{2}{g}a(k)N(x(k))\frac{G(k)\epsilon^2(k)}{D(k)}\\
&-2\gamma\frac{a(k)N(x(k))\beta(k-1)\epsilon(k)}{D(k)}\\
&+\frac{2}{g}a(k)N(x(k))\frac{N(x(k))\psi(k)\beta(k-1)\epsilon(k)}{D(k)}\\
\leq\ & (\gamma^2+2\gamma d_b)\Delta z(k)+2\gamma\Delta\psi(k)-\frac{2}{g}N(x(k))(\Delta z(k)\\
&+\psi(k)\Delta\psi(k)+\frac{[\Delta\psi(k)]^2}{2})+\frac{1}{|g|}|N(x(k))|[\Delta\psi(k)]^2\\
\leq\ & c_1\Delta z(k)+2\gamma\Delta\psi(k)-\frac{2}{g}N(x(k))\Delta x(k) \qquad (21)
\end{aligned}
$$

where $c_1=\gamma^2+2\gamma d_b$.

Noting that $x(k)=z(k)+\frac{\psi^2(k)}{2}$ and taking summation on both hand sides of (21) results in

$$
\begin{aligned}
V(k)&\leq -\frac{2}{g}\sum_{k'=0}^{k}N(x(k'))\Delta x(k')+c_1z(k)+2\gamma\psi(k)+c_2\\
&\leq -\frac{2}{g}\sum_{k'=0}^{k}N(x(k'))\Delta x(k')+c_1x(k)+c_3 \qquad (22)
\end{aligned}
$$

where $c_2=V(-1)$ and $c_3=c_2+\frac{2\gamma^2}{c_1}$ are some finite constants.

Applying Lemma 2.2 to (22) gives the boundedness of $V(k)$ and $x(k)$ and $N(x(k))$ thus the boundedness of $\hat{\Theta}_{fg}(k)$, $\hat{g}_I(k)$, $\int_0^R \hat{p}(r,k)dr$, and $G(k)$. Considering that $z(k)$ is an nondecreasing sequence satisfying $0\leq z(k)\leq x(k)$, thus the boundedness of $x(k)$ means that $z(k)$ and $\psi(k)$ are bounded. The boundedness of $z(k)$ further implies

$$
\lim_{k\to\infty}\Delta z(k)=\lim_{k\to\infty}\frac{a(k)G(k)\epsilon^2(k)}{D(k)}=0 \qquad (23)
$$

Noting that $e(k)=y(k)-y_d(k)$, we have $y(k)=O[e(k)]$ because the reference signal $y_d(k)$ is bounded. Using the Lipschitz condition of function $\Phi_i(\cdot)$, $i=1,2,\ldots,n$ and Lemma 2.1, it follows $\Phi(k-1)=O[e(k)]$, where $\Phi(k-1)$ is defined in (11). According to the definition of $\beta(k)$ in (16), the boundedness of $\hat{\Theta}_{fg}(k)$, and Lemma 2.3, it is clear that $\beta(k)=o[O[e(k-1)]]$. Then, from the boundedness of $N(x(k))$, $\psi(k)$, and $G(k)$, it can be seen that $\epsilon(k)\sim e(k)$. Therefore, we can conclude that $D(k)$ in (17) satisfies $D(k)=O[\epsilon^2(k)]$.

According to the definition of $a(k)$ in (17), the following two cases need to be considered:

- For $a(k)=0$, it is clear from the definition of $a(k)$ that $|\epsilon(k)|\leq\lambda$, which means that $\lim_{k\to\infty}\sup\{\epsilon(k)\}\leq\lambda$.
- For $a(k)=1$, it is clear from the definition of $a(k)$ that $|\epsilon(k)|>\lambda$. Noting that $D(k)=O[a^2(k)\epsilon^2(k)]$, thus applying the Key Technical Lemma [21] to (23) gives $\lim_{k\to\infty}a(k)\epsilon(k)=0$. Let us define a time interval as $Z_1=\{k|a(k)=1\}$ and suppose that Z_1 is an infinite set. Then, we have

$$
\lim_{k\to\infty,k\in Z_1}\epsilon(k)=\lim_{k\to\infty,k\in Z_1}a(k)\epsilon(k)=0 \qquad (24)
$$

which conflicts with $|\epsilon(k)|\geq\lambda$, for $a(k)=1$. Therefore, Z_1 must be a finite set and then, we have

$$
\lim_{k\to\infty}a(k)=0,\ \ \lim_{k\to\infty}\sup\{|\epsilon(k)|\}\leq\lambda \qquad (25)
$$

According to the discussions for the above two cases, we have $\lim_{k\to\infty}a(k)=0$ and $\lim_{k\to\infty}\sup\{|\epsilon(k)|\}\leq\lambda$, which implies $N(x(k))$ will converge to a constant ultimately and thus let denote $\lim_{k\to\infty}G(k)=C$. Noting that $\beta(k)=o[O[e(k-1)]]$ gives $\lim\beta(k)=0$ and using the boundedness of $N(x(k))$ and $\psi(k)$, thus it can be derived from the definition of $\epsilon(k)$ in (17) that

$$
\lim_{k\to\infty}\sup\{|\epsilon(k)|\}=\lim_{k\to\infty}\sup\{|\frac{\gamma e(k)}{G(k)}|\}\leq\lambda
$$

which gives

$$
\lim_{k\to\infty}\sup\{|e(k)|\}\leq\frac{C\lambda}{\gamma}
$$

This implies the boundedness of $y(k)$. From Lemma 2.1, it is clear that the boundedness of $u(k)$ and $\xi_i(k)$, $i=1,2\ldots,n$ is guaranteed. This completes the proof of the boundedness of all the signals in the closed-loop system and the tracking error satisfying $\lim_{k\to\infty}\sup|e(k)|<\frac{C\lambda}{\gamma}$. ■

IV. Simulation Results

The following second order nonlinear plant is used for simulation.

$$
\begin{cases}
\xi_1(k+1) = 0.2\xi_1(k)\cos(\xi_1(k))+0.1\xi_1(k)\sin(\xi_1(k))\\
\qquad\qquad +3\xi_2(k)\\
\xi_2(k+1) = 0.3\xi_2(k)\frac{\xi_1(k)}{1+\xi_1^2(k)}-0.6\frac{\xi_2^3(k)}{2+\xi_2^2(k)}-0.12u(k)\\
\qquad\qquad +d_0(k)\\
y(k) = \xi_1(k)\\
u(k) = \int_0^R p(r)E_r[v](k)dr
\end{cases}
$$

where $d_0(k)=0.2\cos(0.05k)\cos(\xi_1(k))$. Select the density function $p(r)=e^{-0.07(r-1)^2}$ and $R=10$. The control objective is to make the output $y(k)$ track the desired reference trajectory $y_d(k)=1.5\sin(\frac{\pi}{5}kT)+1.5\cos(\frac{\pi}{10}kT)$, $T=0.05$. The initial system states are $\bar{\xi}_2(0)=[1,\ 1]^T$. The tuning factor and the threshold value are chosen as $\gamma=4$ and $\lambda=0.1$. The simulation results are showed in Figs. 2-4. Fig. 2 depicts the output $y(k)$ and the reference signal $y_d(k)$. Figure 3 illustrates the boundedness of the control input $u(k)$, the estimated parameters $\hat{g}_I(k)$, $\hat{\Theta}_{fg}(k)$, and $\hat{p}(r,k)$. Fig. 4 demonstrates the discrete Nussbaum gain $N(x(k))$ and the sequences $x(k)$ and $\beta(k)$. As illustrated in Fig. 4, to detect the control direction, the discrete Nussbaum gain adapts by searching alternately in the two directions: when the control gain g is negative, the sign of $N(x(k))$ changes from positive to negative and remains so for good; when the control gain is positive, the sign of $N(x(k))$ keeps positive without any switch.

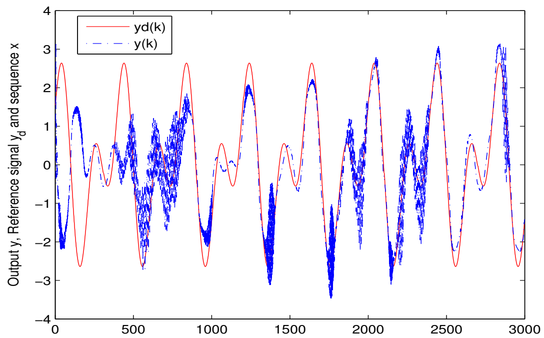

Fig. 2. Reference signal and system output

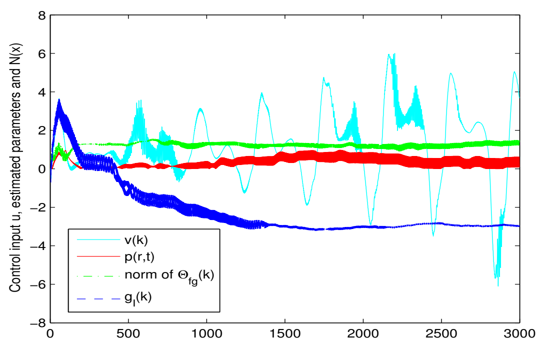

Fig. 3. Control signal and estimated parameters, $r = 1$ for $\hat{p}(r,t)$

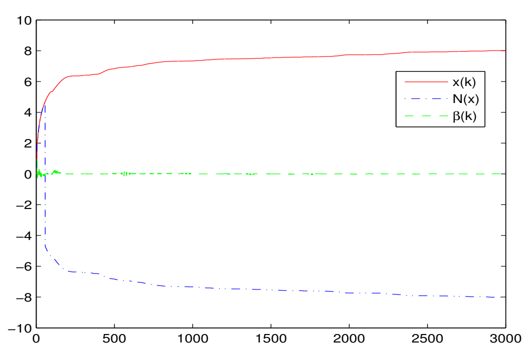

Fig. 4. Nussbaum gain $N(x(k))$ and its argument $x(k)$ and $\beta(k)$

V. Conclusion

This paper has developed adaptive control for a class of strict-feedback discrete-time nonlinear systems with unknown control gains and hysteresis input. Based on the future states prediction, adaptive control has been designed with the discrete Nassbaum gain employed to tackle the unknown control directions problem. Under the proposed adaptive control law, all the signals in the closed-loop system are globally bounded and the output tracking error is made to be within a neighborhood around zero ultimately.

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