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Distributed Fault-tolerant Control Systems Design Against Actuator Faults and Faulty Interconnection Links: an Adaptive Method

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Abstract— **This paper presents an adaptive method to solve the robust fault-tolerant control (FTC) problem for a class of large scale systems against actuator faults and lossy interconnection links. In terms of the special distributed architectures, adaptation laws are proposed to estimate the unknown eventual faults of actuators and interconnections, constant external disturbances, and controller parameters on-line. Then a class of distributed state feedback controllers is constructed to automatically compensate the fault and disturbance effects based on the information from adaptive schemes. On the basis of Lyapunov stability theory, it shows that the resulting adaptive closed-loop large-scale system can be guaranteed to be asymptotically stable in the presence of uncertain faults of actuators and interconnections, and constant disturbances. The proposed design technique is finally evaluated in the light of a simulation example.**

I. Introduction

The problem of distributed control of large-scale interconnected systems has received considerable attention over the past few years, because there are many applications of the control design technique in a lot of practical control systems, such as the control of vehicular platoons [21], cross-directional control in the chemical industry [22], Microelectromechanical system (MEMS) and other large-scale systems composed of a large number of spatially interconnected units. Therefore, many approaches have been developed to synthesize some types of distributed controllers, in which interconnections are also considered between individual controllers to make the dynamic large scale systems well-posed, stable, and contractive (see, e.g., [1]-[7], and the references therein). Many issues which always exist in interconnection channels (communication channels), such as time delays, single attenuations, bandwidth limitations (bit rate limitations) are considered by some researchers [1]-[5]. In [1], the problem of signal attenuations in interconnection channels is considered, and necessary and sufficient conditions for well-posedness, stability, and contractiveness are obtained using integral quadratic constraints (IQCs) methods. Recursive information flow [3] is proposed to compensate for the effects of bandwidth limitations. In [2], [4], [5], distributed controllers are constructed using some special effective approaches for the stability and performance of closed-loop interconnected system with the problem of time-delays in channels. In this paper, we consider the problem related to the issue of single attenuations.

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Recently, fault-tolerant control (FTC) system design, which can make the systems operate in safety and with proper performances whenever components are healthy or faulted has received significant attention (see e.g., $[6],[7]-[19]$). The existing faut-tolerant design approaches can be broadly classified into two groups, namely passive approach $[6],[8]-[10]$ and active approach $[7],[11]-[19]$. Since the active FTC system offers the flexibility to select different controllers, the most suitable controller can be chosen for the situation and the better performance can be obtained than the passive FTC system. There are primary two typical approaches for fault compensations in active fault-tolerant, such as adaptive approach $[11]-[16]$ and fault detection and isolation (FDI) $[17]-[19]$. For the FTC system based on FDI, the controller reconfiguration or restructure is based on the fault diagnostic information, which is provided by a fault detection and isolation mechanism. However, it should be noted that the FDI mechanism might not always give the exact fault information. Different from the FDI method with the need for a mechanism to provide the exact fault information, the new proposed adaptive method is not necessary for the estimations to give the exact fault information.

In this paper, the adaptive compensation design approach will be used for general actuator and interconnection fault models. Each control effectiveness and constant disturbances are assumed to be unknown. An adaptive method is proposed to solve the problem for developing some distributed state feedback controllers. Due to the distributed architectures are different from other architectures [11], [12], we first propose some adaptation laws to estimate the unknown control effectiveness, constant external disturbances, and controller parameters on-line. Then, the distributed controllers are constructed using the updated values of these estimation. Based on Lyapunov stability theory, the adaptive closed-loop large-scale system can be guaranteed to be asymptotically stable in the presence of actuator and interconnection failures, and constant disturbances. The corresponding distributed fault-tolerant controller design via direct adaptive method is presented in [7].

II. Preliminaries and Problem Statement

Notations: R stands for the set of real numbers. Given matrices $M_k, k=1,\dots,n$, the notation $\mathrm{diag}_{k=1}^{n}[M_k]$ denotes

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the block-diagonal matrix with M_k along the diagonal and denoted $\mathrm{diag}_k[M_k]$ for brevity. For signals or vectors x_k, the notation $\mathrm{cat}_{k=1}^n x_k$ denotes the signal or vector $(x_1, x_2, \ldots, x_n)$ formed by concatenating x_k. This is also usually denoted $\mathrm{cat}_k x_k$ for brevity.

In this paper, we consider a large-scale system G composed of N interconnected linear time-invariant continuous time subsystems G_i, $i = 1, 2, \ldots, N$. Each subsystem is captured the following state-space equations:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \end{bmatrix} = \begin{bmatrix} A_{TT}^i & A_{TS}^i & B_{Td}^i & B_{Tu}^i \\ A_{ST}^i & A_{SS}^i & B_{Sd}^i & B_{Su}^i \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i \\ u_i(t) \end{bmatrix} \quad (1)$$

where $x_i(t) \in R^{n_i}$ is the state, $u_i(t) \in R^{m_i}$ is the control input, $d_i \in R^{p_i}$ is the constant external disturbance, and $v_i := \mathrm{cat}_j(v_{ij})$, $v_{ij} \in R^{q_{ji}}$ and $w_i := \mathrm{cat}_j(w_{ij})$, $w_{ij} \in R^{q_{ij}}$ $j = 1, 2, \ldots, N$ is the interconnection input to each subsystem and the interconnection output from each subsystem, respectively. All system matrices are known real constant matrices with appropriate dimensions.

In the normal case, once the relationships between the inputs and outputs at each subsystem have been defined, the distributed system can be described by closing all loops by imposing the constraints of interconnection with the interconnection condition such that

$$v_{ij}(t) = w_{ji}(t). \quad (2)$$

We assume the states of the subsystems are available at every instant, and every state has its interconnection channel interconnected with other subsystems. A state feedback controller with same interconnection structure for this system has subsystems K_i given by

$$\begin{bmatrix} u_i(t) \\ w_i^K(t) \end{bmatrix} = \begin{bmatrix} K_{i1} & K_{i2} \\ K_{i3} & K_{i0} \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i^K(t) \end{bmatrix} \quad (3)$$

where $v_i^K(t) := \mathrm{cat}_j(v_{ij}^K)$, $w_i^K(t) := \mathrm{cat}_j(w_{ij}^K)$ and $v_{ij}^K(t)$, $w_{ji}^K(t) \in R^{q_{ji}^K}$ also have interconnection condition with

$$v_{ij}^K(t) = w_{ji}^K(t). \quad (4)$$

Then, the closed-loop system can be illustrated for example in Fig.1.

We make the following assumption on the plant and controller matrices:

Assumption 1. The subsystems are interconnected only through their states, that means,

$$A_{SS}^i = 0, \quad B_{Su}^i = 0, \quad B_{Sd}^i = 0, \quad K_{i0} = 0. \quad (5)$$

Remark 1. The assumption $A_{SS}^i = 0$ means the large-scale system must not have algebraic loops, and many mechanical systems do belong to this class of systems [1], [10]. The assumption $B_{Su}^i = 0$ can always be satisfied by placing low-pass filters of sufficiently high bandwidth at the control input u_i as has been done in the linear parameter varying (LPV) control literature [1], [5]. In this paper, we consider external

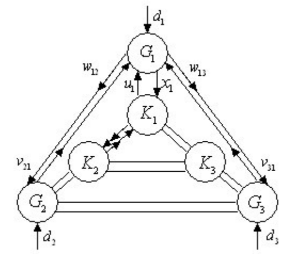

Fig. 1. Example of interconnected closed-loop system with N = 3 subsystems.

disturbance does not exist in interconnection channels. Thus, we assume $B_{Sd}^i = 0$. It's reasonable to assume controller parameter $K_{i0} = 0$ and design other parameters to obtain our control satisfactory objective.

In this paper, we formulate the faults including loss of actuator effectiveness and outage in subsystem G_i and lossy interconnection links between communicating subsystems. Let $u_{if}^{hF}(t)$ and $v_{ij}^{hF}(t)$ represent the signals from the fth actuator in G_i and jth communicating subsystem that have failed in the hth faulty mode, respectively. Then we denote the fault model as follows:

$$\begin{aligned} & u_{if}^{hF}(t) = \rho_{if}^h u_{if}(t),\ 0 < \underline{\rho}_{if}^h \le \rho_{if}^h \le \bar{\rho}_{if}^h \\ & v_{ij}^{hF}(t) = \sigma_{ji}^h v_{ij}(t),\ 0 \le \underline{\sigma}_{ji}^h \le \sigma_{ji}^h \le \bar{\sigma}_{ji}^h \\ & i, j = 1, 2, \ldots, N,\ f = 1, 2, \ldots, m_i,\ h = 1, 2, \ldots, L \end{aligned} \quad (6)$$

where ρ_{if}^h and σ_{ji}^h are unknown constants, the index h denotes the hth faulty mode and L is the total faulty modes. For every faulty mode, $\underline{\rho}_{if}^h$ and $\bar{\rho}_{if}^h$ represent the lower and upper bounds of ρ_{if}^h, respectively. Similarly, $\underline{\sigma}_{ji}^h$ and $\bar{\sigma}_{ji}^h$ represent the lower and upper bounds of σ_{ji}^h respectively. Note the practical case, when $\underline{\rho}_{if}^h = \bar{\rho}_{if}^h = 1$ and $\underline{\sigma}_{ji}^h = \bar{\sigma}_{ji}^h = I$, there are no faults for the fth actuator u_{if} and jth interconnection links v_{ij}. when $\underline{\rho}_{if}^h = \bar{\rho}_{if}^h = 0$ and $\underline{\sigma}_{ji}^h = \bar{\sigma}_{ji}^h = 0$, the fth actuator u_{if} is outage and jth interconnection link is complete disconnection. when $0 < \underline{\rho}_{if}^h \le \bar{\rho}_{if}^h < 1$ and $0 < \underline{\sigma}_{ji}^h \le \bar{\sigma}_{ji}^h < I$, that means the type of two kinds of faults is loss of effectiveness.

Denote

$$\begin{aligned} & u_i^{hF}(t) = [u_{i1}^{hF}(t), u_{i2}^{hF}(t), \cdots, u_{im_i}^{hF}(t)]^T = \rho_i^h u_i(t) \\ & v_{ij}^{hF}(t) = [v_{ij1}^{hF}(t), v_{ij2}^{hF}(t), \cdots, v_{ijq_{ji}}^{hF}(t)]^T = \sigma_{ji}^h v_{ij}(t) \end{aligned} \quad (7)$$

where $\rho_i^h = \mathrm{diag}_f[\rho_{if}^h]$, $\rho_{if}^h \in [\underline{\rho}_{if}^h, \bar{\rho}_{if}^h]$, and $\sigma_{ji}^h = \mathrm{diag}_k[\sigma_{jik}^h]$, $\sigma_{jik}^h \in [\underline{\sigma}_{jik}^h, \bar{\sigma}_{jik}^h]$.

Then, the sets of operators with above structures are denoted by

$$\begin{aligned} & \Delta_{\rho_i^h} = \{\rho_i^h : \rho_i^h = \mathrm{diag}_f[\rho_{if}^h], \rho_{if}^h \in [\underline{\rho}_{if}^h, \bar{\rho}_{if}^h]\}, \\ & \Delta_{\sigma_{ji}^h} = \{\sigma_{ji}^h : \sigma_{ji}^h = \mathrm{diag}_k[\sigma_{jik}^h], \sigma_{jik}^h \in [\underline{\sigma}_{jik}^h, \bar{\sigma}_{jik}^h]\}. \end{aligned} \quad (8)$$

where $f = 1, 2, \ldots, m_i$, $k = 1, 2, \ldots, q_{ji}$.

For convenience in the following sections, for all possible faulty modes L, the following uniform actuator and interconnection links fault model is exploited:

$$\begin{aligned} & u_i^F(t) = \rho_i u_i(t), \quad \rho_i \in \{\rho_i^1 \cdots \rho_i^L\} \\ & v_{ij}^F(t) = \sigma_{ji} v_{ij}(t), \quad \sigma_{ji} \in \{\sigma_{ji}^1 \cdots \sigma_{ji}^L\}. \end{aligned} \quad (9)$$

Based on the above description, the equation (2) can be represented by

$$v_{ij} = \sigma_{ji} w_{ji} \tag{10}$$

for all $i, j = 1 \dots N$, and the dynamics with actuator fault and lossy interconnection links (1) can be rewritten by

$$\dot{x}_i(t) = A^i_{TT} x_i(t) + \sum_{j=1}^{N} A^{ij}_{TS} \sigma_{ji} A^{ji}_{ST} x_j(t) + B^i_{Tu} \rho_i u_i(t) + B^i_{Td} d_i. \tag{11}$$

In terms of (3) and Assumption 1, the controller form can be described as:

$$u_i(t) = \hat{K}_{i1} x_i(t) + \sum_{j=1}^{N} K_{i2} \hat{\sigma}_{ji}(t) \hat{K}_{i3} x_j(t) + \hat{K}_{i4} \hat{d}_i(t) \tag{12}$$

where $f = 1, 2, \dots, m_i$, $k = 1, 2, \dots, q_{ji}$ and $g = 1, 2, \dots, p_i$, $\rho_{if} \in \Delta_{\rho_i^h}$, $\sigma_{ji} \in \Delta_{\sigma_{ji}^h}$, and $\hat{\rho}_i(t)$, $\hat{\sigma}_{ji}(t)$, $\hat{d}_i(t)$, $\hat{K}_{i1}(t)$, $\hat{K}_{i3}(t)$, $\hat{K}_{i4}(t)$ are the estimate of ρ_i, σ_{ji}, d_i, K_{i1}, K_{i3}, K_{i4}, respectively. K_{i2} is an appropriate dimensions matrix chosen by the system designer.

Then, following (11) and (12), the closed-loop subsystem $G_i, i \in \{1, 2, \dots, N\}$ is given by

$$\begin{aligned}\dot{x}_i(t) = (A^i_{TT} + B^i_{Tu} \rho_i \hat{K}_{i1}(t)) x_i + \sum_{j=1}^{N} (A^{ij}_{TS} \sigma_{ji} A^{ji}_{ST} \\ + B^i_{Tu} \rho_i K_{i2} \hat{\sigma}_{ji}(t) \hat{K}_{i3}(t)) x_j(t) + B^i_{Tu} \rho_i \hat{K}_{i4} \hat{d}_i(t) + B^i_{Td} d_i.\end{aligned} \tag{13}$$

We introduce for the system the following standard assumption denotes the internally stabilizability of each nominal isolated subsystem in actuator failure case:

Assumption 2. All pairs $\{A^i_{TT}, B^i_{Tu} \rho_i\}$, $i = 1, 2, \dots, N$, are uniformly completely controllable for any actuator failure mode $\rho_i \in \{\rho_i^1 \dots \rho_i^L\}$ under consideration.

Now, the main objective of this paper is to synthesize the distributed controller $u_i(t)$ given in (12) such that the closed-loop system (13) can be guaranteed to be asymptotically stable even in the cases of failures on actuators and interconnections.

III. Fault-tolerant Control System Design

In this section, we develop the adaptive laws to estimate the fault effect factors of actuators and interconnection links, and also to estimate the constant external disturbances. Then, a method for designing distributed adaptive fault-tolerant controllers to guarantee closed-loop system asymptotically stable via state feedback is presented in Theorem 1.

Denote

$$M_i := x_i^T P_i B^i_{Tu}, \quad R_i := \sum_{j=1}^{N} K_{i2} \hat{\sigma}_{ji} \hat{K}_{i3} x_j, \quad S_i := \hat{K}_{i4} \hat{d}_i, \tag{14}$$

where $i = 1, 2, \dots, N$ and P_i are positive symmetric matrices. M_i, R_i and S_i being bound vector functions described by $M_i = [M_{i1}, M_{i2}, \cdots, M_{im_i}] \in R^{1 \times m_i}$, $R_i = [R_{i1}, R_{i2}, \cdots, R_{im_i}]^T \in R^{m_i}$, $S_i = [S_{i1}, S_{i2}, \cdots, S_{im_i}]^T \in R^{m_i}$ and respectively, which will be used later.

In particular, for any $i \in \{1, 2, \dots, N\}$, $\hat{\rho}_{if}(t)$ is the estimate of the unknown ρ_{if} which is updated by the following adaptive laws:

$$\frac{d\hat{\rho}_{if}(t)}{dt} = -l_{if}(M_{if} R_{if} + M_{if} S_{if}), \quad f = 1, 2, \dots, m_i \tag{15}$$

where the constants $l_{if} > 0$; $\hat{\sigma}_{jik}(t)$ is the estimate of the unknown σ_{jik} which is updated according to the adaptive laws:

$$\frac{d\hat{\sigma}_{jik}(t)}{dt} = r_{jik} x_i^T(t) P_i a^{ijk}_{TS} a^{jik}_{ST} x_j(t), \quad k = 1, 2, \dots, q_{ji} \tag{16}$$

where the constant $r_{jik} > 0$, a^{ijk}_{TS} and a^{jik}_{ST} are the kth low of A^{ij}_{TS} and A^{ji}_{ST}, respectively; $\hat{d}_i(t)$ is the estimate of the unknown constant external disturbances d_i updated according to the following adaptive laws:

$$\frac{d\hat{d}_{ig}(t)}{dt} = s_{ig} x_i^T(t) P_i b^{ig}_{Td}, \quad g = 1, 2, \dots, p_i \tag{17}$$

where the constant $s_{ig} > 0$, and b^{ig}_{Td} is the gth column of B^i_{Td}.

Thus, for the fault-tolerant large-scale system described by (11), we propose the distributed adaptive state feedback controller (12) with the control gain functions $K_{i1}(t) = [\hat{K}_{i1,1}(t), \hat{K}_{i1,2}(t), \dots, \hat{K}_{i1,m_i}(t)]^T \in R^{m_i \times n_i}$ updated by the following adaptive laws:

$$\frac{d\hat{K}_{i1,f}(t)}{dt} = -\Gamma_{i1,f} x_i x_i^T P_i b^{if}_{Tu}, \quad i = 1, 2, \dots, N, f = 1, 2, \dots, m_i \tag{18}$$

where $\Gamma_{i1,f}$ is any positive constant, $\hat{K}_{i1,f}(t_0)$ is finite, and b^{if}_{Tu} is the fth column of B^i_{Tu}; K_{i2} is an appropriate dimensions matrix chosen by the system designer; $\hat{K}_{i3}(t) = [\hat{K}_{i3,1}(t), \hat{K}_{i3,2}(t), \dots, \hat{K}_{i3,m_i}(t)]^T \in R^{m_i \times n_i}$ updated according to the adaptive law:

$$\frac{d\hat{K}_{i3,k}(t)}{dt} = -\Gamma_{i3,k} \sum_{j=1}^{N} x_j x_i^T P_i B^i_{Tu} \hat{\rho}_i k_{i2k} \hat{\sigma}_{jik} \tag{19}$$

where $i, j = 1, 2, \dots, N, k = 1, 2, \dots, q_{ji}$, $\Gamma_{i3,k}$ is any positive constant, $\hat{K}_{i3,k}(t_0)$ is finite, k_{i2k} is the kth column of K_{i2}; $\hat{K}_{i4}(t) = [\hat{K}_{i4,1}(t), \hat{K}_{i4,2}(t), \dots, \hat{K}_{i4,m_i}(t)]^T \in R^{m_i \times n_i}$ updated according to the adaptive law: $f = 1, 2, \dots, m_i, g = 1, 2, \dots, p_i$

$$\frac{d\hat{K}_{i4,g}(t)}{dt} = -\Gamma_{i5,g} \hat{d}_{ig} x_i^T P_i B^i_{Tu} \hat{\rho}_{if} \tag{20}$$

where $\Gamma_{i4,g}$ is any positive constant, $\hat{K}_{i4,g}(t_0)$ are finite.

Letting

$$\begin{array}{ll}\tilde{\rho}_i(t) = \hat{\rho}_i(t) - \rho_i, & \tilde{K}_{i1}(t) = \hat{K}_{i1}(t) - K_{i1}, \\ \tilde{\sigma}_{ji}(t) = \hat{\sigma}_{ji}(t) - \sigma_{ji}, & \tilde{K}_{i3}(t) = \hat{K}_{i3}(t) - K_{i3}, \\ \tilde{d}_i(t) = \hat{d}_i(t) - d_i, & \tilde{K}_{i4}(t) = \hat{K}_{i4}(t) - K_{i4}.\end{array} \tag{21}$$

Due to ρ_{if}, σ_{jik}, d_{ig}, and K_{i1}, K_{i3}, K_{i4} are an unknown constants, the error system can be written as following equations:

$$\begin{array}{l}\dot{\tilde{\rho}}_{if}(t) = \dot{\hat{\rho}}_{if}(t),\ \dot{\tilde{\sigma}}_{jik}(t) = \dot{\hat{\sigma}}_{jik}(t),\ \dot{\tilde{d}}_{ig}(t) = \dot{\hat{d}}_{ig}(t), \\ \dot{\tilde{K}}_{i1}(t) = \dot{\hat{K}}_{i1}(t),\ \dot{\tilde{K}}_{i3}(t) = \dot{\hat{K}}_{i3}(t),\ \dot{\tilde{K}}_{i4}(t) = \dot{\hat{K}}_{i4}(t).\end{array} \tag{22}$$

Then, the following theorem can be obtained, which shows the uniform ultimate boundedness of the closed-loop large-scale system (13) and the error system (22).

Theorem 1. Consider the adaptive closed-loop large-scale interconnected system described by (13), satisfying Assumptions 1 and 2. Then the closed-loop fault-tolerant control system is asymptotically stable for any $\rho_i \in \Delta_{\rho_i^h}$ and $\sigma_{ji} \in \Delta_{\sigma_{ji}^h}$ if there exist positive symmetric matrices $P_i > 0$, $\hat{\rho}_{if}(t)$, $\hat{\sigma}_{jik}(t)$ and $\hat{d}_{ig}(t)$ determined according to the adaptive laws (15), (16) and (17), and control gain $\hat{K}_{i1}(t)$, $\hat{K}_{i3}(t)$, $\hat{K}_{i4}(t)$ updated by adaptive laws (18), (19), (20), respectively.

Proof: For the adaptive closed-loop large-scale system described by (13), we first define a Lyapunov functional candidate as:

$$
\begin{aligned}
V &= \sum_{i=1}^{N} x_i^T P_i x_i + \sum_{i=1}^{N}\sum_{f=1}^{m_i} \frac{\tilde{\rho}_{if}^2}{l_{if}} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} \frac{\tilde{\sigma}_{jik}^2}{r_{jik}} \\
&+ \sum_{i=1}^{N}\sum_{g=1}^{p_i} \frac{\tilde{d}_{ig}^2}{s_{ig}} + \sum_{i=1}^{N}\sum_{f=1}^{m_i} \rho_{if} \tilde{K}_{i1,f}^T \Gamma_{i1,f}^{-1} \tilde{K}_{i1,f} \\
&+ \sum_{i=1}^{N}\sum_{k=1}^{q_{ji}} \tilde{K}_{i3,k}^T \Gamma_{i3,k}^{-1} \tilde{K}_{i3,k} + \sum_{i=1}^{N}\sum_{g=1}^{p_i} \tilde{K}_{i4,g}^T \Gamma_{i4,g}^{-1} \tilde{K}_{i4,g}
\end{aligned} \tag{23}
$$

Then, The time derivative of V for $t > 0$ associated with a certain failure mode $\rho_i \in \Delta_{\rho_i^h}$ and $\sigma_{ji} \in \Delta_{\sigma_{ji}^h}$ is

$$
\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} \{ x_i^T [(A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})] x_i \\
&+ 2x_i^T P_i [\sum_{j=1}^{N} (A_{TS}^{ij} \sigma_{ji} A_{ST}^{ji} + B_{Tu}^i \rho_i K_{i2} \hat{\sigma}_{ji} \hat{K}_{i3}) x_j] \\
&+ 2x_i^T P_i B_{Td}^i d_i + 2x_i^T P_i B_{Tu}^i \rho_i \hat{K}_{i4} \hat{d}_i + \sum_{f=1}^{m_i} \frac{2\tilde{\rho}_{if}\dot{\tilde{\rho}}_{if}}{l_{if}} \\
&+ \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} \frac{2\tilde{\sigma}_{jik}\dot{\tilde{\sigma}}_{jik}}{r_{jik}} + \sum_{g=1}^{p_i} \frac{2\tilde{d}_{ig}\dot{\tilde{d}}_{ig}}{s_{ig}} + \sum_{f=1}^{m} 2\rho_{if} \tilde{K}_{i1,f}^T \Gamma_{i1,f}^{-1} \dot{\tilde{K}}_{i1,f} \\
&+ \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2\tilde{K}_{i3,k}^T \Gamma_{i3,k}^{-1} \dot{\tilde{K}}_{i3,k} + \sum_{g=1}^{p_i} 2\tilde{K}_{i4,g}^T \Gamma_{i4,g}^{-1} \dot{\tilde{K}}_{i4,g} \} \\
&= \sum_{i=1}^{N} \{ x_i^T [(A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})] x_i \\
&+ 2x_i^T P_i \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} a_{TS}^{ijk} \sigma_{jik} a_{ST}^{jik} x_j + \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} \frac{2\tilde{\sigma}_{jik}\dot{\tilde{\sigma}}_{jik}}{r_{jik}} \\
&+ 2x_i^T P_i B_{Tu}^i \rho_i \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} K_{i2,k} \hat{\sigma}_{jik} \hat{K}_{i3,k} x_j + 2x_i^T P_i \sum_{g=1}^{p_i} b_{Td}^{ig} d_{ig} \\
&+ 2x_i^T P_i B_{Tu}^i \rho_i \sum_{g=1}^{p_i} \hat{K}_{i4,g} \hat{d}_{ig} + \sum_{f=1}^{m_i} \frac{2\tilde{\rho}_{if}\dot{\tilde{\rho}}_{if}}{l_{if}} + \sum_{g=1}^{p_i} \frac{2\tilde{d}_{ig}\dot{\tilde{d}}_{ig}}{s_{ig}} \\
&+ \sum_{f=1}^{m} 2\rho_{if} \tilde{K}_{i1,f}^T \Gamma_{i1,f}^{-1} \dot{\tilde{K}}_{i1,f} + \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2\tilde{K}_{i3,k}^T \Gamma_{i3,k}^{-1} \dot{\tilde{K}}_{i3,k} \\
&+ \sum_{g=1}^{p_i} 2\tilde{K}_{i4,g}^T \Gamma_{i4,g}^{-1} \dot{\tilde{K}}_{i4,g} \}
\end{aligned} \tag{24}
$$

Chosen the adaptive laws as (15)-(17), we can adjust $\hat{\rho}_i$, $\hat{\sigma}_{jik}$ and $\hat{d}_{ig}$ to guarantee that there exist constants $K_{i3,k} \in R^{n_i}$ and $K_{i4,g} \in R^{n_i}$, $k = 1, 2, \ldots, q_{ji}$, $g = 1, 2, \ldots, p_i$ make sure the following inequalities hold true:

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i B_{Tu}^i \hat{\rho}_i K_{i2,k} \hat{\sigma}_{jik} K_{i3,k} x_j \\
&\qquad \le -\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i a_{TS}^{ijk} \hat{\sigma}_{jik} a_{ST}^{jik} x_j
\end{aligned} \tag{25}
$$

$$
\sum_{i=1}^{N}\sum_{g=1}^{p_i} 2x_i^T P_i B_{Tu}^i \hat{\rho}_i K_{i4,g} \hat{d}_{ig} \le -\sum_{i=1}^{N}\sum_{g=1}^{p_i} 2x_i^T P_i b_{Td}^{ig} \hat{d}_{ig}. \tag{26}
$$

Then, following the above inequalities and equation (21), we can rewrite (24) as

$$
\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{N} \{ x_i^T [(A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{1i})] x_i \\
&+ \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i a_{TS}^{ijk} \tilde{\sigma}_{jik} a_{ST}^{jik} x_j - \sum_{g=1}^{p_i} 2x_i^T P_i B_{Tu}^i \tilde{\rho}_i \hat{K}_{i4,g} \hat{d}_{ig} \\
&- \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i B_{Tu}^i \tilde{\rho}_i K_{i2,k} \hat{\sigma}_{jik} \hat{K}_{i3,k} x_j + \sum_{g=1}^{p_i} 2x_i^T P_i b_{Td}^{ig} \hat{d}_{ig} \\
&+ \sum_{g=1}^{p_i} 2x_i^T P_i B_{Tu}^i \hat{\rho}_i \hat{K}_{i4,g} \hat{d}_{ig} + \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i B_{Tu}^i \hat{\rho}_i K_{i2,k} \hat{\sigma}_{jik} \hat{K}_{i3,k} x_j \\
&+ \sum_{f=1}^{m_i} \frac{2\tilde{\rho}_{if}\dot{\tilde{\rho}}_{if}}{l_{if}} + \sum_{f=1}^{m} 2\rho_{if} \tilde{K}_{i1,f}^T \Gamma_{i1,f}^{-1} \dot{\tilde{K}}_{i1,f} \\
&+ \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2\tilde{K}_{i3,k}^T \Gamma_{i3,k}^{-1} \dot{\tilde{K}}_{i3,k} + \sum_{g=1}^{p_i} 2\tilde{K}_{i4,g}^T \Gamma_{i4,g}^{-1} \dot{\tilde{K}}_{i4,g} \} \\
&\le \sum_{i=1}^{N} \{ x_i^T [(A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i \hat{K}_{i1})] x_i \\
&+ \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i B_{Tu}^i \hat{\rho}_i K_{i2,k} \hat{\sigma}_{jik} \tilde{K}_{i3,k} x_j + \sum_{g=1}^{p_i} 2x_i^T P_i B_{Tu}^i \hat{\rho}_i \tilde{K}_{i4,g} \hat{d}_{ig} \\
&- \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2x_i^T P_i B_{Tu}^i \tilde{\rho}_i K_{i2,k} \hat{\sigma}_{jik} \hat{K}_{i3,k} x_j - \sum_{g=1}^{p_i} 2x_i^T P_i B_{Tu}^i \tilde{\rho}_i \hat{K}_{i4,g} \hat{d}_{ig} \\
&+ \sum_{f=1}^{m_i} \frac{2\tilde{\rho}_{if}\dot{\tilde{\rho}}_{if}}{l_{if}} + \sum_{f=1}^{m} 2\rho_{if} \tilde{K}_{i1,f}^T \Gamma_{i1,f}^{-1} \dot{\tilde{K}}_{i1,f} \\
&+ \sum_{j=1}^{N}\sum_{k=1}^{q_{ji}} 2\tilde{K}_{i3,k}^T \Gamma_{i3,k}^{-1} \dot{\tilde{K}}_{i3,k} + \sum_{g=1}^{p_i} 2\tilde{K}_{i4,g}^T \Gamma_{i4,g}^{-1} \dot{\tilde{K}}_{i4,g} \}
\end{aligned} \tag{27}
$$

Following Assumption 2, for any $\rho_i \in \Delta_{\rho_i^h}$, there exist $P_i > 0$ and K_{i1} satisfying

$$
(A_{TT}^i + B_{Tu}^i \rho_i K_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i K_{i1}) < 0. \tag{28}
$$

Thus, (27) can be rewritten as

$$
\begin{aligned}
\dot{V}(t) &\le \sum_{i=1}^{N} \{ x_i^T [(A_{TT}^i + B_{Tu}^i \rho_i K_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i K_{i1})] x_i \\
&+ 2x_i^T P B_{Tu}^i \rho_i \tilde{K}_{i1} x + \sum_{f=1}^{m} 2\tilde{K}_{i1,f}^T \Gamma_{i1,f}^{-1} \dot{\tilde{K}}_{i1,f} \\
&+ 2\sum_{f=1}^{m_i} \tilde{\rho}_{if} M_{if} R_{if} + 2\sum_{f=1}^{m_i} \tilde{\rho}_{if} M_{if} S_{if} + \sum_{f=1}^{m_i} \frac{2\tilde{\rho}_{if}\dot{\tilde{\rho}}_{if}}{l_{if}} \}
\end{aligned} \tag{29}
$$

where M_{if}, R_{if} and S_{if} are denoted in (14).

Defining

$$
Q_i = -[(A_{TT}^i + B_{Tu}^i \rho_i K_{i1})^T P_i + P_i (A_{TT}^i + B_{Tu}^i \rho_i K_{i1})] > 0. \tag{30}
$$

Then, note that the adaptive law is chosen as (15), (18), we can rewrite (27) as

$$\dot{V}(t) \leq -\sum_{i=1}^{N} x_i^T Q_i x_i. \qquad (31)$$

Hence, it is easy see that $\dot{V} < 0$ for any $x \neq 0$. Thus, the state $x(t)$ converges asymptotically to zero. Furthermore, all the signals are bounded. ■

IV. An Example and Simulation Results

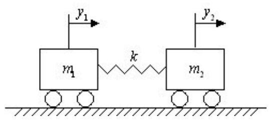

Fig. 2. two cars connected by spring.

In this section, we will study a multiple vehicle fault-tolerant control problem using distributed adaptive method. Consider a group of two vehicles connected by a spring, see Fig.2. The ith car's dynamics is governed by:

$$m_i \ddot{y}_i = k(y_j - y_i) + d_i + u_i$$

where y_i is the position of ith vehicle from its equilibrium position. d_i is the constant disturbance acting on ith vehicle, such as friction. u_i is its multi-input control. Then, the vehicle dynamics can be converted to the interconnected system format as:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_{ij}(t) \end{bmatrix} = \begin{bmatrix} A_{TT}^i & A_{TS}^{ij} & B_{Td}^i & B_{Tu}^i \\ A_{ST}^{ji} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_i(t) \\ \sigma_{ji} v_{ij}(t) \\ d_i \\ \rho_i u_i(t) \end{bmatrix} \qquad (32)$$

where $i, j \in \{1,2\}$, $x_i = [\dot{y}, \ddot{y}]^T$, and $A_{TT}^i = \begin{bmatrix} 0 & 1 \\ -\frac{2k}{m_i} & 0 \end{bmatrix}$, $A_{TS}^{ij} = \begin{bmatrix} 0 \\ \frac{k}{m_i} \end{bmatrix}$, $B_{Tu}^i = \begin{bmatrix} 0 & 0 \\ \frac{1}{a_i m_i} & \frac{1}{b_i m_i} \end{bmatrix}$, $B_{Td}^i = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_i} & 0 \end{bmatrix}$, $A_{ST}^{ji} = \begin{bmatrix} 1 & 0 \end{bmatrix}$, and we assume that $m_1 = 1$, $m_2 = 1.5$, $k = 1.25$, $a_1 = 1$, $b_1 = 0.5$, $a_2 = 1$, $b_1 = 2$.

To verify the effectiveness of the proposed adaptive method, the simulations are given with the following parameters and initial conditions:

$l_{if} = 0.5$, $r_{jik} = 10$, $s_{ig} = 10$, $\hat{\rho}_{if}(0) = 1$, $\hat{\sigma}_{jik}(0) = 1$, $\hat{d}_{ig}(0) = 0, \Gamma_{i1,f} = 10$, $\Gamma_{i3,k} = 10$, $\Gamma_{15,g} = 50$, $\Gamma_{25,g} = 100$, $K_{i2} = [1\ 1]^T$, $i = 1,2$, $f, g = 1,2$, $k = 1$.

We consider the following two possible faulty modes:

Normal mode 1: Both of the two subsystems' actuators and interconnection links are normal, that is, $\rho_{11}^1 = \rho_{12}^1 = \rho_{21}^1 = \rho_{22}^1 = 1$ and $\sigma_1^1 = \sigma_2^1 = 1$.

Faulty mode 2: The first actuators of subsystem G_1 and G_2 are outage, the second actuators of them may be normal or loss of effectiveness, and the maximum loss of effectiveness

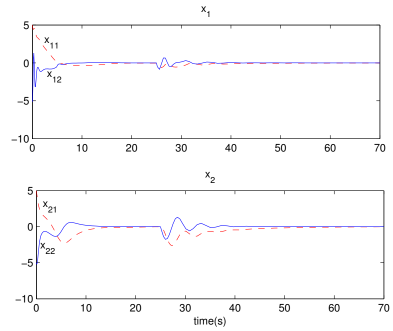

Fig. 3. Response curves of the first and the second vehicle states $x_1(t)$, $x_2(t)$ with distributed controllers.

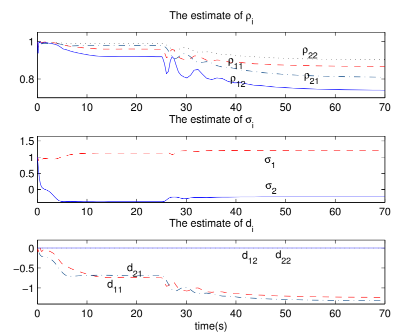

Fig. 4. Response curves of the estimate of actuator fault effect factors, attenuation factors in interconnection links and constant disturbances with two vehicles.

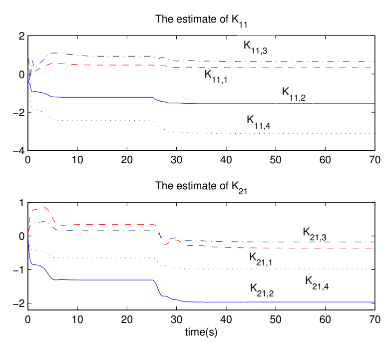

Fig. 5. Response curves of the estimate of controller parameters K_{i1}, $i = 1,2$.

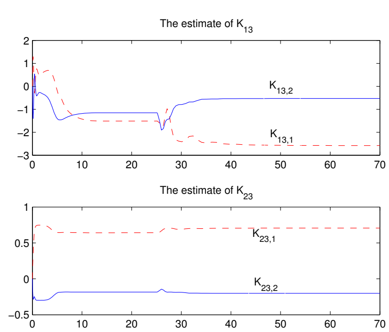

Fig. 6. Response curves of the estimate of controller parameters K_{i3}, $i = 1,2$.

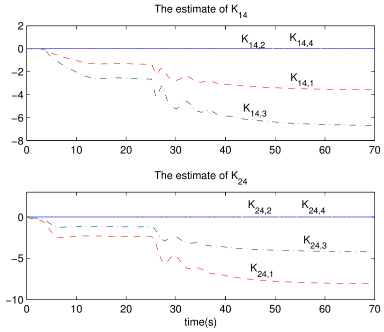

Fig. 7. Response curves of the estimate of controller parameters K_{i4}, $i = 1, 2$.

for the second actuators of G_1 and G_2 are 0.3 and 0.5, respectively. The spring may be normal or loss of effectiveness, described by $a \leq \sigma_1^2 = \sigma_2^2 \leq 1$, $a = 0.1$, which denotes the maximum loss of effectiveness for the interconnections.

The following faulty case is considered in the simulations, that is, before 5 second, the interconnected systems operate in normal case, and the constant external disturbances $d_1 = d_2 = [-5,\ 5]^T$ enter into the systems at the beginning ($t \geq 0$). At 5 second, some faults in interconnection channels have occur, described by $\sigma_{12} = \sigma_{21} = 0.1$. At 25 second, the actuator faults occur, described by $\rho_1 = \mathrm{diag}[0, 0.3], \rho_2 = \mathrm{diag}[0.5, 0]$.

Fig.3 is the response curves of the states of subsystems G_1, G_2 with adaptive state feedback controller in above-mentioned faulty case, respectively. Fig.4 is the response curves of the estimation of fault effect factors ρ_i, σ_{ji}, $i, j \in \{1, 2\}$, and constant disturbances, respectively. Fig.5-Fig.7 are the response curves of the estimation of controller parameters K_{i1}, K_{i3}, K_{i4}, $i \in \{1, 2\}$, respectively. It is easy to see the estimations can converge but not converge to theirs true values. In our adaptive fault-tolerant control design, there is no need for the estimations of ρ_{if} and σ_i, $i \in \{1, 2\}$ to converge to their true values.

V. Conclusion

In this paper, an adaptive method for FTC system design against actuator faults and interconnection signal attenuations for a class of large-scale systems have been proposed. We have presented the adaptation laws to estimate the unknown eventual faults on actuators and interconnections, constant external disturbances, and controller parameters on-line. By making use of their updated values, a class of distributed state feedback controllers has been constructed for automatically compensating the fault and disturbance effects based on the information from adaptive schemes. According to the Lyapunov stability theory, it has shown that the resulting adaptive closed-loop large-scale system can be guaranteed to be asymptotically stable even in the cases of faults on actuators and interconnections, and constant disturbances. A numerical example has shown the effectiveness of the proposed adaptive method.

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