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Output Agreement in high-dimensional multi-agent systems

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Abstract— **In this paper, the output agreement problem for networks of dynamic agents based on high-order state-space descriptions is formulated and investigated. The agent dynamics is described by a time-invariant linear system with matrices A, B and C. The agreement problem, concerning the calculation of an agreed value based on the agent outputs, is proposed for such a class of networks. A linear state feedback control protocol for such networks is established for solving such an agreement problem. The control protocol includes two parts: a local state feedback controller and the interactions among the finite neighbors. A sufficient condition for the existence of such a control protocol is given. The corresponding design algorithm is presented, as well. Some numerical simulations are presented, which are consistent with our theoretical results.**

I. INTRODUCTION

In recent years, decentralized control of communicating-agent systems has emerged as a challenging new research area. It has attracted multi-disciplinary researchers in a wide range including physics, biophysics, neurobiology, systems biology, apply mathematics, mechanics, computer science and control theory. The applications of multi-agent systems are diverse, ranging from cooperative control of unmanned air vehicles, formation control of mobile robots, control of communication networks, design of sensor-network, to flocking of social insects, swarm-based computing, etc. A common characteristics of the relevant analytical techniques is that they are deeply connected with decentralized, or networked control theory.

Agreement and consensus protocol design is one of the important problems encountered in decentralized control of communicating-agent systems. It has been paid attention for a long time by computer scientists, particularly in the field of automata theory and distributed computation [1]. Agreement upon certain quantities of interest is required in many applications such as multivehicle systems, multirobot systems, groups of agents and so on.

In the past decade, quite a tremendous amount of interesting results have been addressed for agreement and consensus problems in different formulations due to different type of agent dynamics and different type of tasks of interest. In [2], the problem of cooperation among a collection of vehicles performing a shared task using intervehicle communication to coordinate their actions was considered. The agents in the group were with linear dynamics. Tools from algebraic graph theory were used to prove the formation stability. In [3], a dynamic graph structure was provided as a convenient framework for modeling distributed dynamic systems where the topology of the interaction among its elements evolves in time. Some promising directions were highlighted as well.

Following the pioneering work in [4], there are many researchers have worked in analysis of swarms [5]-[9],[13]-[24]. In [5], the stability analysis for swarms with continuous-time model in n-dimensional space was addressed. Following this direction, stability analysis of social foraging swarms that move in an n-dimensional space according to an attractant/repellent or a nutrient profile was addressed in [6]. The corresponding results in the case of noisy environment was given in [7].

Differently from the above disciplinary, in [8] and [9], a model of coordinated dynamical swarms with physical size and asynchronous communication was introduced and analysis of stability properties of such swarms were presented with a fixed communication topology. A potential application of these theoretical results is in the field of the leader-follower formation control of multi-robot systems [10]-[12].

In [13], a simple discrete-time model of finite autonomous agents all moving in the plane with same speed but with different heading was proposed. Moreover, the concept of Neighbors of agents was introduced. Some simulation results to demonstrate the nearest neighbor rule were obtained. Based on this model, theoretical explanations were first given in [14] for the simulation results in [13]. Some sufficient conditions for coordination of the system of agents in the point of view of statistical mechanics. Another qualitative analysis for this model under certain simplifying assumption was given in [15].

In [16], a systematical framework of consensus problem in networks of dynamic agents with fixed/switching topology and communication time-delays was addressed. Under the assumption that the dynamic of the agent is a simple scalar continuous-time integrator $\dot{x} = u$, three consensus problems were discussed. They are directed networks with fixed topology, directed networks with switching topology and undirected networks with communication time-delays and fixed topology. Moreover, a disagreement function was introduced for disagreement dynamics of a directed network with switching topology. The undirected networks case was discussed by the same authors in [17]. Some other interesting results can be seen in [18]-[24] and the references therein.

In this paper, we follow the work in [16][17] and consider consensus problem for a more general class of networks. In

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our network model, the dynamic of the agents is given by a time-invariant linear system (A, B, C) with single input and single output. Such a dynamic is more general and complex than a scalar integrator in [16][17] and can be used to model more processes in reality. The main contribution in this paper is to pose and address output agreement problems for such networks with fixed topology.

An outline of this paper is as follows. In Section II, we recall the algebraic graph theory. In Section III, the output agreement problem is formulated. The control protocol and the whole network dynamics are given in Section IV. In Section V, the existence of the protocol and the design algorithm are discussed. Some simulation results are given in Section VI. Finally, Section VII concluded the whole paper.

II. Algebraic Graph Theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a undirected graph with the set of vertices $\mathcal{V} = \{\mathrm{v}_1, \mathrm{v}_2, \cdots, \mathrm{v}_M\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. The node indexes of $\mathcal{G}$ belong to a finite index set $\mathcal{I} = \{1, 2, \cdots, M\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (\mathrm{v}_i, \mathrm{v}_j)$. The adjacency elements associated with the edges are positive, i.e., $e_{ij} \in \mathcal{E} \iff a_{ij} > 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in \mathcal{I}$. Since the graph considered is undirected, it means once e_{ij} is an edge of $\mathcal{G}$, e_{ji} is an edge of $\mathcal{G}$ as well. As a result, the adjacency matrix $\mathcal{A}$ is a symmetric nonnegative matrix.

The set of *neighbors* of node v_i is denoted by $N_i = \{\mathrm{v}_j \in \mathcal{V} : (\mathrm{v}_i, \mathrm{v}_j) \in \mathcal{E}\}$. A *cluster* is any subset $J \subseteq \mathcal{V}$ of the nodes of the graph. The set of neighbors of a cluster N_J is defined by

$$N_J = \bigcup_{\mathrm{v}_i \in J} N_i. \tag{1}$$

The *degree* of node v_i is the sum $\sum_{j \neq i} a_{ij}$, denoted by $\deg(\mathrm{v}_i)$. The *degree matrix* is an $M \times M$ matrix define as $\Delta = [\Delta_{ij}]$ where

$$\Delta_{ij} = \begin{cases} \deg(\mathrm{v}_i), & i = j; \\ 0, & i \neq j. \end{cases}$$

The *Laplacian* of graph $\mathcal{G}$ is defined by

$$L = \Delta - A \tag{2}$$

An important fact of L is that all the row sums of L are zero and thus $\mathbf{1}_M = [1, 1, \cdots, 1]^T \in \mathbb{R}^M$ is an eigenvector of L associated with the eigenvalue $\lambda = 0$.

A *path* between each distinct vertices i and j is meant a sequence of distinct edges of $\mathcal{G}$ of the form $(\mathrm{v}_i, \mathrm{v}_{k_1}), (\mathrm{v}_{k_1}, \mathrm{v}_{k_2}), \cdots, (\mathrm{v}_{k_l}, \mathrm{v}_j)$. A graph is called *connected* if there exist a path between any two distinct vertices of the graph.

Lemma 1: [25] The graph $\mathcal{G}$ is connected if and only if $\mathrm{rank}(L) = M - 1$.

By Lemma 1, for a connected graph, there is only one zero eigenvalue of L, all the other ones are positive and real.

III. Output Agreement Problems On Network

Given a graph $\mathcal{G}$, let $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}$ denote the state vector and the output of node v_i, respectively. Suppose each node of the graph $\mathcal{G}$ is a dynamic agent given by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i + Bu_i, \\ y_i(t) &= Cx_i,\ i = 1, \cdots, M. \end{aligned} \tag{3}$$

where $x_i \in \mathbb{R}^n$ is the state of the agent i, $u_i \in \mathbb{R}$ is the single control input of the agent i, and $y_i \in \mathbb{R}$ is the single output of the agent i. A, B, C are constant matrices with proper dimensions. We refer to $(\mathcal{G}, A, B, C, x, y)$ with $x = (x_1^T, x_2^T, \cdots, x_M^T)^T$ and $y = (y_1, y_2, \cdots, y_M)^T$ as a *network* with state $x \in \mathbb{R}^{nM}$, output $y \in \mathbb{R}^M$, the linear constant dynamics (A, B, C) and the topology $\mathcal{G}$.

We assume the pair (A, B) is controllable. Without loss of generality, we assume that (A, B, C) is in controllable canonical form, i.e.,

$$A = \begin{bmatrix} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{n-1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and

$$C = [c_0, c_1, \cdots, c_{n-1}].$$

We say a state feedback

$$u_i = k_i(x_i, x_{j_1}, x_{j_2}, \cdots, x_{j_{l_i}}) \tag{4}$$

is a *protocol* with topology $\mathcal{G}$ if the cluster $J_i = \{\mathrm{v}_{j_1}, \mathrm{v}_{j_2}, \cdots, \mathrm{v}_{j_{l_i}}\}$ of nodes with indexes $j_1, j_2, \cdots, j_{l_i} \in \mathcal{I}$ satisfies the property $J_i \subseteq N_i$. In addition, if $|J_i| < M$ for all $i \in \mathcal{I}$, (4) is called a *distributed protocol*.

We say the protocol (4) asymptotically solves the *output agreement problem* if there exists a scalar $y_f \in \mathbb{R}$ such that the closed-loop network with the protocol (4) satisfies that $\lim_{t \to \infty} y_i(t) = y_f$, $i = 1, \cdots, M$.

In this paper, we are interested in finding under what conditions there exists a protocol (4) asymptotically solves the *output agreement problem* and the design algorithm of such a protocol.

IV. Control Protocol And Network Dynamics

In this section, we present the control protocol that solve the aforementioned output agreement problem. We will use a linear protocol with fixed topology and no communication time-delay:

$$u_i = u_{i1} + u_{i2} \tag{5}$$

where

$$u_{i1} = K_{self} x_i \tag{6}$$

is the feedback from the agent i itself and

$$u_{i2} = \sum_{j \in N_i} a_{ij} K_{nghb}(x_j - x_i) \tag{7}$$

is the feedback from the neighbors, where the constant matrices $K_{self} \in \mathbb{R}^{1 \times n}$ and $K_{nghb} \in \mathbb{R}^{1 \times n}$ need to be designed.

By applying the above protocol (5), the agent i's dynamics is formed as follows:

$$\begin{aligned}\dot{x}_i(t) &= (A+BK_{self})x_i + BK_{nghb}\sum_{j\in N_i} a_{ij}(x_j - x_i) \\ y_i(t) &= Cx_i. \end{aligned} \tag{8}$$

Then the whole network dynamics is summarized as follows:

$$\begin{array}{l}\dot{x}(t) = \Phi x(t) \\ y(t) = \Gamma x(t)\end{array} \tag{9}$$

where

$$\Phi = I_M \otimes (A + BK_{self}) - L \otimes BK_{nghb}, \tag{10}$$

and

$$\Gamma = I_M \otimes C \tag{11}$$

with L the aforementioned Laplacian associate with the graph $\mathcal{G}$.

Then finding a control protocol to solve the output agreement problem of the network $(\mathcal{G}, A, B, C, x, y)$ is formulized as *to find suitable matrices* K_{self} *and* K_{nghb} *such that the system (9) satisfies that* $\lim_{t\to\infty} y = \mathbf{1}_M y_f$.

V. Protocol Existent Condition and Design

Here we present a sufficient condition for existence of protocol (5) that asymptotically solves the output agreement problem of the network $(\mathcal{G}, A, B, C, x, y)$.

Lemma 2: [26] Given a polynomial $f(s) = \sum_{i=0}^{n} \beta_i s^i \in \mathbb{R}[s]$, $n \geq 3$, $f(s)$ is Hurwitz stable if

$$\beta_i\beta_{i+1} > 3\beta_{i+2}\beta_{i-1},\ i = 1, 2, \cdots, n-2. \tag{12}$$

Theorem 1: If the topology $\mathcal{G}$ is connected, then there exist state feedback gains K_{self}, K_{nghb} such the protocol (5) asymptotically solves the output agreement problem of the network $(\mathcal{G}, A, B, C, x, y)$.

Proof: First, we select γ_i, $i = 0, \cdots, n$ such that

$$\gamma_i\gamma_{i+1} > 3\gamma_{i+2}\gamma_{i-1},\ i = 1, 2, \cdots, n-2. \tag{13}$$

with $\gamma_n = 1$. By Lemma 2, the polynomial $g(s) = \sum_{i=0}^{n} \gamma_i s^i$ is Hurwitz stable. Then we take

$$K_0 = [\alpha_0 - \gamma_0, \alpha_1 - \gamma_1, \cdots, \alpha_{n-1} - \gamma_{n-1}] \tag{14}$$

It follows that the matrix

$$A + BK_0 = \begin{bmatrix} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ -\gamma_0 & -\gamma_1 & \cdots & -\gamma_{n-1} \end{bmatrix}$$

is Hurwitz stable.

Next, let

$$K_{self} = K_0[0, e_2, \cdots, e_n] + \alpha_0 e_1^T, \tag{15}$$

where e_i is the ith column vector of I_n, $i = 1, 2, \cdots, n$. It follows that

$$A + BK_{self} = \begin{bmatrix} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ 0 & -\gamma_1 & \cdots & -\gamma_{n-1} \end{bmatrix}$$

Denote the eigenvalues of L are $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_M$. Let

$$K_{nghb} = \lambda_M^{-1}\gamma_0 e_1^T \tag{16}$$

Then we have

$$\begin{aligned} & A + BK_{self} - \lambda_i BK_{nghb} \\ = \ & A + B(K_0[0, e_2, \cdots, e_n] + \alpha_0 e_1^T) \\ & -\lambda_i B\lambda_M^{-1}\gamma_0 e_1^T \\ = \ & A + B(\alpha_0 - \frac{\lambda_i}{\lambda_M}\gamma_0)e_1^T \\ & +BK_0[0, e_2, \cdots, e_n] \\ = \ & \begin{bmatrix} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ -\frac{\lambda_i}{\lambda_M}\gamma_0 & -\gamma_1 & \cdots & -\gamma_{n-1} \end{bmatrix} \end{aligned}$$

Then the characteristic polynomial of the matrix $A + BK_{self} - \lambda_i BK_{nghb}$ is

$$g_i(s) = \sum_{i=1}^{n} \gamma_i s^i + \frac{\lambda_i}{\lambda_M}\gamma_0,\ i = 1, 2, \cdots, M. \tag{17}$$

For matrix $A + BK_{self}$, its characteristic polynomial is

$$g_1(s) = \sum_{i=1}^{n} \gamma_i s^i = sg_1'(s)$$

where $g_1'(s) = \sum_{i=1}^{n} \gamma_i s^{i-1}$. By (13), it is to see that $g_1(s)$ is Hurwitz stable. It follows that matrix $A+BK_{self}$ has only one zero eigenvalue and all the others are with negative real part.

For matrix $A + BK_{self} - \lambda_i BK_{nghb}$, $i = 2, \cdots, M$, it is easy to see that

$$\gamma_1\gamma_2 > 3\gamma_0\gamma_3 > 3\frac{\lambda_i}{\lambda_M}\gamma_0\gamma_3$$

compounding with (13), it is to see that $g_i(s)$ is Hurwitz stable, $i = 2, \cdots, M$. It follows that matrix $A + BK_{self} - \lambda_i BK_{nghb}$ is Hurwitz stable, $i = 2, \cdots, M$.

Thirdly, there exists a nonsingular matrix W such that

$$W^{-1}LW = D = diag\{\lambda_1, \lambda_2, \cdots, \lambda_M\}$$

It follows that

$$W^{-1} \otimes I_n \Phi W \otimes I_n = J = diag\{J_1, J_2, \cdots, J_M\} \tag{18}$$

where

$$J_i = A + BK_{self} - \lambda_i BK_{nghb},\ i = 1, 2, \cdots, M. \tag{19}$$

It is obvious that $\lim_{t\to\infty}\exp(J_i t) = 0$, $i = 2, \cdots, M$. As to $J_1 = A + BK_{self}$, we have

$$\lim_{t\to\infty}\exp(J_i t) = e_1 w_l^T. \tag{20}$$

where

$$w_l = \frac{1}{\gamma_1}[\gamma_1, \cdots, \gamma_{n-1}, 1]^T \tag{21}$$

is the left eigenvector of J_1 associated with the eigenvalue zero. Furthermore, $w_l^T e_1 = 1$. Thus, we have

$$\lim_{t\to\infty}\exp(\Phi t) = \frac{1}{M}\mathbf{1}_M \otimes e_1(\mathbf{1}_M \otimes w_l)^T. \tag{22}$$

It follows that

$$\begin{aligned}\lim_{t\to\infty} y(t) &= I_M \otimes C \lim_{t\to\infty}\exp(\Phi t)x(0) \\ &= I_M \otimes C\frac{1}{M}\mathbf{1}_M \otimes e_1(\mathbf{1}_M \otimes w_l)^T x(0) \\ &= \mathbf{1}_M \otimes y_f \end{aligned} \tag{23}$$

where

$$y_f = \frac{1}{M}\sum_{i=1}^{M} Ce_1 w_l^T x_i(0) = c_0[1, \frac{\gamma_2}{\gamma_1}, \cdots, \frac{\gamma_n}{\gamma_1}]\frac{1}{M}\sum_{i=1}^{M} x_i(0) \tag{24}$$

■

According to Theorem 1, we present the following protocol design algorithm.

Algorithm 1: For the network $(\mathcal{G}, A, B, C, x, y)$ with (3), assuming $\mathcal{G}$ is connected, then we may design state feedback gains K_{self}, K_{nghb} to construct a control protocol (5) that asymptotically solves the output agreement problem by the following steps:

1) according to (12), determine γ_i, $i = 0, 1, \cdots, n-1$;
2) according to (14), determine K_0;
3) take $K_{self} = K_0[0, e_2, \cdots, e_n] + \alpha_0 e_1^T$;
4) determine the maximum eigenvalue of L;
5) take $K_{nghb} = \lambda_M^{-1}\gamma_0 e_1^T$.

Remark 1: In fact, if we select sufficient small μ substitute for λ_M^{-1}, then $K_{nghb} = \mu\gamma_0 e_1^T$ is acceptable as well.

VI. Simulation

In this section, we present two examples to illustrate our obtained results.

Example 1: Consider a network $(\mathcal{G}, A, B, C, x, y)$. The graph $\mathcal{G}$ is shown in Fig.1 and the adjacency matrix are limited to $0, 1$ matrices. Moreover, let $n = 3$ and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}, \; C = [1,\ 0,\ 0].$$

First, we select $\gamma_0 = 1$, $\gamma_1 = 2$ and $\gamma_2 = 2$. Then we get $K_0 = [-2, -4, -5]$. It follows that $K_{self} = [-1, -4, -5]$. Next, we get

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

the maximum eigenvalue of L is 4.4142. Then we let $K_{nghb} = [0.2,\ 0,\ 0]$. Fig. 2 shows the simulation results for the control protocol (5) for the network with random set of initial conditions. it is shown that the protocol asymptotically solves the output agreement problem for the network in Example 1.

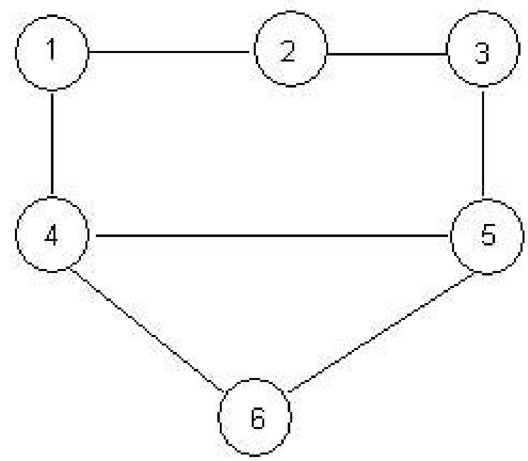

Fig. 1. Undirected graph $\mathcal{G}$ with $M = 6$ nodes for Example 1.

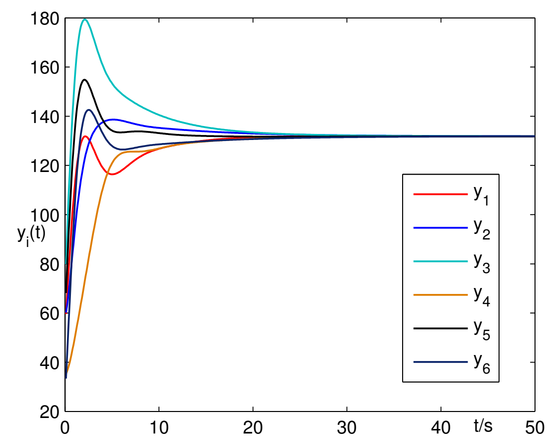

Fig. 2. Output of the network in Example 1.

Example 2: Consider a network $(\mathcal{G}, A, B, C, x, y)$. The graph $\mathcal{G}$ is shown in Fig.3 and the adjacency matrix are limited to $0, 1$ matrices. Moreover, let $n = 5$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; C = [1,\ 1,\ 1,\ 1,\ 1].$$

First, we select $\gamma_0 = 20$, $\gamma_1 = 30$, $\gamma_2 = 50$, $\gamma_3 = 20$ and $\gamma_4 = 10$. Then we get $K_0 = [-20, -30, -50, -20, -10]$. It follows that $K_{self} = [0, -30, -50, -20, -10]$. Next, we

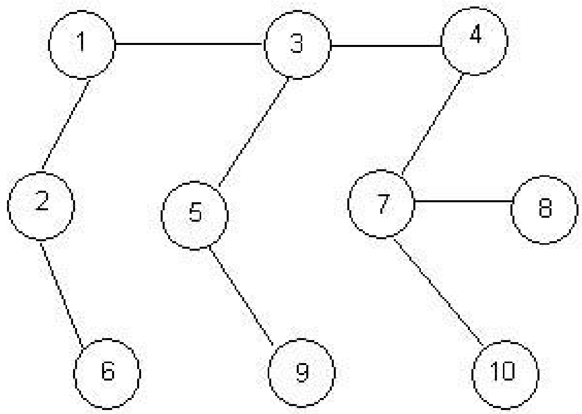

Fig. 3. Undirected graph $\mathcal{G}$ with $M = 10$ nodes.

get

$$L=\begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 3 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

the maximum eigenvalue of L is 4.4142. Then we let $K_{nghb} = [0.2, 0, 0]$. Fig. 4 and Fig. 5 show the simulation results for the control protocol (5) for the network with random set of initial conditions. By simple calculation, we get

$$y(10000) = \begin{bmatrix} 179.8784 \\ 179.8779 \\ 179.8790 \\ 179.8798 \\ 179.8789 \\ 179.8776 \\ 179.8805 \\ 179.8808 \\ 179.8788 \\ 179.8808 \end{bmatrix}$$

These facts show that the protocol asymptotically solves the output agreement problem for the network in Example 2.

VII. Conclusion

In this paper, the output agreement problem for networks of high-dimensional dynamic agents has been investigated. A sufficient condition for the existence of such a control protocol and the design algorithm have been obtained. The future work includes output agreement problem in the switching topology and communication time-delay case. The authors are focusing all there attention on these open problems.

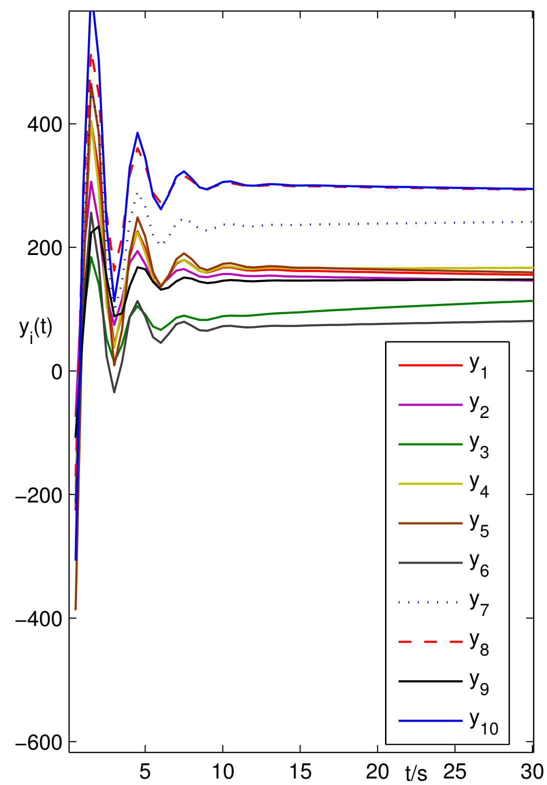

Fig. 4. Output of the network in Example 2 for $t \in$[0, 30].

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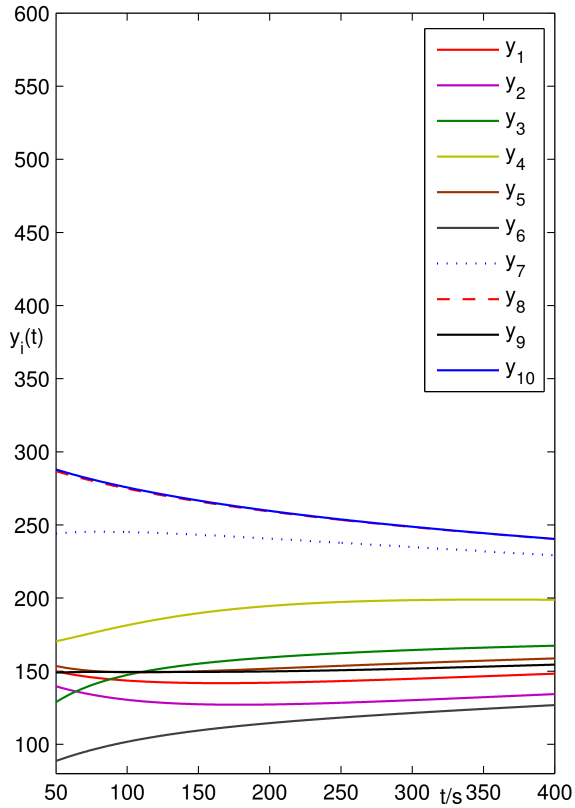

Fig. 5. Output of the network in Example 2 for $t \in [50, 400]$.

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